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Pandey-Sharma's Geometry

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Abstract: We show that the Pandey-Sharma's spacetime is a counterexample for a conjecture of Boyer-Plebañski and for the Sen's theorem.

Key words: Boyer-Plebañski's conjecture・Sen's theorem・4-space of class one・Pandey-Sharma spacetime

INTRODUCTION

Pandey-Sharma [1] considered the following geometry with spherical symmetry [2]:

$$ds^2 = dr^2 + r^2(d\theta^2 + sin^2\theta \, d\varphi^2) - (1 + B(t)\, r^2)^2 \, dt^2, \tag{1}$$

for a conformally flat perfect fluid distribution with zero density, where $B(t) \neq 0$ is an arbitrary function. The corresponding non-null components of Riemann and Ricci tensors and the scalar curvature are given by $[(x^k) = (r, \theta, \varphi, t)]$:

$$\begin{aligned} R_{44} &= -3R_{1414} = -6B(1 + Br^2), \qquad R_{33} = sin^2\theta \, R_{22} = r^2 sin^2\,\theta \, R_{11}, \\ R &= 6R_{11} = \frac{12B}{1 + Br^2}, \qquad R_{3434} = sin^2\theta \, R_{2424} = r^2 sin^2\,\theta \, R_{1414}, \end{aligned} \tag{2}$$

with the Weyl tensor equals to zero.

On the other hand, we have the Boyer-Plebañski's conjecture [3]:

"If R_4 is conformally flat with spherical symmetry, then it has class one", (3)

therefore (1) should be of class one, however, Pandey-Sharma [1] proved that this is false, that is, (1) does not accepts local and isometric embedding into E_5 In other words, (1) is a counterexample for the Boyer-Plebañski's conjecture; besides, this spacetime also shows the insufficiency of the conditions of Karmarkar [4] because it verifies such conditions but has not class one.

The theorem of Sen [5] is given by:

"If R_4 is conformally flat and its curvature tensor has the structure:

$$R_{ijkm} = E(R_{ik}R_{jm} - R_{im}R_{jk}) + F(g_{ik}g_{jm} - g_{im}\, g_{jk}), \tag{4}$$

with $E \neq 0$ and F scalars, then R_4 has class one".

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The quantities (2) permit to show that the Riemann tensor verifies the expression (4) for $E = -\frac{3}{R}$ and $F = \frac{R}{12}$, however, (1) is not of class one and thus it is a counterexample for the Sen's theorem.

The perfect fluid for (1) has zero density, but it is interesting to note that Gupta-Pandey [6] constructed spherical symmetric conformally flat metrics of class one for perfect fluid distribution with density $\neq 0$.

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