

Calculation of Transformer Losses under Non-Sinusoidal Currents Using: Two Analytic Methods and Finite Element Analysis

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Abstract: Transformers are usually designed for nominal frequency and sinusoidal load currents. Nowadays, usage of non-linear loads, such as power electronic loads, has extremely increased. These loads can lead to create heat and extra loss in transformer parts and ultimately can cause insulation deterioration, ageing and finally fast failure. In this study, the effectual parameters on the loss of a single phase transformer under harmonic condition have been evaluated. Then, power reduction rate, maximum permissible current and also, transformer losses have been considered and calculated by analytic (using IEEE C57.110 standard and corrected harmonic loss factor) and Finite Element (FE) methods. In this study, Finite Element Method (FEM) has been used as a very precise method for calculating the loss of the transformer under non-sinusoidal currents.

Key words: Capacity reduction • Finite element method • Loss evaluation • Transformer

INTRODUCTION

Transformers are the most important component in power system and are interfaces between consumers and suppliers. Contemporary with ever-increasing electrical energy demand, the number and capacity of installed transmission transformers and especially distribution transformers are increasing. However, considering the point that the efficiency of these components is 97-99%, there was not enough attention to the amount of loss and performance of transformers. By considering the large number of transformers in transmission and distribution networks, it can be seen that the total power loss of these components is high. So, any reduction in loss of transformers would considerably reduce the total loss of electrical network and this will lead to optimum utilization of energy resources and helps environmental preservation.

In recent decades many of transformers that was installed for supplying linear loads with full sinusoidal current at the first, gradually, their main loads have been exchanged with non-linear loads that producing harmonic current [1]. Increasing the amount of harmonic contents of load current create extra loss in windings and thus, lead to increase in their hot spot temperature, increase of

stress on their insulations, reduction of insulation useful life and ultimately, reduction of transformer capacity [2].

Loads that produce harmonics in electrical system are: rectifiers, electronic phase control, PWM drivers, UPS, cyclo converters, arc furnaces, adjustable speed drives, computers and ballasts of fluorescent lamps in official center and large ventilation systems in commercial center [3].

Problems caused by non-linear loads and their effect on increasing the temperature of transformer was presented in the IEEE transformer committee meeting in 1980 and after discussion was approved a user guideline. This IEEE C57.110 standard entitled as "Recommended Practice for Establishing Transformer Capability When Supplying Non-Sinusoidal Load Currents". The purpose for publication of this standard was providing a procedure to determine the capacity reduction and permissible current of transformer under non-sinusoidal currents [4, 5].

This paper reviews IEEE C57.110 standard in order to evaluate loss of a 15 kVA single phase transformer under harmonic load currents and also, calculate capacity reduction. To verify obtained results from two calculation methods, the transformer is modeled and simulated using FEM.

Transformer Losses under Non-Sinusoidal Currents:

Transformer manufacturers usually try to design transformers in a way that their minimum losses occur in rated voltage, rated frequency and sinusoidal current. However, by increasing the number of non-linear loads in recent years, the load current is no longer sinusoidal. This non-sinusoidal current causes extra loss and temperature in transformer [6].

Transformer loss is divided into two major groups, no load and load loss. As a following:

$$P_T = P_{NL} + P_{LL} \quad (1)$$

P_{NL} = No load loss;

P_{LL} = Load loss;

P_T = Total loss of transformer.

A brief description of transformer losses and harmonic effects on them is presented in following:

No Load Loss: No load loss or core loss appears because of time variable nature of electromagnetic flux passing through the core and its arrangement is affected the amount of this loss. Since distribution transformers are always under service, considering the number of this type of transformer in network, the amount of no load loss is high but constant. This type of loss is caused by hysteresis phenomenon and eddy currents into the core. These losses are proportional to frequency and maximum flux density of the core and are separated from load current [7].

Many experiments have shown that core temperature increase is not a limiting parameter in determination of transformers permissible current in the non-sinusoidal currents [4, 5, 8]. Furthermore, considering that the value of voltage harmonic component is less than 5%, only the main component of the voltage is considered to calculate no load loss, the error of ignoring the harmonic component is negligible. So, IEEE C57.110 standard has not considered the core loss increase due to non-linear loads and has supposed this loss constant, under non-sinusoidal currents.

Load Loss: Load loss includes dc or Ohmic loss, eddy loss in windings and other stray loss and it can be obtained from short circuit test:

$$P_{LL} = P_{dc} + P_{EC} + P_{OSL} \quad (2)$$

In above equation:

P_{dc} = Loss due to resistance of windings;

P_{EC} = Windings eddy current loss;

P_{OSL} = The other stray loss in structural parts of transformer such as tank, clamps.

The sum of P_{EC} and P_{OSL} is called total stray loss. According to Eq. 3, we can calculate its value from the difference of load loss and Ohmic loss:

$$P_{TSL} = P_{EC} + P_{OSL} = P_{LL} - P_{dc} \quad (3)$$

It should be mentioned that there is no practical or experimental process to separate windings eddy loss and other stray loss yet [3].

Ohmic Loss: This loss can be calculated by measuring winding dc resistance and load current. If RMS value of load current increases due to harmonic component, this loss will increase by square of RMS of load current [4]. The windings Ohmic loss under harmonic condition is shown in Eq. 4:

$$P_{dc} = R_{dc} \times I^2 = R_{dc} \times \sum_{h=1}^{h=h_{max}} I_{h,max}^2 \quad (4)$$

Eddy Current Loss in Windings: This loss is caused by time variable electromagnetic flux that covers windings. Skin effect and proximity effect are the most important phenomenon in creating these losses.

Also, the most amount of loss is in the last layer of conductors in winding, which is due to high radial flux density in this region [1, 8]:

$$P_{EC} = \frac{\pi \tau^2 \mu^2}{3\rho} f^2 \times H^2 \propto f^2 \times I^2 \quad (5)$$

In this equation:

τ = A conductor width perpendicular to field line;

ρ = Conductor's resistance.

According to IEEE C57.110 standard, the amount of rated eddy current loss of windings is about 33% of total stray loss for oil-filled and about 67% of total stray loss for dry-type transformers.

As mentioned above, eddy current loss of windings is proportional to square of current and square of harmonic frequency in harmonic condition. In following equation, this loss is calculated:

$$P_{EC} = P_{EC-R} \times \sum_{h=1}^{h=h_{max}} h^2 \left[\frac{I_h}{I_R} \right]^2 \quad (6)$$

In which:

P_{EC-R} = Rated eddy current loss of windings;
 I_h = The current related to h_{th} harmonic;
 I_R = Rated load current;
 h = Order of harmonic.

Also, the harmonic loss factor for eddy current loss of winding can be defined according to the following equation:

$$F_{HL} = \frac{\sum_{h=1}^{h=h_{max}} h^2 I_h^2}{\sum_{h=1}^{h=h_{max}} I_h^2} = \frac{\sum_{h=1}^{h=h_{max}} h^2 \left[\frac{I_h}{I_R}\right]^2}{\sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R}\right]^2} \quad (7)$$

Therefore, according to the above equation, rated eddy current loss of transformer winding under non-sinusoidal currents must be multiplied by the coefficient F_{HL} .

According to the results obtained from test step and various researches, harmonic loss factor presented in Eq. 7 is acceptable only for transformers which the dimensions of their string of conductors are less than 3 mm. For conductors with dimensions more than 3 mm, considering coefficient F_{HL} like above will lead to results with less accuracy. Therefore, a corrected harmonic loss factor (F'_{HL}) is presented in [1] that shows more accurate results.

Due to skin effect in strings of conductor with dimensions more than 3 mm, electromagnetic flux cannot fully penetrate in string of winding conductor in high frequency. Therefore, field permeability (δ), that depend on frequency, can be defined in different harmonic frequency as:

$$\delta = \sqrt{\frac{\rho}{\mu \pi f h}} = \frac{\delta_R}{\sqrt{h}} \quad (8)$$

In this equation:

δ_R = Penetration depth in rated frequency, which is about 10 mm for copper and about 13 mm for aluminum in 50 Hz;
 ρ = Conductor's resistance;
 μ = Its permeability;
 f = Fundamental frequency.

In mentioned condition, eddy current loss and corrected harmonic loss factor, are calculated according to following equations [1]:

$$P_{EC} = \mu_0 \omega \times H^2 \times F(\xi) \quad (9)$$

$$F(\xi) = \frac{1}{\xi} \times \frac{\sinh \xi - \sin \xi}{\cosh \xi + \cos \xi} \quad (10)$$

$$\xi = \frac{\tau}{\delta} \quad (11)$$

$$\xi_h = \xi_R \times \sqrt{h} \quad (12)$$

$$F'_{HL} = \frac{\sum_{h=1}^{h=h_{max}} h \left[\frac{F(\xi_h)}{F(\xi_R)} \right] \left[\frac{I_h}{I_R} \right]^2}{\sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R} \right]^2} \quad (13)$$

According to the above equations and under non-sinusoidal currents, rated eddy current loss of winding must be multiplied by coefficient F'_{HL} [1].

Other Stray Loss: Due to the linkage between electromagnetic flux and conductor, a voltage induces in the conductor and this will lead to producing eddy current. Eddy current produces loss and increases temperature. A part of eddy current loss which is produced in structural parts of transformers (except in the windings) is called other stray loss [9, 10].

This loss is created in metallic parts such as tank, enclosure and clamps and depends on many factors such as size of core, class of voltage of transformer and construction of materials used to build tank and clamps [11, 12].

To determine the effect of frequency on the value of other stray loss, different tests have been fulfilled. Considering the results derived from [13], the resistance of other stray loss in low frequency (0-360 Hz) is equal to:

$$R_{AC}^{lf} = 0.00129 \left(\frac{f_h}{f_1} \right)^{0.8} \quad (14)$$

Of course for frequencies in the range of (420-1200 Hz), the resistance will be calculated by Eq. 15:

$$R_{AC}^{hf} = 0.33358 \left(\frac{f_h}{f_1} \right)^{-1.87} \quad (15)$$

Also, at first, other stray loss is increased to the determined frequency and after that will decrease. Meanwhile, the frequency that maximum other stray loss occurs is in it, depends on the kind of material used in structural parts of transformer.

According to IEEE C57.110 standard, the rated value of other stray loss (P_{OSL-R}) is about 67% of total stray loss for oil-filled transformer, while it is about 33% of total stray loss for dry-type transformer.

Also, in presence of non-sinusoidal current, the value of other stray loss changes by the square of RMS current and harmonic frequency by exponent factor of 0.8:

$$P_{OSL} = P_{OSL-R} \times \sum_{h=1}^{h=h_{max}} h^{0.8} \left[\frac{I_h}{I_R} \right]^2 \quad (16)$$

Harmonic loss factor for other stray loss is:

$$F_{HL-STR} = \frac{P_{OSL}}{P_{OSL-R}} = \frac{\sum_{h=1}^{h=h_{max}} h^{0.8} \left[\frac{I_h}{I_R} \right]^2}{\sum_{h=1}^{h=h_{max}} \left[\frac{I_h}{I_R} \right]^2} \quad (17)$$

So, under non-sinusoidal currents it is only necessary to multiply the rated other stray loss by harmonic loss factor, F_{HL-STR} .

When a transformer is utilized under non-sinusoidal voltages and currents, due to loss increase and, as a result, increase of temperature, its rated power must decrease. This action will be possible by limiting total transformer loss under non-sinusoidal current to the amount of loss in sinusoidal voltage and load current. In other word, maximum permissible current of transformer in harmonic load must be determined as its loss would be equal to the loss in hot spot and under sinusoidal current condition [2].

Load loss in linear load state and rated condition in per unit is equal to:

$$P_{LL-R}(pu) = 1 + P_{EC-R}(pu) + P_{OSL-R}(pu) \quad (18)$$

P_{LL-R} = Rated load loss of transformer;

P_{EC-R} = Eddy current loss of winding in rated condition;

P_{OSL-R} = Other stray loss in rated condition;

The number 1 in this equation is per unit amount of Ohmic loss.

Now, if we have non-sinusoidal current:

$$P_{LL}(pu) = I^2(pu) \times [1 + F_{HL} \cdot P_{EC-R}(pu) + F_{HL-STR} \cdot P_{OSL-R}(pu)] \quad (19)$$

So, maximum permissible load current to determine the capacity reduction of transformer is [4]:

$$I_{max}(pu) = \left[\frac{P_{LL-R}(pu)}{1 + F_{HL} \cdot P_{EC-R}(pu) + F_{HL-STR} \cdot P_{OSL-R}(pu)} \right]^{0.5} \quad (20)$$

Base on this equation, we can determine maximum permissible load current of transformer and also, determine its capacity reduction under non-sinusoidal current.

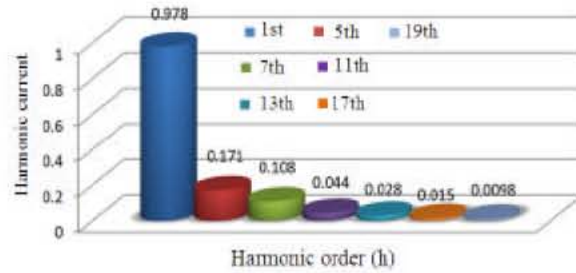


Fig. 1: Non-Linear load specification for studied transformer

An Illustrative Example-Determination of Loss in a Single Phase Transformer: In this section, a 15 kVA single phase distribution oil-filled transformer is considered and generic parameters of this transformer are summarized in Table 1. Then, the amount of loss, permissible current and reduction of capacity are calculated with two methods, analytical and Finite Element (FE) methods. Non-sinusoidal current content has been presented in Fig. 1.

Two Analytical Methods: Using IEEE C57.110 standard: First, total stray loss is obtained by difference dc loss from load loss:

$$P_{TSL} + P_{LL} - P_{dc} = 550.82 - [(435.12 \times 0.75^2) + (0.0412 \times 65.2^2)] = 130.92 \text{ watt}$$

Since a single phase transformer is oil-filled kind, rated eddy current loss of winding and other rated stray loss are calculated from following equation:

$$P_{EC-R} = 0.33 \times P_{TSL} = 0.33 \times 130.92 = 43.2 \text{ watt}$$

$$P_{OSL-R} = 0.67 \times P_{TSL} = 0.67 \times 130.92 = 87.72 \text{ watt}$$

Also, by use of Eq. 7 and 17 harmonic loss coefficient F_{HL} and F_{HL-STR} are calculated that are Presented in Table 2 based on Eq. 20:

$$I_{max}(pu) = \left[\frac{1 + 0.312}{1 + (2.73 \times 0.103) + (1.14 \times 0.209)} \right]^{0.5} = 0.91 \text{ pu}$$

So:

$$\text{Equivalent k VA} = 15 \times 0.91 = 13.65 \text{ kVA}$$

Using Corrected Harmonic Loss Factor Method: By use of Eq. 13 harmonic loss coefficient, F'_{HL} is calculated which is shown in Table 3.

Table 1: Technical specification of a 15 kVA single phase distribution transformer

Rated kVA	Rated frequency	Voltage HV	Voltage LV
15 kVA	50 Hertz	20000 volt	231 volt
Rated current HV	Rated current LV	Ohmic resistance HV	Ohmic resistance LV
0.75 A	65.2 A	435.12 ohm	0.0412 ohm
	Load loss	No load loss	
	550.82 watt	120.12 watt	

Table 2: Computation table for harmonic loss factors (F_{HL} , F_{HL-STR})

h	$(\frac{I_h}{I_R})$	h^2	h^2	$(\frac{I_h}{I_R})^2$	$h^2(\frac{I_h}{I_R})^2$	$h^{0.8}(\frac{I_h}{I_R})^2$
1	0.9780	1.000	1	0.95600	0.956	0.95600
5	0.1710	3.624	25	0.02900	0.730	0.10500
7	0.1080	4.743	49	0.01200	0.588	0.05690
11	0.0440	6.809	121	0.00200	0.242	0.01360
13	0.0280	7.783	169	0.00078	0.132	0.00607
17	0.0150	9.646	289	0.00023	0.066	0.00222
19	0.0098	10.544	361	0.00010	0.036	0.00105
\sum				1	2.730	1.14000

Table 3: Computation table for corrected harmonic loss factor (F'_{HL})

h	$(\frac{I_h}{I_R})$	$(\frac{I_h}{I_R})^2$	$F(\xi)$	$h[\frac{F(\xi_h)}{F(\xi_R)}]$	$h[\frac{F(\xi_h)}{F(\xi_R)}][\frac{I_h}{I_R}]^2$
1	0.9780	0.95600	0.04156	1.000	0.95600
5	0.1710	0.02900	0.19590	23.570	0.68350
7	0.1080	0.01200	0.25957	43.270	0.52464
11	0.0440	0.00200	0.35113	92.936	0.18587
13	0.0280	0.00078	0.38015	118.900	0.09274
17	0.0150	0.00023	0.41083	168.050	0.03865
19	0.0098	0.00010	0.41632	190.330	0.01900
\sum		1			2.5

Table 4: Transformer losses under non-sinusoidal current

Type of transformer loss	Sinusoidal load current loss (watt)	Harmonic loss factor	Non-Sinusoidal load current loss (watt)
Ohmic loss	419.90		419.9
Eddy current loss of winding	43.20	$F_{HL} = 2.73$	Using IEEE C57.110 = 118
		$F'_{HL} = 2.75$	and
		$F_{HL-STR} = 1.14$	Using corrected harmonic loss factor = 108
Other stray loss	87.72		100
Total load loss	550.82		Using IEEE C57.110 = 637.9
			and
			Using corrected harmonic loss factor = 627.9
No load loss	120.12		120.12
Total transformer loss	670.94		Using IEEE C57.110 = 758
			and
			Using corrected harmonic loss factor = 748

Based on Eq. 20, permissible current for determining transformer capacity under harmonic load is:

$$I_{max}(\text{pu}) = \left[\frac{1 + 0.312}{1 + (2.5 \times 0.103) + (1.14 \times 0.209)} \right]^{0.5} = 0.94 \text{ pu}$$

And:

$$\text{Equivalent kVA} = 15 \times 0.94 = 14.1 \text{ kVA}$$

Now, if harmonic component of non-linear load changes considerably, this equivalent power must be calculated again. For example, if ITHD become very high, this equivalent power will extremely decrease.

Transformer loss component under harmonic load has been calculated and shown in Table 4. Under harmonic load current that inserted in Table 2, total load loss of transformer has increased about 16% by using standard method and has increased about 14% by use of corrected harmonic loss factor method.

Calculation Transformer Losses Using FEA: In this section, for modeling transformer by Finite Element Method, software Maxwell v.11 is used. The FEM is a numerical technique for obtaining approximation solution to boundary value problems of mathematical physics.

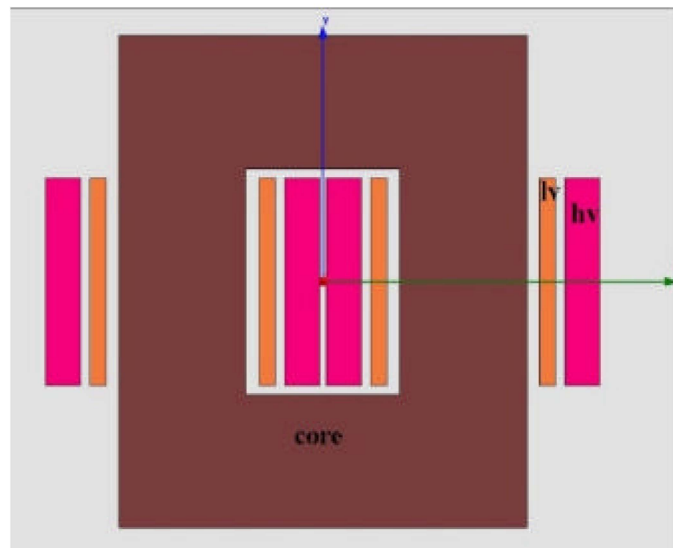


Fig. 2: Two-dimensional cross-section of the transformer

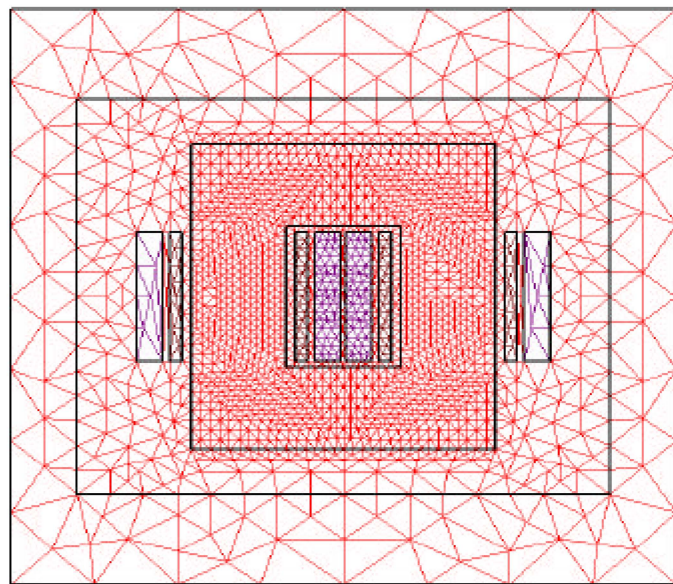


Fig. 3: Meshed model of the transformer

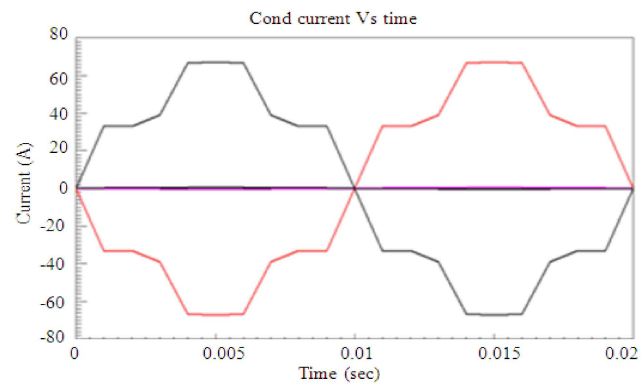


Fig. 4: Non-sinusoidal load currents

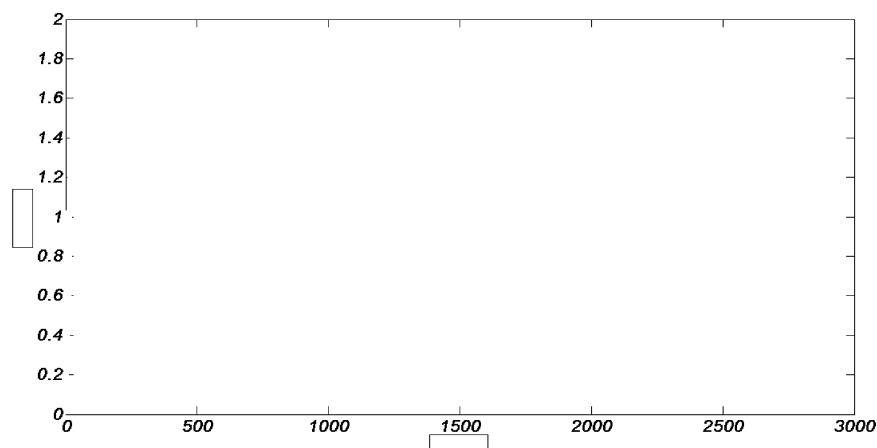


Fig. 5: B-H curve of core

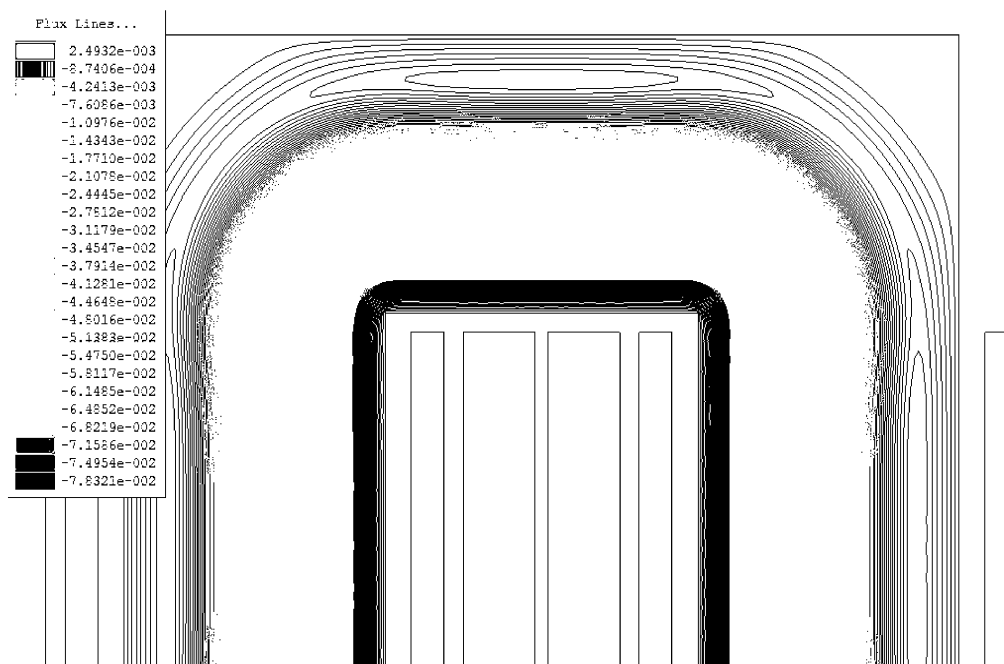


Fig. 6: Flux distribution in part of the core of the transformer

Especially it has become a very important tool for solving electromagnetic problems because of its ability to model geometrically and compositionally complex problems.

Utilizing this software, a 2 Dimension (2D) model of transformer is presented. Also, the behavior of transformer under linear and non-linear (harmonic) load currents is simulated. Load currents are presented in Fig. 1. First, the rated sinusoidal 50 Hz current is applied and simulated and also, load loss has been calculated. Then, a non-sinusoidal current including harmonics is applied to the transformer. Each harmonic current with its frequency apply to windings and after simulation, load loss of this harmonic component calculated.

Two-dimensional cross-section of the transformer, meshed model for the transformer, considered non-linear load current and saturation curve of core are respectively shown in Figs. 2-5, respectively.

In Fig. 6, flux distribution has been shown in part of the transformer core. Also, Fig. 7 shows flux density in core of transformer. The current passing through windings is near the rated current of transformer, which means that this condition is very similar to short circuit condition. Therefore, magnetic current and flux amplitude will be low in the core.

After simulation and passing the solution stages, the results of loss calculation using FEA have been shown in Table 5.

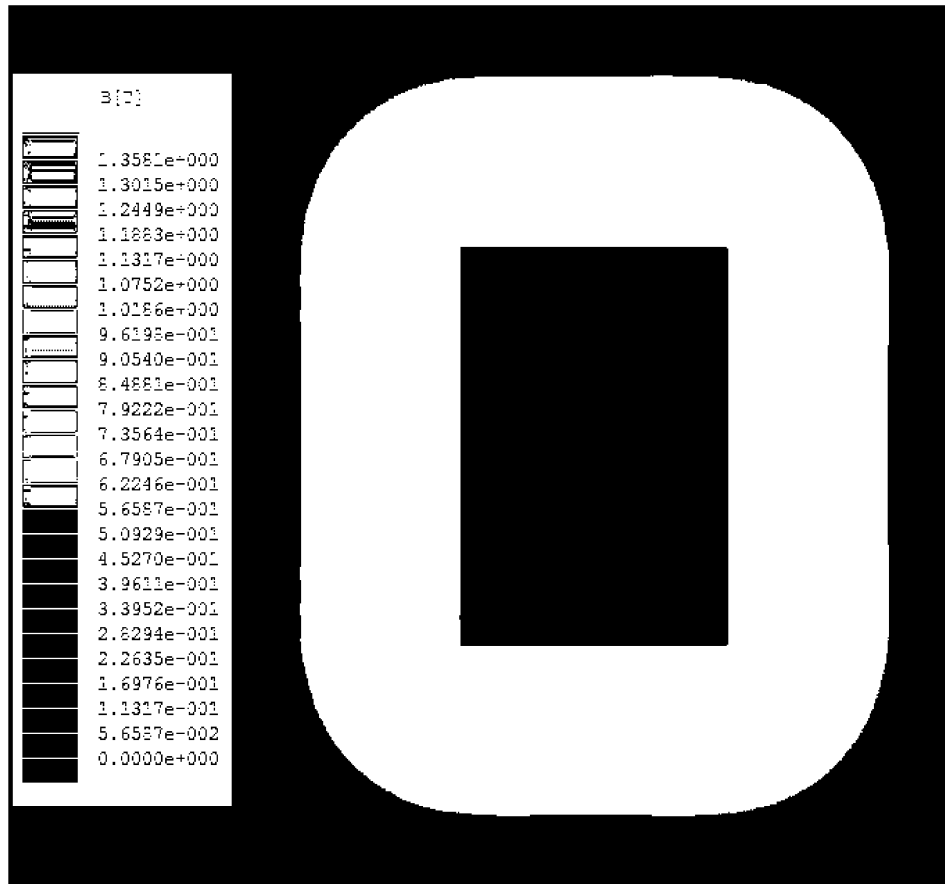


Fig. 7: Flux density in core of the transformer

Table 5: Comparing total loss of transformer

The method of total loss determination	Sinusoidal current	Non-Sinusoidal current
By standard method	670.94 watt	758 watt
By corrected harmonic loss factor method	670.94 watt	748 watt
By FEA method	707.62 watt	781.32 watt

Comparing the Results of Loss Calculation Between Analytic and Finite Element Methods: The results of analytic methods and FEA are presented in Table 5. A simple comparison between results obtained from analytic and estimated value obtained by FEA shows that the results of these two methods are very close to each other. Also, simulation with software Maxwell v.11 and finite element method has a very good accuracy. Therefore, we can use Finite Element Method to estimate the rate of loss in transformer designing, building and operation stages.

CONCLUSION

One of the most important issues in loss reduction of transformer under linear and non-linear loads is to determine the component of loss in different parts of

transformer, exactly. In this study, Finite Element Method (FEM) has been used as a very precise method for calculating the loss of the transformer under linear and non-linear load currents. Also, the effect of non-linear loads on transformer loss and capacity have been carried out and then evaluated by use of analytical method and applying IEEE C57.110 standard and corrected harmonic loss factor. Results have shown that the effect of non-linear loads on Ohmic loss and other stray loss is low, while its effect on winding eddy current loss is high.

Also, the values of loss and capacity reduction of a single phase transformer have been calculated. Then, loss of this transformer, supplying a non-linear load, was estimated by use of Finite Element Method (FEM). Comparing the results of methods shows that the values estimated by Finite Element Method (FEM) are very

similar to the values obtained from analytic method. This shows that the simulation with the Finite element method has a very good coordination with the values obtained from applying the standard.

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