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Mesoscopic Circuit

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Abstract: In this paper we study the quantization of the energy spectrum of a mesoscopic circuit, analyzing the semi classical-quantum correspondence between the modulus k^2 and $a_m(q)$, which is solution of the Schrodinger equation of the mesoscopic circuit. The values of q which provides the quantum requirement for oscillatory motion to be $a_m(q) \geq 2q$, so the term $2k^2 - 1$ must be less than one.

Key words: Energy spectrum · Mesoscopic circuit · Quantization

INTRODUCTION

In a series of articles and chapters [1-7], several authors have developed a theory of quantum electrical LC circuits, that is, electrical systems described by two parameters: an inductance L, and a capacitance C, and also by the discrete nature of electric charge and the magnetic flux ϕ. Now, in a recent work [8] we have proposed a semiclassical theory of quantum electrical circuits. The solution of the obtained differential equation is similar to that deduced for nonlinear equation of the pendulum [9-12]. The solution is given in terms of the Jacobi elliptic functions sn(z, k) and cn(z,k). The semiclassical theory of quantum LC circuits [1] starts from the quantum Hamiltonian of the LC circuit [1-8, 15]. The resulting equations become:

$$H^* = \frac{2\hbar^2}{q_e^2 L}\sin^2\left(\frac{q_e\phi}{2\hbar}\right) + \frac{Q^2}{2C}, \quad \frac{\partial H^*}{\partial Q} = \frac{Q}{C} = -\dot{\phi}, \frac{\partial H^*}{\partial \phi} = \dot{Q} = \frac{\hbar}{q_e L}\sin(\phi/\phi_0),$$
$$\ddot{\phi} + \phi_0 / LC\sin(\phi/\phi_0) = 0, \quad \ddot{\alpha} + \omega_0^2 \sin(\alpha) = 0, \tag{1}$$

where $\omega_0^2 = 1/LC$, $\alpha = \phi/\phi_0$. The equations above are considered, mathematically, as classical equations, but they include quantum effects, the quantized nature of electric charge through of the parameter $\hbar/q_e = \phi_0$.

Base on this information the Hamiltonian (1) minus a constant can be put in the form:

$$H = \frac{1}{2}C\phi_0^2(\dot{\alpha})^2 + \frac{\hbar^2}{Lq_e^2}\cos\alpha, \quad p_\alpha = C\phi_0^2\dot{\alpha}, \quad E = \frac{1}{2}\frac{p_\alpha^2}{C\phi_0^2} + \frac{\phi_0^2}{L}\cos\alpha, \tag{2}$$

where E is the energy; we define $\varepsilon = \frac{E + \phi_0^2/L}{C\phi_0^2}$, then $\frac{1}{2}\frac{p_\alpha^2}{C\phi_0^2} = \varepsilon C\phi_0^2 - \frac{\phi_0^2}{L}(1-\cos\alpha)$ which gives the generalized momentum:

$$p_\alpha = \sqrt{2(C\phi_0^2)^2\left[\varepsilon - \omega_0^2(1-\cos\alpha)\right]}. \tag{3}$$

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For a conservative system of one degree of freedom the semiclassical quantization rule is given by the action integral $I = \frac{1}{2\pi}\oint p dq$, such that p and q are the generalized momentum and coordinate, respectively. In order that the wave function be single-valued, the quantization condition reduces to:

$$I = n\,(1 + r/4)\hbar \tag{4}$$

in particular r = 2 for oscillatory motion and r = 0 for rotational motion.

Energy Spectrum and Semi Classical-quantum Correspondence: For oscillatory motion of charge, from (3) and (4) we have $I = \oint_{\alpha_1} p_\alpha d\alpha = (n + 1/2)\pi\hbar$

where $\alpha_1 = -\alpha_2$ is the maximum amplitude; thus $I_{osc} = 2C\phi_0^2 \int_0^{\alpha_1} \sqrt{2\left[\varepsilon_0 - \omega_0^2(1-\cos\alpha)\right]} d\alpha$ such that $\varepsilon_0 = \omega_0^2(1 - \cos\alpha_0)$ is the total energy of the mesoscopic circuit, then:

$$I_{osc} = 4C\phi_0^2\omega_0 \int_0^{\alpha_1} \sqrt{\sin^2\alpha_0/2 - \sin^2\alpha/2}\, d\alpha \tag{5}$$

Let $\sin(\alpha/2) = \sin(\alpha_0/2)\sin\chi$, $\chi \in [0, \pi/2]$,, therefore:

$$I_{osc} = 8C\phi_0^2\omega_0 \sin^2(\alpha_0/2) \int_0^{\pi/2} \frac{1}{\sqrt{1 - \sin^2(\alpha_0/2)\sin^2\chi}} d\chi; \tag{6}$$

defining the modulus $k^2 = \sin^2(\alpha_0/2) = \varepsilon/2\,\omega_0^2$, $k^2 \in [0,1]$, equation (6) is simplified to:

$$I_{osc} = (n + 1/2)\pi\hbar = 8C\phi_0^2\omega_0\left[(k^2 - 1)K(\pi/2, k) + E(\pi/2, k)\right] \tag{7}$$

where $K(\pi/2,k)$, $E(\pi/2,k)$ are the complete elliptic integrals of the first kind and second kind, respectively [13, 14]. In the limit of α_0 being very small, $k^2 \to 0$, (7) becomes $I_{osc} = C\phi_0^2\pi\sqrt{\varepsilon/2}$; when $\alpha_0 \to \pi$, $k^2 \to 1$, (7) is equivalent to $I_{osc} = C\phi_0^2\sqrt{2\varepsilon}/\pi$.

The Schrödinger equation for the mesoscopic circuit is given by $\frac{\partial^2\psi(\alpha)}{\partial\alpha^2} + \frac{2C\phi_0^2}{\hbar^2}(E + \phi_0^2/L\cos\alpha)\psi(\alpha) = 0$, which by the change of variable $\alpha = 2z$, takes the form:

$$\frac{\partial^2\psi(z)}{\partial z^2} + \frac{8C\phi_0^2}{\hbar^2}(E + \phi_0^2/L\cos 2z)\psi(z) = 0 \tag{8}$$

Comparing with the standard Mathieu's equation $\frac{\partial^2 y}{\partial z^2} + (a_m - 2q\cos 2z)y = 0$, we obtain $a_m(q) = \frac{8C\phi_0^2 E}{\hbar^2}$ and $q = \frac{4C\phi_0^4}{\hbar^2 L}$. Since the wave function has the same value when α goes through 2π period, we impose the boundary condition $\psi(2z + 2\pi) = \psi(2z)$ to the wave function (8). As solution of (8) are the even Mathieu function $ce_{2n}(z, q)$ and $se_{2n+2}(z, q)$, n = 0,1,2...

In order to compare the quantum and the semiclassical energy spectrum, we observe the modulus k^2 and the values a_m (q) so $a_m(q) = \frac{8C\phi_0^2}{\hbar^2}(C\phi_0^2\varepsilon - \frac{\phi_0^2}{L}) = \frac{8C\phi_0^2}{\hbar^2}(2k^2 - 1),$ and from the definition of q we get:

$$a_m(q) = 2q(2k^2 - 1). \quad (9)$$

The expression $(2k^2 - 1) = a_m(q)/2q$ implies a correspondence one-to-one between values of k^2 and a_m (q). A more fundamental view of the correspondence comes from the study of the values of q which provides the quantum requirement for oscillatory motion to be $a_m(q) \geq 2q$, so the term $2k^2 - 1$ must be less than one.

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