American-Eurasian Journal of Scientific Research 13 (3): 60-61, 2018
ISSN 1818-6785

DOI: 10.5829/idosi.aejsr.2018.60.61

Pseudoinverse of a Square Matrix

J. López-Bonilla, R. López-Vázquez and S. Vidal-Beltrán

ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso,
Col. Lindavista CP 07738, CDMX, México

Abstract: We exhibit that the Faddeev-Sominsky's algorithm to construct the usual inverse A^{-1} when det $A_{nxn} \neq 0$, then if $1 \leq rank\, A \leq n-1$ it gives a natural inverse A^{+} with the properties $AA^{+}A = A$ and $A^{+}AA^{+} = A^{+}$.

Key words: Gower's inverse · Faddeev-Sominsky's technique · Generalized inverse

INTRODUCTION

If A_{nxn} is non-singular, the Faddeev-Sominsky's algorithm gives the corresponding inverse matrix, in fact [1-3]:

$$\begin{array}{lll} B_1 & a_1 = -trB_1, & C_1 = B_1 + a_1 I, \\ B_2 = C_1 A, & a_2 = -\frac{1}{2} trB_2, & C_2 = B_2 + a_2 I, \\ \vdots & \vdots & \vdots \\ B_{n-1} = C_{n-2}A, & a_{n-1} = -\frac{1}{n-1} trB_{n-1}, & C_{n-1} = B_{n-1} + a_{n-1} I, \\ B_n = C_{n-1}A, & a_n = -\frac{1}{n} trB_n, & C_n = 0 \end{array} \qquad (1)$$

[Cayley-Hamilton theorem],

then:

$$A^{-1} = -\frac{1}{a_j} C_{j-1}, \; j = n; \; p = rank\; A = n, \qquad (2)$$

where a_r, $r = 1,..., n$ are the coefficients of the characteristic polynomial of A [4, 5]:

$$\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^2 + ... + a_{n-1}\lambda + a_n = 0, \qquad (3)$$

in particular, $a_n = (-1)^n \det A$

In Sec. 2 we accept that is non-defective, that is, that it has independent eigenvectors, with the constraints:

$$1 \leq p \leq n-1, \quad a_p \neq 0, \qquad (4)$$

and we find that (2) gives a pseudoinverse of A if $j = p$,

Generalized Inverse: First we remember that a non-defective matrix verifying the conditions (4) satisfies the identity [6, 7]:

$$A^{p+1} + a_1 A^p + ... + a_{p-1} A^2 + a_p A = 0 \qquad (5)$$

that in the Faddeev-Sominsky's method implies the values:

$$a_k = 0, \quad B_k = C_k = 0, \quad k = p+1,...,n. \qquad (6)$$

Now the remarkable fact is that (2) for $j = p$ allows construct a pseudoinverse with the properties [8]:

$$AA^{+}A = A, \quad A^{+}AA^{+} = A^{+} \qquad (7)$$

such that:

$$A^{+} = -\frac{1}{a_p} C_{p-1} \qquad (8)$$

which is more simple than the Gower's inverse [6, 7]:

$$A_G^{+} = -\frac{1}{a_p}\left(B_{p-1} - \frac{a_{p-1}}{a_p} B_P \right); \qquad (9)$$

with (1), (5), (6) and (8) is easy to prove the fulfilling of the relations (7).

Hence the Faddeev-Sominsky's procedure gives the usual inverse A^{-1} if det $A \neq 0$ [$p = rank\; A = n$], and when $1 \leq p \leq n-1$ it generates the pseudoinverse A^{+} indicated in (8).

Corresponding Author: J. López-Bonilla, ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. 4, 1er. Piso, Col. Lindavista CP 07738, CDMX, México.

REFERENCES

1. Faddeev, D.K. and I.S. Sominsky, 1949. Collection of problems on higher algebra, Moscow.
2. Faddeev, D.K. and V.N. Faddeeva, 1963. Methods in linear algebra, W. H. Freeman, San Francisco, USA.
3. López-Bonilla, J., H. Torres-Silva and S. Vidal-Beltrán, 2018. On the Faddeev-Sominsky's algorithm, World Scientific News, 106: 238-244.
4. Guerrero-Moreno, I., J. López-Bonilla and J. Rivera-Rebolledo, 2011. Leverrier-Takeno coefficients for the characteristic polynomial of a matrix, J. Inst. Eng., 8(1-2): 255-258.
5. Horn, R.A. and Ch. R. Johnson, 2013. Matrix analysis, Cambridge University Press.
6. Gower, J.C., 1980. A modified Leverrier-Faddeev algorithm for matrices with multiple eigenvalues, Linear Algebra and its Appls., 31(1): 61-70.
7. López-Bonilla, J., R. López-Vázquez, S. Vidal-Beltrán, 2018. An alternative to Gower's inverse matrix, World Scientific News, 102: 166-172.
8. Ben-Israel, A. and T.N.E. Greville, 2003. Generalized inverses: Theory and applications, Springer-Verlag, New York.