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Alfvén Waves for Massive Photons

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Abstract: We study in this paper the coronal heating by Alfvén waves considering massive photons. To model the problem we use the Proca's electromagnetic field equations. We show that Alfvén waves are evanescents if theirs frequencies are smaller than the critical frequency, which is related to the photon's mass.

Key words: Proca Lagrangian · Massive photons · Alfvén waves · Coronal heating

INTRODUCTION

The metrology to determine the photon mass is not that easy, see for instance [1] about photon and graviton mass limits; the discussion on photon mass, or at least its mention, is not new [2] . In general, Maxwell's equations are based on the assumption of zero photon rest mass, but here we discuss the non-vanishing nature of the photon mass in solar corona. In order to access the problem of the non-vanishing nature of the photon mass, the conventional density Lagrangian must be modified, which considering now the photon mass, known as Proca Lagrangian, was conceived first by Proca as early as 1930/1936 [2].

In order to derive the Alfvén wave dispersion relation out of this new approach, we do not need to go too far, but we can just use the basic equations presented in the original paper of Alfvén [3]:

$$\nabla \times H = \frac{4\pi}{c} J, \quad \nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}, \quad B = \mu H, \quad J = \sigma (E + \frac{v}{c} \times B); \tag{1}$$

this system is closed using the "fluid" equation:

$$\rho \frac{\partial v}{\partial t} = \frac{1}{c}(J \times B) - \nabla p. \tag{2}$$

Here σ is the electric conductivity, μ is the magnetic permeability, ρ is the plasma mass density and p is the pressure. For the sake of simplicity, Alfvén considered $\sigma = \infty$, $\mu = 1$ and found the wave equation:

$$\frac{\partial^2 H_1}{\partial z^2} = \frac{4\pi\rho}{H_0^2}\frac{\partial H_1}{\partial t^2} = \frac{1}{V_A^2}\frac{\partial^2 H_1}{\partial t^2}. \tag{3}$$

Note that he expanded all the relevant quantities around the equilibrium ones, for example $H = H_0 + H_1 + ...$, and just neglected the second order terms, so H_1 is a first order term (perturbation).

In order to involve the photon mass in the Alfvén wave dispersion relation we have to introduce the photon mass contribution into the Maxwell's basic field equations. We show here only the equations that are modified by the non photon mass contribution, that is, the Ampere and Gauss laws:

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$$\nabla \times H - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{4\pi}{c}J - \lambda^2 A, \quad \nabla E = 4\pi\rho - \lambda^2 \phi, \tag{4}$$

where $\lambda = \frac{\hbar}{m_\gamma c}$ has dimension of inverse of length, it is indeed the inverse length of the Compton wavelength of the photon $(\lambda_{Compton} = \frac{\hbar}{m_\gamma c})$ or electromagnetic mass scenery at high temperature T as solar corona where coexist electron-positron and neutrino-antineutrino pairs [4]. The other relations are:

$$\nabla . H = 0,\ H = \nabla \times A,\ E = -\nabla\phi - \frac{1}{c}\frac{\partial A}{\partial t},\ \nabla . J + \frac{\partial \rho}{\partial t} = 0,\ \nabla . A + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0; \tag{5}$$

Note that when λ = 0, Proca´s equations reduce to the Maxwell's ones. It can be shown that the dispersion relation for light, considering harmonic variation in space and time, is given by [5]:

$$\omega^2 = c^2k^2 + \lambda^2 c. \tag{6}$$

If we go now to the "Proca Alfvén wave", we have the basic equations:

$$\nabla \times H - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{4\pi}{c}J - \lambda^2 A,\ \nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t},\ B = \mu H,\ J = \sigma(E + \frac{v}{c} \times B), \tag{7}$$

closing the system with the "fluid" equation:

$$\rho\frac{\partial v}{\partial t} = \frac{1}{c}(J \times B) - \nabla p. \tag{8}$$

Considering again no displacement current $(\frac{\partial E}{\partial t} = 0)$ and assuming infinity electric conductivity (ideal plasma), we get the following dispersion relation for the two known Alfvén waves, but with finite photon mass correction:

$$\omega^2 = k^2 v_A^2 - \lambda^2 v_A^2 = v_A^2(k^2 - \lambda^2), \tag{9}$$

(this mode is called compressional or fast Alfvén wave) and,

$$\omega^2 = k_z^2 v_A^2 - \lambda^2 v_A^2 = v_A^2(k_z^2 - \lambda^2), \tag{10}$$

these are the shear – non-compression - Alfvén wave.

Here we assumed that the ambient – background - magnetic field is aligned with the z axis, k is the total wave number and the Alfvén velocity is:

$$v_A = \frac{H}{\sqrt{4\pi\rho}}. \tag{11}$$

We see that for $\omega = \omega_{critical}^2 = \lambda^2 v_A^2,$ the wave number k vanishes and the Alfvén mode is a non-propagating mode, for $\omega = \omega_{critical}$ (it means that $k^2 > 0$), the Alfvén (both) propagates without damping or decay and for $\omega = \omega_{critical}$(which means that $k^2 < 0$), the Alfvén mode (both) decay exponentially, it is a cutoff mode under this condition; it dissipates its energy close to the excitation place.

Coronal Heating Analysis: In order to see the importance of this result to coronal heating, we have to use coronal data to estimate the relevant parameters. In order to see the finite photon mass effect we need to estimate the relation $k_z^2 \approx \lambda^2 \approx (\frac{2\pi}{\lambda_z})^2 \approx (\frac{6}{230{,}000\ kilometers})^2$, thus $\lambda^2 \approx (\frac{6}{230{,}000\ kilometers})^2, \approx 9x10^{-20} cm^{-2}$, and since $\lambda = \frac{\hbar}{m_\gamma c}$, we have to have a photon mass of $m_\gamma \approx \sqrt{9x10^{-10} km^{-2}}(\frac{\hbar}{c})$.

To make the Alfvén wave evanescent (note that for electrons the Compton wavelength is $\lambda_e = \frac{\hbar}{m_e c} = 2.4263x10^{-10} cm$), thus:

$$m_\gamma \approx \sqrt{9x10^{-20} cm^{-2}}(2.4263x10^{-10} cm) m_e \approx 7x10^{-20} m_e. \quad (12)$$

Therefore, we get for the photon mass in order to the Alfvén wave be evanescent, in solar corona, the value of $m_\gamma \approx 7x10^{-49} g$. This is a new result that we can compare with experimental results; to proceed now, let us see what the experiment says about this mass.

Experimental Checks on Photon Mass: Now it is important to say some words on the photon rest mass, [2, 6, 7] present a list of experiments that establishes limits for the photon mass, these experiments started as late as 1936. The number obtained for the various experiments range in the interval – from the highest limit to the lowest limit:

$$m_\gamma \leq (0.8x10^{-39} - 4.0x10^{-59}), \quad (13)$$

where the former number is gotten from the dispersion of starlight and the latter used the method of the dimension of magnetic field in arms of galaxy. There are several values in between these extremes; the photon mass we found is exactly inside this interval, showing that the finite photon mass can indeed be responsible for Alfvén wave to power the solar corona causing its anomalous not yet fully explained heating.

The Magnetohydrodinamic Effects: To study the limits of the photon mass using the MHD formalism is been around for a while, one could see, for instance [8] which discuss magnetohydrodynamic waves in the magnetosphere and the photon rest mass, [9] presents limit on the photon mass deduced from Pioneer-10 observations of Jupiter's magnetic field, [10, 11] indicate improved upper limit on the photon rest mass. Examining all these papers we can assure that the critical photon mass found in our study - $m_\gamma \approx 7x10^{-49} g$ - is a rather than reasonable value to be expected and so it might be possible to find evanescent Alfvén waves in solar corona and consequently this can be a possible explanation for coronal heating. This value totally agrees with the Table for photon mass presented at: http://pdg.lbl.gov/2009/listings/rpp2009-list-photon.

Coronal Heating: Coronal heating by Alfvén wave has been studied by several authors [8, 12] and heating of the solar corona [13] and references therein. In these papers it is shown that there are several mechanisms that can dissipate the energy of the Alfvén wave into the coronal plasma. Therefore, if the Alfvén wave is evanescent – a non propagating mode -, all the energy carried in the wave field is dissipated, by some mechanisms, into the corona, before being carried to out the coronal region. There are several mechanisms that can dissipate this energy such as viscosity and Cherenkov dissipation [13]. Even if the energy is deposited in a small layer close to the evanescent region, other mechanisms can act to re-distribute it into all coronal structure. It is worth to mention that the scale length of coronal plasma is such that the plasma is colisional and has a mean free path much smaller that the loop´s total length [13].

Since Alfvén waves are abundantly generated at the solar environment due to different mechanisms such as Kelvin - Helmholtz instabilities, ion/helium beam instabilities, line tied phenomenon, mode coupling and so on, it is to expect that the dissipation of Alfvén waves continuously due to its inability to always propagate without evanescence makes it possible for one to argue if this is not really the reason for the corona to be so hot above Sun's atmosphere [14]. Of course, not all Alfvén modes in Sun have to be evanescent in corona, but due to the peculiar – strong local gravity, highly turbulent plasma, complex local and global background magnetic field structure and an enormous amount of particles beams - the generation of all types of MHD wave eigenmodes, compressional and non compressional, bulk and surface should be therefore copious; therefore any twist in the local magnetic field or compression can create Alfvén waves [15].

The spectral energy density carried by an Alfvén wave is:

$$U_k = \frac{1}{16\pi}(\vec{E}_k \cdot \frac{\partial}{\partial\omega}(\omega\vec{\varepsilon}).E_k^*) + \vec{B}_k.\vec{B}_k), \quad (14)$$

In this scenario this energy will be totally absorbed by the coronal plasma. When the mode is a propagated one, the dissipation quality Q factor in corona is in general too large and the energy generated is carried to out of the corona:

$$\frac{1}{2}Q^{-1} = \frac{\gamma_k}{\omega_k} = -\frac{(\vec{E}_k.\vec{\varepsilon}AH..E_k^*)}{(\vec{E}_k \cdot \frac{\partial}{\partial\omega}(\omega\vec{\varepsilon}\vec{H})..E_k^*)}, \quad (15)$$

where γ_k is the damping rate due any linear dissipation mechanism. In general for propagating modes or the dissipation factor is low or the Q factor is too large (no dissipation).

The needed energy power to sustain the coronal temperature is [13] $P_{corona}^{needed} \approx 10^{-3} ergs^{-1} cm^{-3}$. In general, the propagating Alfvén modes dissipate around 10% of the need power, since not all its energy is dissipated in the structure, the kinetic Alfvén wave can power more. Therefore, solar Alfvén waves carry enough energy that when fully dissipated can sustain the observed coronal temperature [13].

CONCLUSIONS

In conclusion, we can say that the finite photon mass corrections to MHD plasma wave dispersion relation – Alfvén waves – permits the possibility for these eigenmodes to be evanescent for solar coronal plasma conditions. The evanescence makes it possible that the energy carried by the MHD electromagnetic fields and fluid be dissipated into the solar corona once it cannot be pumped out of the corona, since the excited mode is not a propagating one. Therefore, it might well be that the finite mass of the photons in these electromagnetic-hydrodynamic wave modes explain why the solar corona is far hotter than the solar atmosphere - that is, chromospheres and photosphere - below it.

Even if the solar corona (and the solar atmosphere in general) is known to be a highly nonhomogeneous plasma structure, confined by a background highly non-uniform magnetic field as well, the photon mass effect on coronal heating deserves indeed further investigation, since most of the model trying to explain (using wave-particle and wave-wave interactions, for instance) the coronal heating do not take also into consideration the real complicated geometry of the solar magnetic field, those models can even to borrow results from fusion confinement structure [16, 17]. In the range suggested could dominate over much stronger local gradients in the Alfvén speed.

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