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# Efficient Vlsi Implementation of Modified Dual Tree Discrete Wavelet Transform Based on Lifting Scheme

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**Abstract:** In past few decades, dual tree discrete wavelet transform (DT-DWT) has become an open active research area for image processing applications. Nowadays, lifting based DT-DWT seeks more attention due to its lower computational complexity and shift invariance property. Nevertheless, VLSI implementation of lifting based DT-DWT further needs to be improved for low power and high speed image processing applications. In this article, an efficient VLSI implementation of modified lifting based DT-DWT is proposed. In proposed approach, two real valued lifting based filters are used to implement dual tree structure in which six high pass bands (i.e., two HH, two HL and two LH) and two low pass bands (LL) are obtained. Next, high frequency components (HH1 and HH2) as well as low pass bands (LL1 and LL2) are merged togetheras single sub bands HHm and LLm respectively. In synthesis section, the original image is recovered using LL<sub>mb</sub> HH<sub>mb</sub>LH2and HL2 without considering the two high pass bands (HL1and LH1).Hence, the proposed approach greatly reduces the hardware requirements and consumes less power than the DT-DWT without affecting the image quality. Experiment wascarried out and tested on various test images using Xilinx ISE and MATLAB R2010a. The experiment results inferred that the proposed approach is well suited for image compression.

Key words: Discrete wavelet transforms • Modified dual tree discrete wavelet transforms • Lifting Algorithm • Image compression

## **INTRODUCTION**

In current years, discrete wavelet transform (DWT) is widely used for image processing applications because of its ability to represent the signal in different scale, highest energy compaction, multi resolution and higher compression ratio. However, therealization of DWT demands higher computationally complexity and more memory storage which are notappropriate forhigh speed and low power image/video processing applications [1]. Sweldens and Daubechies [2, 3] proposed a lifting based DWT which requiresfewer arithmetic computations.In lifting-based DWT (LDWT) scheme, the wavelet filter matrix is divided into successive matrices including diagonal, uppertriangular and lower triangular matrices. Then, thewavelet filtersare implemented using banded matrices multiplications [2, 3]. Several lifting based DWT architectureshave been proposed for image processing

applications. These architectures may include folded lifting based DWT [4], systolic array DWT [5] and recursive lifting based DWT [6]. Conversely, the lifting based architecturesclaim large amount of frame buffers to store the intermediate LL output and also require complex control path.In [4], the author proposed block based architecture which requires large embedded memory andlonger output latency. In combined line-based architecture [7], hardware resources are not effectively utilized and it needs to be further optimized. In addition, the DWT and lifting based DWT are suffered by shift invariance and lack of directional selectivity [8] which affects the quality of reproduced image. In [9, 10], a DT-DWT has been proposed to solve the shift variance problem and to improve directional selectivity. Furthermore, a fully shift-invariant lifting based stationary wavelet transform (LSWT) has been proposed by Lee.et.al [11]. On the other hand, both DT-DWT and LSWT needs

Corresponding Author: S. Senthil Kumar, Department of Electronics and Communication Engineering, SVS College of Engineering, Coimbatore, Tamil Nadu, India - 642 109. more lifting hardware and consume more power, which are undesirable for high speed and low power image processing applications. Hence, an effective VLSI implementation of modified lifting based dual tree discrete wavelet transformhas been proposed in this article to improve the system performance of dual tree wavelet transform by suppressing the frequency components including LH1 and HL1 for reconstruction.

The remaining part of this article ispresented as follows. Basic principle and lifting based dual tree wavelet transform are discussed in the section 2. Section 3 demonstrates the modified lifting based dual tree structure. Experiment results and discussionsare described in the section 4. At the end, the article is successfully concluded in the section 5.

#### Lifting Scheme and Dual Tree Wavelet Transform

Lifting Scheme: The fundamental idea behind the lifting scheme is that instead of using scaling functions on the finer level to build a wavelet, it uses an old, simple wavelet and scaling functions on the same level to synthesize a new wavelet. The wavelet filters are implemented using lifting steps such as split, predict, update and normalize. Here, the split phase splits the original signal X(i) into even and odd parts. The predictionstep predicts the high pass detailed sub band coefficients and the update stage gives low pass approximation sub band filter coefficients. In 9/7 lifting scheme, the implementation of 2D-DWT requirestwo predictions, two updates and one scaling.A basic 9/7 lifting-based DWT structure is described in Fig. 1. The input signal X(i) is decomposed into two parts such as X(2i) and X(2i + 1). Then, the first stage of lifting operation is executed as follows [1]:

$$p(i)^{1} = \alpha \left( X(2i) + X(2i+2) \right) + X(2i+1)$$
<sup>(1)</sup>

$$u(i)^{1} = \beta \left( p(i)^{1} + p(i-1)^{1} \right) + X(2i)$$
<sup>(2)</sup>

where,  $p(i)^1$  and  $u(i)^1$  represent the first predicted and updated coefficients,  $\alpha$  and  $\beta$  are the first level lifting constants. In proposed approach,  $\alpha = -1.586134342$  and  $\beta = 0.052980118$  are used for first stage lifting process. The second stage of predict and update of lifting process is given by:

$$p(i)^{2} = \gamma \left( u(i)^{1} + u(i+1)^{1} \right) + p(i)^{1}$$
(3)

$$u(i)^{2} = \delta\left(p(i)^{2} + p(i-1)^{2}\right) + u(i)^{1}$$
(4)

where,  $p(i)^2$  and  $u(i)^2$  represent the second lifting predicted and updated coefficients,  $\alpha$  and  $\beta$  are the second level lifting constants and its typical values are 0.8829110762, 0.4435068522 respectively. The scaling is performed using the normalization parameters *K* and  $K^{-1}$ . In practice, the typical value of normalization factor K is 1.149604398. The normalization is done as follows:

$$u(i) = Ku(i)^2 \tag{5}$$

$$p(i) = p(i)^2 K^{-1}$$
(6)

Lifting Based Dual Tree Discrete Wavelet Transform: In case of DWT, small shifts in the input signal or image can cause significant changes in the distribution of signal/image energy across scales in the DWT coefficients.In addition to that the DWT is not able to discriminate spectral features in different orientations [12, 13]. In DT-DWT, directional selectivity and shift variance problems are eliminated by processing the same input signal using two different real valued lifting based DWT filters and ne of the wavelet filters is an approximate Hilbert transform of the other. In this approach, DWT algorithm is implemented using real filters without complex arithmetic operations. The reverse operation is done at the synthesis part to recover the original image. The analysis and synthesis sections of lifting based dual tree discrete transform (DT-DWT) are illustrated in Fig 2-3.

Modified Lifting Based Dual Tree Discrete Wavelet Transform: The objective of image compression is to exploit the redundant features to reduce the memory storage. As DT-DWT is a redundant transform, it introduces additional redundancy though it achieves good directional selectivity. Hence, the redundancy must



Fig. 1: 9/7 Lifting structure for 9/7 filter



Fig. 2 Analysis Section of Lifting based DT-DWT structure



Fig. 3: Synthesis Section of Lifting based DT-DWT structure



Fig. 4: Analysis section of modified Lifting based dual tree discrete wavelet transform (Proposed)

be reduced without sacrificing perfect reconstruction and it can be accomplished by simply removing the insignificant complex coefficients [12]. Though, noise shaping algorithm (NSA) helps to reduce the number of coefficients, it requires more binary data for representation [12]. Therefore, the modified lifting based dual tree structure has been modified and presented in this article. At the analysis part, the low frequency sub bands (LL1 &LL2) as well as high frequency sub bands (HH1 & HH2) are merged as single sub bands such as LL<sub>m</sub> and HH<sub>m</sub> respectively and the original image is reconstructed at the synthesis part by using LL<sub>m</sub>, HH<sub>m</sub>, HL2 and LH2 sub bands. This structure requires less number of lifting hardware and thereby reducing thepower consumptionas well asarithmetic computations than the 2D-DWT. Fig. 4 demonstrates the analysis section of modified lifting based dual tree structure.

In proposed approach, initially an input image is decomposed up to a desired level by two separable 2D DWT branches using lifting scheme such as, tree 1 and tree 2, whose filters are specifically designed to meet the Hilbert pair requirement. At each level of decomposition, six high-pass sub bands as well as two low-pass sub bands are generated. Next, the HH1 and HH2sub bands are linearly combined as Hhm [13, 14]. Similarly the lowpass sub bands of each tree are combined as Llm. Finally, the reconstructed image is obtained at the synthesis part by inverting the process performed at the analysis section. Let us assume that  $\phi_r$  and  $\Psi_r$  be the scaling and wavelet functions of real tree. Let  $\phi_r$  and  $\Psi_r$  be the scaling and wavelet functions of imaginarytree. The product of  $\varphi(.)$  along the first coordinate and  $\Psi(.)$  along the secondcoordinate correspond to LH sub band. Similarly, wavelets (LH, HL, HH) associated with DWT are given by [15, 16],

$$\begin{split} \phi(i)\psi(j) &= \left[\phi_r(i) + j\phi_i(i)\right] \left[\psi_r(j) + j\psi_i(j)\right] \\ &= \left[\phi_r(i)\psi_r(j) - \phi_i(i)\psi_i(j)\right] + j\left[\phi_i(i)\psi_r(j) + \phi_r(i)\psi_i(j)\right] \end{split}$$

$$\phi(i)\psi^{*}(j) = [\phi_{r}(i) + j\phi_{i}(i)][\psi_{r}(j) - j\psi_{i}(j)]$$
  
=  $[\phi_{r}(i)\psi_{r}(j) + \phi_{i}(i)\psi_{i}(j)] + j[\phi_{i}(i)\psi_{r}(j) - \phi_{r}(i)\psi_{i}(j)]$ 

(7)

$$\phi^*(i)\psi(j) = \left[\phi_r(i) - j\phi_i(i)\right] \left[\psi_r(j) + j\psi_i(j)\right]$$
$$= \left[\phi_r(i)\psi_r(j) + \phi_i(i)\psi_i(j)\right] - j\left[\phi_i(i)\psi_r(j) - \phi_r(i)\psi_i(j)\right]$$



Fig. 5: Synthesis section of modified Lifting based dual tree discrete wavelet transform (Proposed)

$$\phi^{*}(i)\psi^{*}(j) = [\phi_{r}(i) - j\phi_{i}(i)][\psi_{r}(j) - j\psi_{i}(j)]$$
  
=  $[\phi_{r}(i)\psi_{r}(j) - \phi_{i}(i)\psi_{i}(j)] - j[\phi_{i}(i)\psi_{r}(j) + \phi_{r}(i)\psi_{i}(j)]$ 

$$\psi(i)\phi(j) = [\psi_r(i) + j\psi_i(i)][\phi_r(j) + j\phi_i(j)]$$
  
=  $[\psi_r(i)\phi_r(j) - \psi_i(i)\phi_i(j)] + j[\psi_r(i)\phi_i(j) + \psi_i(i)\phi_r(j)]$ 

$$\psi(i)\phi^{*}(j) = [\psi_{r}(i) + j\psi_{i}(i)][\phi_{r}(j) - j\phi_{i}(j)]$$
  
=  $[\psi_{r}(i)\phi_{r}(j) + \psi_{i}(i)\phi_{i}(j)] - j[\psi_{r}(i)\phi_{i}(j) - \psi_{i}(i)\phi_{r}(j)]$ 

(10)

(11)

$$\psi^{*}(i)\phi(j) = [\psi_{r}(i) - j\psi_{i}(i)][\phi_{r}(j) + j\phi_{i}(j)]$$
  
= [\psi\_{r}(i)\phi\_{r}(j) + \psi\_{i}(i)\phi\_{i}(j)] + j[\psi\_{r}(i)\phi\_{i}(j) - \psi\_{i}(i)\phi\_{r}(j)]

$$\psi^{*}(i)\phi^{*}(j) = [\psi_{r}(i) - j\psi_{i}(i)][\phi_{r}(j) - j\phi_{i}(j)]$$
  
=  $[\psi_{r}(i)\phi_{r}(j) - \psi_{i}(i)\phi_{i}(j)] - j[\psi_{r}(i)\phi_{i}(j) + \psi_{i}(i)\phi_{r}(j)]$   
(14)

$$\begin{aligned} \psi(i)\psi(j) &= \left[\psi_{r}(i) + j\psi_{i}(i)\right] \left[\psi_{r}(j) + j\psi_{i}(j)\right] \\ &= \left[\psi_{r}(i)\psi_{r}(j) - \psi_{i}(i)\psi_{i}(j)\right] + j\left[\psi_{r}(i)\psi_{i}(i) + \psi_{i}(i)\psi_{r}(j)\right] \end{aligned}$$
(15)

$$\psi(i)\psi^{*}(j) = [\psi_{r}(i) + j\psi_{i}(i)][\psi_{r}(j) - j\psi_{i}(j)]$$
  
=  $[\psi_{r}(i)\psi_{r}(j) + \psi_{i}(i)\psi_{i}(j)] - j[\psi_{r}(i)\psi_{i}(i) - \psi_{i}(i)\psi_{r}(j)]$   
(16)

$$\psi^{*}(i)\psi(j) = [\psi_{r}(i) - j\psi_{i}(i)][\psi_{r}(j) + j\psi_{i}(j)]$$
  
= [\psi\_{r}(i)\psi\_{r}(j) + \psi\_{i}(i)\psi\_{i}(j)] + j[\psi\_{r}(i)\psi\_{i}(i) - \psi\_{i}(i)\psi\_{r}(j)] (17)

$$\psi^{*}(i)\psi^{*}(j) = [\psi_{r}(i) - j\psi_{i}(i)][\psi_{r}(j) - j\psi_{i}(j)]$$
  
= [\psi\_{r}(i)\psi\_{r}(j) - \psi\_{i}(i)\psi\_{i}(j)] - j[\psi\_{r}(i)\psi\_{i}(i) + \psi\_{i}(i)\psi\_{r}(j)] (18)

Real-dual tree discrete wavelet transformconsiders only the real part of the complex wavelets [16] for the analysis section and it is implemented using two separate lifting based 2DDWT. It produces eightsub bands such as HH2, HL1, LH2, HL2, LH1, HH1, LL1 and LL2 as follows:

$$\begin{aligned} \psi_{1,1}(i,j) &= \phi_r(i)\psi_r(j), \ \psi_{2,1}(i,j) = \phi_i(i)\psi_i(j) \\ \psi_{1,2}(i,j) &= \psi_r(i)\phi_r(j), \ \psi_{2,2}(i,j) = \psi_i(i)\phi_i(j) \\ \psi_{1,3}(i,j) &= \psi_r(i)\psi_r(j), \ \psi_{2,3}(i,j) = \psi_i(i)\psi_i(j) \end{aligned}$$
(19)

Real-dual tree discrete wavelet transform is a redundant and two times expensive. Thus, the proposed structure combines the two high pass bands as well two low pass bands as a single component such as HHm and LLm respectively. Further, it performs the synthesis operation without considering two high pass bands and as a result, the hardware requirement is reduced by a factor of 2 for synthesis operation. The modified synthesis section is shown in Fig. 5.

# **RESULTS AND DISCUSSIONS**

In this section, the performance of proposed approach is qualitatively analyzed and the numerical results of performance metrics are discussed. Experiment was conducted on Intel Pentium® IV 3.0-GHz CPU and 2-GB RAM using Xilinx ISE design suit 14.5 and MATLAB 2010. The proposed approach was tested on test images including "living room.tiff", "house.tiff", "lake.tiff" and "Jet plane.tiff" with image size of 512× 512 and the parameters including Average Absolute Difference (ABSDIFF), Peak Signal to Noise Ratio (PSNR) [14, 15], Image Fidelity (IMFID) and Mean Square Error (MSE) [14, 15] are considered for performance comparison. These parameters are calculated as follows.

$$ABSDIFF = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} |o(x,y) - r(x,y)|$$
(20)

1

$$PSNR = 20\log_{10} \left( \frac{255^2 MN}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} |o(x, y) - r(x, y)|^2} \right)$$
(21)



Fig. 6: Original and reconstructed images of proposed approach







Fig. 7(b): PSNR vs. Sample images



Fig. 7(c): Image Fidelity vs. Sample images



Fig. 7(c): Mean Square Error vs. Sample images Fig. 7(a)-(d): Comparison of test results

Table I: Performance comparison of ABSDIFF (Absolute difference)						Table III: Comparison of image fidelity				
	Absolu	Absolute difference(Absolute difference)					Image Fidelity			
Images	living r	living room		lake	Jetplane	Images	Living room	House	Lake	Jet plane
DWT	3.3771		0.4953	3.8376	1.8326	DWT	-5.48E-04	-0.00012	-0.0013	-0.00022
LWT	4.0565		1.0356	4.252	1.9746	LWT	-7.79E-04	-0.00012	-0.0016	-0.00028
DTWT	3.3771		0.7226	3.7916	1.8702	DTWT	-5.48E-04	-0.00012	-0.0012	-0.00022
Proposed	2.9106		0.4515	3.6836	1.8051	Proposed	-4.05E-04	-1.2 E-04	-1.2 E-04	-2.9 E-04
Table II: I	Performance c	omparison o	of PSNR			Table IV: P	erformance comp	arison of MSE		
	PSNR						Mean Square Error (MSE)			
Images	living r	room	house	lake	Jetplane	Images	Living room	House	Lake	Jet plane
DWT	39.872	8	38	33.7895	39.9336	DWT	6.6958	0.4565	25.092	5.5391
LWT	38.3497		44.6125	32.94	39.2967	LWT	9.5084	2.2344	30.9707	6.55699
DTWT	39.872	39.8728		33.9318	39.6779	DTWT	6.6958	1.0648	24.1367	5.8662
Proposed	41.1856		52.1822	34.2946	39.9468	Proposed	4.9494	0.3926	23.0834	5.4534
Table V: (	Overall perform	mance com	parison of prop	osed approac	h with existin	g System				
Method	ABSDIFF PSNR		IMFID	MSE	Time Complexity (Sec) Analysis section			Time Complexity (Sec) Synthesis Section		
DWT	2.38565	37.89898	-5.47E-04	9.44585	0.6543			0.4487		
LWT	2.829675	38.79973	-6.95E-04	12.31762	0.6734		0.4580			
DTWT	2.44	40.33	-5.22E-04	9.44	1.23		0.8983			
Proposed	2.21	41.90	-5.04E-04	8.45	1.28			0.4379		
Table VI:	Parameter Co	mparison of	f Proposed and	Existing Syst	tem					
	Lifting based DT-DWT						Modified lifting based DT-DWT			
Parameter	r		Analysis		Sy	Synthesis		Analysis		Synthesis
Power (mW)			42.41		42.41		42.05			42.14
Delay (nS)			2.265		2.334		1.915			2.334
Registers			187	187		149		187		
Memory			48		36	5	2	18		20
Buffers			115		11	1	8	33		79
FF/Latche	s		187		14	9	1	87		108

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$$IMFID = \frac{\left(1 - \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left|o(x,y) - r(x,y)\right|^2\right)}{o(x,y)^2}$$
(22)

$$MSE = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} |o(x,y) - r(x,y)|^2$$
(23)

The quality of reconstructed image is analyzed using the above metrics. Peak Signal-to-Noise Ratio (PSNR) measures the noise power with respect to peak signal power and it is often preferred as performance metric for image compression. PSNR must be high for better image quality. The original and reconstructed images of proposed method are shown in Fig. 6. From Fig. 6, it can be seen that, the quality of reconstructed image is appreciable and it is an exact replica of the original image. The performance of proposed structure is quantitatively evaluated based on the metrics including ABSDIFF, PSNR, IMFID and MSE and plotted in Fig.7 (a)-(d). The performance of proposed approach is compared with the other existing techniques and it is noted in Table I-V. It is observed that the modified lifting based DT-DWToutperforms than other existing techniques. The proposed approach is also simulated using Xilinx ISE design suit 14.5 and the results are reported in Table VI. It is noted that the proposed structure is well suited for low power and high speed image processing applications [16].

# CONCLUSION

In this paper, a novel modified lifting based DT-DWT has been proposed for low power and high speed image compression applications. An input image is divided into two branches of trees and lifting based DWT using 9/7 filter is applied to each tree at the analysis section. Eight sub bands such asHH2, HL1, LH2, HL2, LH1, HH1, LL1 and LL2 are generated. The high-pass sub bands (HH1 and HH2) and low-pass sub bands (LL1 and LL2) are linearly combined to produce HHm and LLm. The reconstructed image is obtained at the synthesis part without considering two high pass bands (LH1 and HL1) and thus the hardware complexity, power and arithmetic computations are reduced to some extent. Experiment was conducted on four test images using Xilinx ISE design suit 14.5 and MATLAB 2010. The performance was evaluated based on the compression and architectural parameters. Test results show that the modified lifting based DT-DWT is best suited for low power and high speed image processing applications without sacrificing the directional and shift invariant features.

### REFERENCES

- TinkuAcharya and ChaitaliChakrabarti, 2006. A Survey on Lifting-based Discrete Wavelet Transform Architectures, Journal of VLSI Signal Processing, (42): 321-339.
- Sweldens, W., 1996. The Lifting Scheme: A Custom-Design Construction of Biorthogonal Wavelets, Applied and Computational Harmonic Analysis, 3(15): 186-200.
- Daubechies, I. and W. Sweldens, 1998. FactoringWavelet Transformsinto Lifting Schemes, The Journal of Fourier analysis and Applications, 4: 247-269.
- Andra K., C. Chakrabarti and T. Acharya, 2002.A VLSI architecture for lifting-basedforward and inverse wavelet transform, IEEE Trans. Signal Process,4 (50):966-977.
- Huang C.T, P.C Tseng, L.G Chen,2002. Efficient VLSI architectures oflifting-based discrete wavelet transform by systematic design method.InProceedings of the IEEE International Symposium on Circuits and Systems, 5: 565-568.
- Xiong C.Y, J.W Tian and J. Liu,2006. Efficient highspeed/low-power line-based architecture for twodimensional discrete wavelet transform using lifting scheme. IEEE Trans. Circuits Syst. Video Technol,16(2): 309-316.

- Dillen G., B. Georis, J. D. Legat and O. Cantineau, 2003. Combined linebased architecture for the 5-3 and 9-7 wavelet transform of JPEG2000, IEEE Trans. Circuits Syst. Video Technol., 13(9): 944-950.
- Mohanty B.K and P.K Meher, 2011. Memory efficient modular VLSI architecture forhighthroughput and low-latency implementation of multilevel lifting 2-DDWT, IEEE Trans. Signal Process, 59(5): 2072-2084.
- Kingsbury N. G., 2000. The dual-tree complex wavelet transform with improved orthogonality and symmetry properties. In the proceedings of IEEE international Conference on Image processing, 375-378.
- Selesnick I.W., R.G.Baraniuk and N.G. Kingsbury, 2005. The dual-tree complex wavelet transform. IEEE Signal Process, (22): 123-151.
- Lee C.S, C.K Lee and K.Y Yoo, 2000. New lifting based structure for un-decimatedwavelet transform. Electron Letters, 36(22): 1894-1895.
- 12. Reeves, T. H. and N. G. Kingsbury, 2002. Overcomplete image coding using iterative projection-based noise shaping. In the proceedings of IEEE International Conf. Image Processing (ICIP02), Rochester.
- 13. UfukBal, 2012. Dual tree complex wavelet transform basedde-noising of optical microscopy images.Biomedical Optics Express, 3 (12): 3231-3239.
- Lilian N. Faria, Leila M. G. Fonseca and Max H. M. Costa, 2012. Performance Evaluation of Data Compression Systems Applied to Satellite Imagery, Journal of Electrical and Computer Engineering, Article ID 471857, 15.
- AmhamedSaffor, Abdul RahmanRamli and Kwan-Hoong Ng,2001. A comparative study of image compression between JPEG and Wavelet, Malaysian Journal of Computer Science, 14(1): 39-45.
- Ankit, A. and Bhurane,2011.Face Recognition using Dual-TreeDiscrete Wavelet Transform,M.Tech Thesis,Shri Guru GobindSinghji Institute of Engineering & TechnologyVishnupuri, Nanded, Maharashtra, India.