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Investigation of Chaotic Nature of Sunspot Data by Nonlinear Analysis Techniques

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Abstract: In this work, an attempt is made to detect the chaotic nature of smooth monthly sunspot (SSN) time series using various nonlinear analysis techniques to quantify the uncertainty involved. Different nonlinear dynamic methods, with varying levels of complexity, are employed such as average mutual information and embedding dimension method to construct a phase space. The correlation dimension method is used to identify the minimum number of variables required to forecast and the Lyapunov exponent method is used to confirm the presence of chaos and the maximal Lyapunov exponent is used to calculate the predictable time. These methods provide either direct or indirect identification of chaotic behavior in the sunspot number. From the analysis results, we arrive at a conclusion that the dynamical behavior of SSNs is a low-dimensional chaotic attractor and the solar activity is a chaotic phenomenon but not a stochastic behavior. It can be inferred that SSN is deterministic; hence, long term prediction of the SSN is impossible.

Key words: Average mutual information · False nearest neighbor · Phase space reconstruction · Correlation dimension · Maximal Lyapunov exponent

INTRODUCTION

India is an agricultural country and monsoon dependent. Rainfall is the vital source for the basic needs. Sunspot and rainfall have a relationship and it is analyzed by many researchers. Recently Hiremath, (2006) made a correlation study between sunspot number and Indian rainfall. According to him, the weak solar activity leads to heavy rainfall in Indian region. Sunspot number is one of the best indicators of solar activity [1]. So, in this work we concentrated on the chaotic nature of the sunspot data. It is well known that both chaos and fractal are a ubiquitous phenomenon in nature and they establish that complex nonlinear irregular and unpredictable behavior could be the fruit of an ordinary deterministic system under several dominant interdependent variables [2, 3, 4]. Moreover, they are applied to deterministic dynamical systems which are a periodic and exhibit sensitive dependence on initial conditions. To better identify a nonlinear behavior, it is suitable to apply appropriate techniques based on theories of chaos and fractal conceptions. There is no doubt that the sun exhibits complex nonlinear and non-stationary behavior. During the last two hundred years, the most important problem of a possible link between long-term solar activity and the earth's environment has attracted considerable attention [5, 6]. The frequently used indicator of long-term solar activity has been the sunspot number, which has taken as the most important data set that is used to represent secular solar activity [7, 8].

Long term prediction of solar activity is an important part of the solar activity prediction. The variation in sunspot number is a good indicator of the general level of solar activity. However, so far it has not been clear whether solar irregularity is chaotic or stochastic. In recent years, more and more results have been revealed

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about deterministic chaos of solar activity [9]. On the other hand, several authors found that there is no evidence to prove that the solar activity is governed by a chaotic low-dimensional process [10]. The present paper attempts to use a variety of techniques for characterizing the dynamical behavior of monthly sunspot numbers.

The importance of this paper is that the solar activity is maximum when more sunspots are present on the surface of the sun. Sunspot number time series is multi variable strong coupling and nonlinear time series. The smoothed monthly sunspot time series can be decided by many factors and a number of variables will be required when we use equations to describe the process of its evolution. The mutual information approach, phase space portrait and maximal Lyapunov exponent are employed for comprehensive characterization. To identify the minimum number of variables required to forecast sunspot time series data is found using correlation dimension. The embedding dimension value gives the number of required equations and the Lyapunov exponent predictability time gives the forecasting period, up to which we can get reliable forecast. More specifically, we attempt to identify the possible presence of chaotic systems in the monthly sunspot observational data.

MATERIALS AND METHODS

In the present work, the continuous time series of smoothed monthly sunspot numbers, which has been widely and frequently used to characterize the long-term solar activity, can be downloaded from NOAA website (http://www.ngdc.noaa.gov). Since the monthly sunspot data series is noisy, thus before the phase space could be reconstructed, this data set must be smoothed. So, we have considered the smoothed kind of the data series which cover the time interval from 1874 May to 2013 November and the plot of the same are shown in Fig. 1.

Four nonlinear dynamic methods, with varying levels of complexity, are employed: (1) phase space reconstruction; (2) average mutual information and embedding dimension method (3) correlation dimension method; and (4) Lyapunov exponent method. These methods provide either direct or indirect identification of chaotic behaviors.

Phase Space Reconstruction: The first step in the process of chaotic analysis is that of attempting to reconstruct the dynamics in phase-space. Reconstructing a phase space is an important research topic on chaotic time series analysis. Two parameters also need to be given: the embedding dimension (m) and the time lag (τ). The nature of the dynamics of a real-world system may be stochastic, deterministic or in between. This can be identified, at least as a preliminary indicator, by using the phase space concept. Generally, the nonlinear time series is analyzed by its phase space portrait. The phase portrait of a dynamical system can be reconstructed from the observation of a single variable by the method of delays as proposed by [11]. According to Takens, almost all d-dimensional sub-manifolds could be embedded in a (m=2d+1) dimensional space while preserving geometrical invariants. The observational time series X(1), X(2),….X(N) is represented by the set of vectors Y (j), the phase space can be reconstructed according to:

$$Y_j = (X_j, X_{j+\tau}, X_{j+2\tau}, \bullet \bullet\bullet, X_{j+(m-1)\tau})$$

j = 1, 2,...., N- (m-l) τ, m is the dimension of the vector Y_j, called as an embedding dimension; and τ is a delay time [11,12].

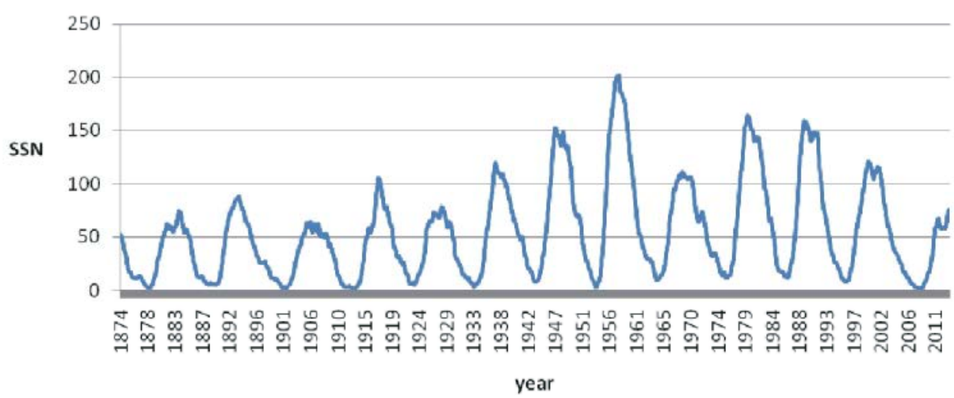

Fig. 1: Plot of the smoothed monthly sunspot numbers.

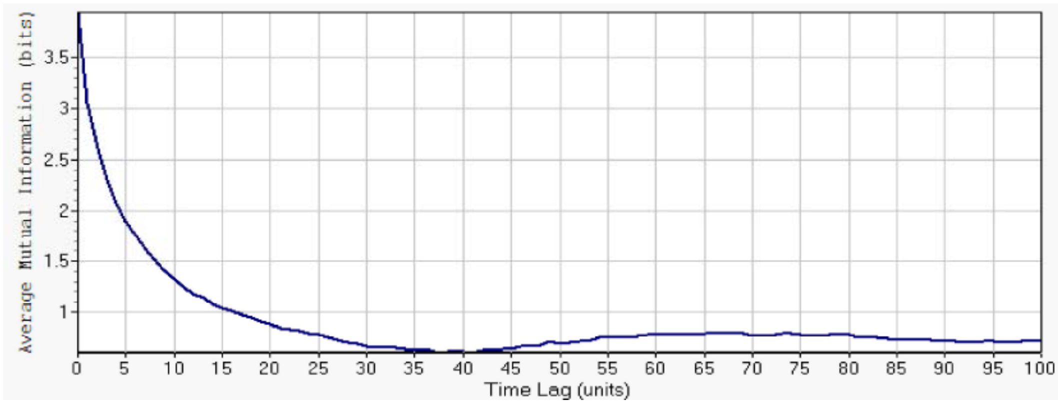

Fig. 2: Average Mutual Information as a Function of Time Lag for Smoothed Sunspot Numbers.

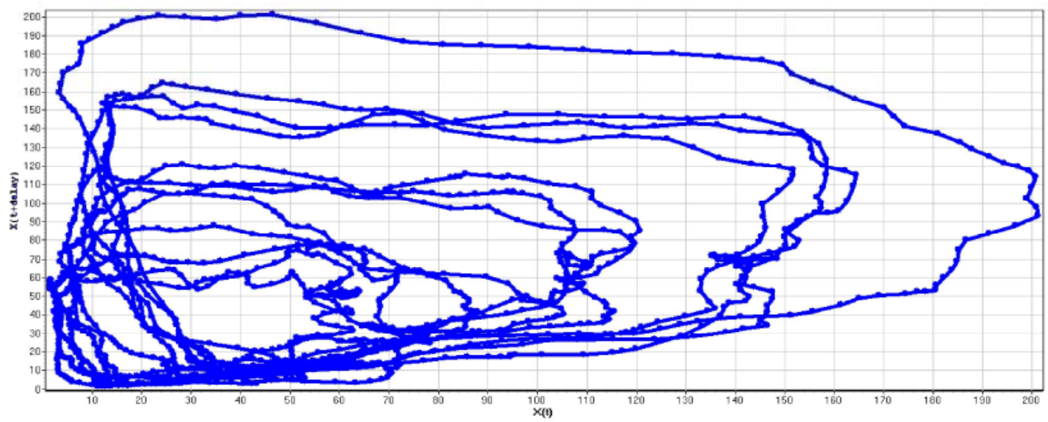

Fig. 3: Reconstructed Phase Space with Estimated Parameters for Sunspot numbers.

The embedding dimension (m) is an important parameter for the analysis of time series of the dynamical system approach and determines the dimension of the Euclidean pseudo-space in which supposedly the attractor is reconstructed from the time series. The embedding dimension, *m* that is small but sufficiently large to unfold the attractor is sought and a popular method to estimate it is the false nearest neighbors. This method increases the embedding dimension by one at each step and counts the percentage of points for which its nearest neighbor falls apart with the addition of a new component (from embedding dimension m to m+1) and therefore these points are called false nearest neighbors. The estimated minimum embedding dimension is the one that first gives an insignificant percentage of false nearest neighbors. Though the minimum m is not an invariant measure, as it depends, for example on the delay parameter τ, it is used in some applications as a measure to discriminate different dynamical regimes and it is therefore included here as well.

The time delay τ is usually chosen by autocorrelation function or average mutual information (AMI). The time delay is estimated where the value of the autocorrelation function is close to zero, thus minimizing the statistical dependence among the coordinates of the vectors while the AMI is the standard way to calculate time delay τ. In practice, one does not know the dimension of the dynamical system and the embedding dimension, which is necessary for the phase space reconstruction. So, the dimensional estimate is computed for increasing embedding dimensions until the dimensional estimate stabilizes as shown in Fig. 2.

To estimate a best possible value of embedding dimension (m) is to check for the closed false nearest neighbors (FNN) in the trajectory of phase space at different value of m, Kennel *et al*., (1992) developed an algorithm that estimates the sufficient dimension for phase space reconstruction and it is known as the false nearest neighbor method [13]. The false nearest neighbor algorithm identifies points within a nonlinear time series that looks to correlate, or relate, at a certain point in time.

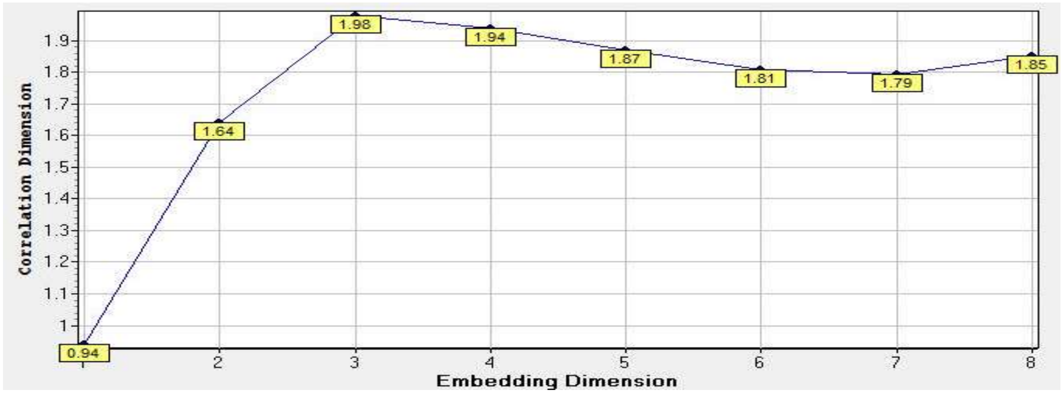

Fig. 4: Relation between correlation dimension and embedding dimension for the smoothed monthly sunspot time series.

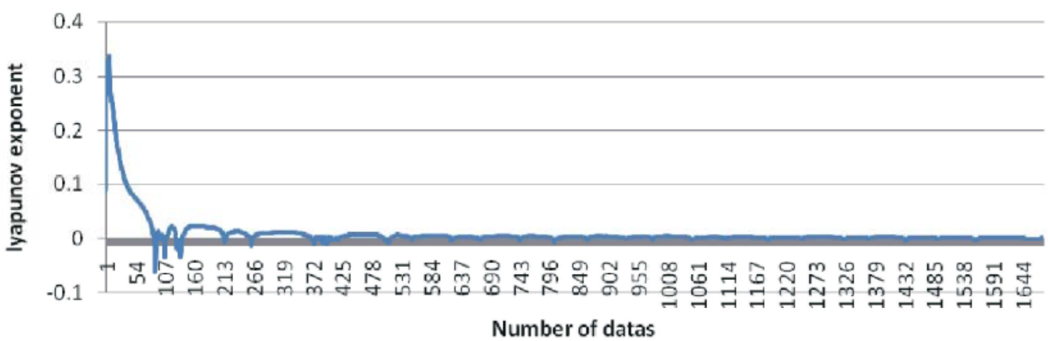

Fig. 5: Maximal Lyapunov exponent for the smoothed monthly sunspot number.

By increasing the embedding dimension, m, it is possible to detect ¯ false neighbors, within the vectors because once the attractors unfold; the vectors close in dimension m, move a significant distance apart in the next state. This indicates that the attractors of the system have not been accurately identified, then embedding dimension is increased by one or both the vectors and its neighbor by increasing the appropriate value of the data. The FNN method checks the neighbors in successive embedding dimensions until a negligible percentage of false neighbors are found. The phase space is reconstructed with the estimated parameters of embedding dimension and time delay as shown in Fig. 3.

Correlation Dimension: There exist a few numbers of methods for the evaluation of the dimension of time series such as box counting [14], Haussdorff dimension [15] and Grassberger and Procaccia [16]. To estimate the fractal dimension of a time series, the concept of correction dimension is useful. Correlation dimension is a nonlinear measure of the correlation between pairs lying on the attractor. Correlation dimension estimation is related to the relative frequency with which the attractor visits each covering element. Correlation dimension is generally a lower bound measure of the fractal dimension. The algorithm proposed by Grassberger and Procaccia [16] computes the correlation integral from which the power-law behavior can be used to estimate the dimension of the attractor. It gives a measure of the complexity for the underlying attractor of the system. The correlation dimension is an estimate of the fractal dimension of a chaotic dynamical systems attracting set. Convergence of the correlation dimension signifies a chaotic time series. The number of variables needed to describe the system effectively can be obtained from the correlation dimension as shown in Fig. 4.

Lyapunov Index and Predictable Scale: The phase space trajectory alone does imply a chaotic system, hence, the need for further investigation. An indicator of nonlinearity in dynamical or time series data is the Lyapunov exponent. A positive Lyapunov value is an indicator of chaos. A Chaotic system is strongly sensitive to initial value. Even there's only a trivial change in initial conditions, the system's evolving pathway will deviate with previous pathway at exponential speed and after a certain period, covers the system's true status completely. Therefore, this indicates unpredictability of the system's long-term movement. When the attractor in a chaotic system is partially unstable, the pathway finally falls on the same chaotic attractor in phase space and the system's biggest Lyapunov index is bigger than zero (ë>0), it indicates chaotic attractor exists which can be used to measure the chaotic level as shown in Fig. 5. In addition, the inverse of the biggest Lyapunov index also shows the system's longest predictable time.

RESULTS AND DISCUSSIONS

Fig. 2 shows the graph of average mutual information of sunspot numbers (SSNs) as a function of time delay. As pointed out by Fraser and Swinney [17], the reasonable value of the time delay is the value when the mutual information exhibits a marked first minimum. From the figure one can see that the reasonable value of time delay for a SSN is 31. The embedding dimension is calculated using False nearest neighbor (FNN) method and it is identified as 3. Fig. 3 shows the reconstructed phase space by applying the time delay and embedding dimension from SSNs. From this figure one can easily see that the phase space is totally unfolded, namely, it does not look like a 'ball of wool'. In this figure, we can clearly see the infinite self-similarity which characterizes the strange attractor, implying that the solar activity is a typical characteristic of a chaotic system [18]. The correlation dimension of the sunspot time series data is shown in Fig. 4. Convergence of the correlation dimension signifies a chaotic time series. The number of variables needed to describe the system effectively, can be obtained from the correlation dimension which is 1.98. Fig. 5 displays the reliable value of the maximal Lyapunov exponent is 0.337258/month. We know that the strange attractor has a length which can be described as the Lyapunov time or predictability time [19]. We estimate that the Lyapunov time is equal to about 2.965 years (1 month/0.337258). Therefore, we conclude that solar-activity forecast can be predicted from short- to mid-term due to its intrinsic complexity.

CONCLUSIONS

In this paper, we attempt to determine whether the smoothed sunspot monthly time series data is chaotic. This was tested using the method of phase space reconstruction, Lyapunov Exponent estimation and Correlation Dimension. To reconstruct the phase space, the time delay and embedding dimensions were obtained at 31 and 3 respectively, using the method of mutual information and false nearest neighbor. From the reconstructed phase space evidence of nonlinearity was seen. The presence of chaotic signals in the data was further confirmed by the correlation dimension method which yield a dimension of 1.98 and by the Lyapunov exponent which was positive and the largest Lyapunov exponent obtained as 0.337258. The predictability of the SSN is evaluated from the inverse of the largest Lyapunov exponent as 2.965 years. From the analysis results, we arrive at a conclusion that the dynamical behavior of SSNs is a low-dimensional chaotic attractor and the solar activity is a chaotic phenomenon but not a stochastic behavior. It can be inferred that SSN is deterministic; hence, long term prediction of the SSN is impossible.

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