

Binary Logistic Regression Analysis Technique Used in Analyzing the Categorical Data In Education Sciences: A Case Study of Terengganu State, Malaysia

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Abstract: Multiple logistic regression is a technique for modeling and studying an association between several variables. It is observed that both of multiple linear regression and multiple logistic regression methods are frequently used in social sciences, medical sciences, education sciences and many more. Similar with the multiple linear regression cases, it's focused on the relationship between a dependent variable and one or more independent variables. However, to use of this method it's depends on some circumstances. The problems always arising at the stage of doing the analysis and interpreting the computer output. In this study, the practical applications of the above mentioned method are discussed. Then, an in-depth analysis follows on advantages and utilization of logistic regression, which is suggested as the technique that resolves the issues related to the statistical.

Key words: Multiple logistic regression • Categorical data variables • And Receiver Operating Characteristic (ROC)

INTRODUCTION

Data can be reached in various modes depending on the core principle of the research. For example, while in certain cases it is necessary to measure, elaborate or analyzes a feature; in other circumstances it is necessary to classify the data into categorical groups (or categories). On other occasions data is gathered in ordinal form. Statistical interpretation of these data requires different statistics methods[1].

According to Akturk [2] and Baspinar and Mendes [3], behavioral sciences, response variables are often consist of categorical or categorized data. Related analysis that widely used is Chi-Square analysis, Fischer's exact test, G-Statistics and Ratio Test (Z-test). However, the use of these statistical techniques depend on several assumption and sometimes these assumption cannot be satisfied to use these techniques or even if the assumption are met results of the analysis are too general for interpretation.

MATERIAL AND METHODS

Logistic regression is a type of predictive model that can be used when the target variable is a categorical variable with two categories for instance live or die, has cancer or no cancer, having coronary heart disease or not having coronary heart disease, patient survives or dies and many more.

In logistic regression, the dependent variable is binary or dichotomous; its only contains data coded as 1 or 0. The objective of logistic regression is to find the best fitting model to illustrate the relationship between the dichotomous characteristic of interest (dependent variable = response or outcome variable) and a set of independent (predictor or explanatory) variables.

Let us consider that, there are P is independent variables which will be denoted as vector $x^1 = \{x_1, x_2, \dots, x_p\}$. We assume that each of these variables is at least interval scaled. Let the conditional probability that the outcome is present be denoted by:

Table 1: Explanation of the variables

| Variables | Code | Explanation of the variables | Categorical |
|----------------|--------|--|---------------------------|
| y | Useful | eBook is very useful to help students | 0 = disagree 1 = agree |
| X ₂ | Hard | Students more hardworking doing their homework using eBook | 0 = disagree 1 = agree |
| X ₁ | Good | eBook designed to help student to have a good performance in education | 0 = disagree 1 = agree |

$$P(Y=1|x)=\pi(x) \quad (1)$$

Then the logit of the multiple regression models is given by the formula as follows:

$$\text{logit}(\pi)=b_0+b_1x_1+b_2x_2+b_3x_3+\dots+b_px_p \quad (2)$$

The specific form of the logistics regression model is given by:

$$\pi(x)=\frac{e^{g(x)}}{1+e^{g(x)}} \quad (3)$$

Where $\pi(x)$ is the probability of occurrence of the characteristic of interest. Since the model produce by logistic regression is nonlinear, the equations used to describe the outcomes are slightly more complex than those for multiple regression. This linear regression equation creates the logit transformation. This transformation is defined, in terms of $\pi(x)$, as:

$$\ln\left(\frac{\pi}{1-\pi}\right)=\sum_{j=1}^p b_j x_{ij} \quad (4)$$

The objective of this study is to discuss multiple logistic analyses, which can be used in the analysis of categorical or categorized data.

Material of this study is a hypothetical sample which is composed of seven variables. Namely variables are as in Table 1. Multiple logistic regression technique was used in the analysis of relationship between variables. Data of 70 respondents (teachers) were collected. The sample size calculations for the data are given by single proportion method.

Table 2: Variables X₂ in the Equation

| Variable | B | S.E. | Wald | df | Sig | R | Exp(B) |
|----------------|--------|--------|---------|----|--------|--------|---------|
| X ₁ | 3.3242 | 0.7874 | 17.8235 | 1 | 0.0000 | 0.4868 | 27.7778 |
| Constant | -.5108 | 0.5164 | 0.9785 | 1 | 0.3226 | | |

Table 3: Variables X₁ and X₂ in the Equation

| Variable | B | S.E. | Wald | df | Sig | R | Exp(B) |
|----------------|---------|--------|--------|----|--------|--------|---------|
| X ₂ | 3.1463 | 1.1850 | 7.0494 | 1 | 0.0079 | 0.2823 | 23.2508 |
| X ₁ | 2.7359 | 1.0000 | 7.4849 | 1 | 0.0062 | 0.2942 | 15.4229 |
| Constant | -1.3100 | 0.6739 | 3.7789 | 1 | 0.0519 | | |

Sample Size Required: The sample sizes required at analysis stage are as follows:

Anticipated population proportion (P) = 0.953

Level of significance= 5% (0.05)

$$\begin{aligned} \text{Absolute precision } (\Delta) &= \pm 5\% \\ &= \left(\frac{196^2}{005} \right) 0.953(1-0.953) \\ &= 68.8 \approx 70 \text{ respondents.} \end{aligned}$$

The sample of 70 respondents required at the analysis stage.

RESULTS AND DISCUSSION

Step 1 : Perform Univariable Analysis:

Table 1: Variables X₂ in the Equation

| Variable | B | S.E. | Wald | df | Sig | R | Exp(B) |
|----------------|---------|--------|---------|----|--------|--------|---------|
| X ₂ | 3.9444 | 1.1005 | 12.8457 | 1 | 0.0003 | 0.4124 | 51.6473 |
| Constant | -0.0953 | 0.4369 | .0476 | 1 | 0.8273 | | |

We found that x_2 is an important factor (p-value < 0.001) at univariable analysis. The crude (unadjusted) OR is 51.6473. At univariable analysis, teachers with “agree status” (for the statement, x_2) has 52 times the odds (chance) than teachers with “disagree status” to have variable (eBook is very useful to help students).

We found that x_1 is an important factor (p-value < 0.001) at univariable analysis. The crude (unadjusted) OR is 27.778. At univariable analysis, teachers with “agree status” (for the statement, x_2) has 28 times the odds (chance) than teachers with “disagree status” to have

Table 4: Correlation between X_1 and X_2

| Correlation between X_1 and X_2 | |
|-------------------------------------|-----------------------|
| X_2 | X_1 $r=-0.08500$ |

Table 5: Variables X_1 and X_2 in the Equation

| Variable | B | S.E. | Wald | df | Sig. | R | Exp(B) |
|------------------|---------|---------|--------|----|--------|--------|--------|
| X_2 | 2.1972 | 1.3333 | 2.7156 | 1 | 0.0994 | 0.1063 | 9.0000 |
| X_1 | 2.1972 | 1.0541 | 4.3450 | 1 | 0.0371 | 0.1924 | 9.0000 |
| $X_1 \times X_2$ | 7.9070 | 40.8574 | 0.0375 | 1 | 0.8465 | 0.0000 | 2716.3 |
| Constant | -1.0986 | 0.6667 | 2.7156 | 1 | 0.0994 | | |

variable y . From the above analysis, we found that the variables x_1 and x_2 are important and we have to include them into the model. So, we run the model again and gained the results as follows:

Step 2: Perform Variable Selection: We review all the results in univariable analysis and select the variables based on their p value (<0.25). In our cases, we select the two variables that's x_1 and x_2 .

Step 3: Check Multicollinearity: In step 3, we check multicollinearity in order to assess which variables are correlated highly. There are two methods that can be used and they are as follows:

The first step is by referring to the Table 4, from that table, we found that correlation between x_1 and x_2 are correlated lowly with one another. The second step is by referring to the standard errors of the variables. Table 3 indicates that the standard errors of the variables are small and this means that there are no multi-collinearity effects.

Step 4: Checking for the Interaction: We have checked all two way interaction between two important independent variables in the equation and the results shows that the interaction part is not significant ($p = 0.8465$). So we make decision not to include the interaction in the model while the others two variables alone are significant.

Step 5: Asses the Goodness-of-fit: In order to know the goodness-of-fit of the model, we tested the model with the three method. They are

- The classification Table
- Hosmer-Lemeshow test
- Area under the ROC curve

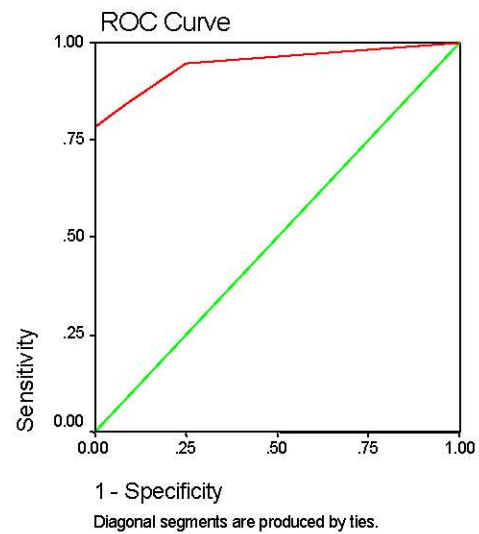


Fig. 1: Receiver Operating Characteristic (ROC) Curve

Table 6: Summary under the Curve

| | | | |
|-----------------|-----------------|-----------------|-------|
| Area | Predicted Value | 0.946 | |
| Std. Error | Predicted Value | 0.026 | |
| Asymptotic Sig. | Predicted Value | 0.000 | |
| Asymptotic 95 % | | | |
| Confidence | Lower | Predicted Value | 0.895 |
| | Upper | Predicted Value | 0.998 |

The area under the ROC curve is 0.946, it is significantly different from 0.5 (with p-value is less than 0.05). This information tells us that the model can discriminate 94.6% of the cases.

In order to assess the goodness-of-fit, we tested the hypothesis that data fit the model well versus data do not fit the model well. From the point of view of Hosmer and Lemeshow Test, we accept the hypothesis (with $P=0.5642$) that the model can predict well. p-value is greater than 0.05 indicated that the model is good enough to use.

Table 7: Summary of Hosmer and Lemeshow Test

| | Chi-Square | df | Sig |
|----------------------|------------|----|--------|
| Goodness-of-fit test | 1.1448 | 2 | 0.5642 |

Table 8: Classification Table

| | | Predicted | | % Correct |
|-------------------|-------|-----------|----------|-----------|
| | | Agree | Disagree | |
| Observed | | 1 | 0 | |
| Agree | 1 | 53 | 3 | 94.64% |
| Disagree | 0 | 3 | 9 | 75.00% |
| Overall % correct | 91.18 | | | |

The classification table in Table 8 shows that the overall percentage correct is very good (above 90%). 91.18% of all respondent said that “eBook is very useful to help students” can be predicted accurately by the model. Ideally, a valid test needs to be high sensitivity and high in specificity.

Step 6: Establish Final Model: Respondents with an agree opinion with the statement “eBook designed

to help student to have a good performance in education” has 15 times the odds to agree with the opinion “eBook is very useful to help students”(CI: 2.27 to 237.21, p-value <0.001). A respondent with an agree opinion with the statement “students more hardworking doing their homework using eBook” has 23 times the odds to agree with the opinion “eBook is very useful to help students”(CI: 2.17 to 109.49, p-value <0.001).

Table 9: Sensitivity and Specificity Table

| | |
|--|---|
| Sensitivity = $53/(53+3)\% = 94.6\%$ | percentage of occurrences correctly predicted |
| Specificity = $9/(3+9)\% = 75\%$ | percentage of nonoccurrence correctly predicted |
| False Positive Rate = $3/53\% = 5.67\%$ | percentage of predicted occurrences which are incorrect |
| False Negative Rate = $3/(3+9)\% = 25\%$ | percentage of predicted nonoccurrence which are incorrect |

Table 10: Variables X_1 and X_2 in the Equation

| Variable | B | Sig | R | Exp(B) | 95% CI for Exp(B) | |
|----------|---------|--------|--------|---------|-------------------|----------|
| | | | | | Lower | Upper |
| X_2 | 3.1463 | 0.0079 | 0.2823 | 23.2508 | 2.1725 | 109.4872 |
| X_1 | 2.7359 | 0.0062 | 0.2942 | 15.4229 | 2.2790 | 237.2149 |
| Constant | -1.3100 | 0.0519 | | | | |

Results Presentation:

Table 11: Logistic Regression Analysis of 70 eBook Respondents

| Predictors | B | S.E. | Sig | df | Wald | Exp(B) | 95% CI for Exp(B) | |
|---|---------|--------|--------|----|--------|---------|-------------------|----------|
| | | | | | | | Lower | Upper |
| Constant | -1.3100 | 0.6739 | 0.0519 | 1 | 3.7789 | | | |
| X_2 (1= Agree)(0= Disagree) | 3.1463 | 1.1850 | 0.0079 | 1 | 7.0494 | 23.2508 | 2.1725 | 109.4872 |
| X_1 (1= Agree)(0= Disagree) | 2.7359 | 1.0000 | 0.0062 | 1 | 7.4849 | 15.4229 | 2.2790 | 237.2149 |
| Test | | X_2 | | | df | | | p |
| Overall model evaluation-2 Log Likelihood | | 28.503 | | | 2 | | | 0.000 |
| Goodness-of-fit test | | | | | | | | |
| Pearson | | 1.145 | | | 1 | | | 0.285 |
| Deviance | | 1.511 | | | 1 | | | 0.219 |
| Hosmer-Lemeshow | | 0.263 | | | 1 | | | 0.608 |

DISCUSSION AND CONCLUSION

In this paper, we show that logistic regression is a very powerful analytical technique when the outcome variable is dichotomous. In order to know more about effectiveness of the logistic model we used the guideline as follow:

- Checking significance test for each predictor
- Checking significance tests of the model against the null model
- Descriptive and inferential goodness-of-fit indices
- Predicted probabilities

We hoped that this article give some guidelines and illustration of how logistic regression is applied to a dataset. As a research finding, from this analysis, we found that respondents with an agree opinion with the statement “eBook designed to help student to have a good performance in education” has 15 times the odds to agree with the opinion “eBook is very useful to help students” and the same interpretation goes to the statement that “students more hardworking doing their homework using eBook” have 23 times the odds to agree with the opinion “eBook is very useful to help students”.

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