World Applied Sciences Journal 9 (6): 681-688, 2010
ISSN 1818-4952

Application of Homotopy Perturbation Method and Differential Transformation Method to Determine Displacement of a Damped System with Nonlinear Spring

[1]A. Fereidoon, [1]M.R. Davoudabadi, [1]H.Yaghoobi and [2]D.D. Ganji

[1]Department of Mechanical Engineering, Semnan University, Semnan, Iran
[2]Department of Mechanical Engineering, Babol University of Technology, Babol, Iran, P.O. Box 484

Abstract: In this paper, Differential Transformation Method (DTM) and He's homotopy perturbation method (HPM) have been applied to determine the displacement of nonlinear oscillator problem with third order non linearities. the concepts of Differential Transformation Method and He's homotopy perturbation method are briefly introduced for applying these methods for solving problem. The results of DTM are compared with HPM as analytical solution and exact solution to verify the accuracy of proposed method. The results reveal that Differential Transformation method very effective and convenient in predicting the solution of such problems. Finally, we analyze effect of physical parameter A (initial amplitude) in this problem.

Key words: Differential transformation method · Homotopy perturbation method · Nonlinear oscillator · Nonlinear spring · Damped system

INTRODUCTION

This paper considers the following general nonlinear oscillator equation [1]:

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+k_1x+k_2x^3=0 \quad (1)$$

Where m $[kg]$ is mass, c $\left[\frac{kg}{\sec}\right]$ is damping coefficient, k_1 $\left[\frac{N}{m}\right]$ and k_2 $\left[\frac{N}{m^3}\right]$ are spring stiffness coefficients.

With Initial Conditions:

$$x(0)=A,\frac{dx}{dt}(0)=1.5 \quad (2)$$

Where $k_2\ x^3$ is a nonlinear term and A is initial amplitude There are many mechanical systems that modeled by mass and spring. The development of numerical techniques for solving nonlinear algebraic equations is a subject of considerable interest. There are many papers that deal with nonlinear algebraic equations. Nonlinear oscillation systems are such phenomena that mostly occurs nonlinearity. These systems are important in engineering because many practical engineering components consist of vibrating systems that can be modeled using oscillator systems. Generally, analysis of actual engineering problem involves solution of nonlinear differential equations. Except for a limited number, these problems cannot be solved explicitly and normally fails to yield exact solutions. Consequently, so many methods have been developed for approximating or numerical solutions. Perturbation method is one of the well-known methods based on the existence of small parameters. In order to overcome the problems associated with finding the small parameter, different new methods have been proposed to eliminate the small parameter, for example, homotopy perturbation method (HPM) [2-7], variational iteration method (VIM) [8-10], Adomian's decomposition method (ADM) [11] and differential transformation method (DTM) [12-22], He's homotopy perturbation method which doesn't need small parameter is implemented for solving the nonlinear differential equations. Homotopy perturbation method yields very rapid convergence of the solution series in most cases and usually only a few iterations leading to high accuracy solutions. Thus HPM is a universal one which can solve various kinds of nonlinear equations. The differential transformation method was first applied in the engineer domain by Zhou [23]. The DTM method is based on the

Corresponding Author: H.Yaghoobi, Department of Mechanical Engineering, Semnan University, Semnan, Iran.
E-mail: yaghoobi.hessam@gmail.com.

Taylor's series expansion and provides an effective numerical means of solving linear and nonlinear initial value problems. A review of the related literature reveals that Chiou [24] make use of the intrinsic ability of differential transforms to solve nonlinear problems. The differential transformation method may be used to solve both ODE and PDE. For example, Chen and Ho [25-26], solve the free vibration problems using DTM. In general, the previous studies have verified that the differential transformation method is an efficient technique for solving nonlinear equation as numerical method. In current study uses the differential transformation method to investigate the displacement of the vibration of a nonlinear oscillator system and illustrates how the corresponding nonlinear equation with nonlinear spring should be converted in to differential transformation and then solved by a process of inverse transformation. Something new in this research is using sub-domain technique in DTM program code to achieve more accuracy toward ordinary domain that will be explained bellow. We applied DTM and HPM for solving the nonlinear oscillator equation. A comparison of the present results shows that fourth order Runge Kutta numerical solution and HPM confirm the high accuracy of DTM.

Basic Idea of Homotopy Perturbation Method: To explain this method, let us consider the following function [27]:

$$A(u)-f(r)=0, r\in\Omega \tag{3}$$

With the boundary conditions of:

$$B\left(u,\frac{\partial u}{\partial n}\right)=0, r\in\Gamma \tag{4}$$

Where A is a general differential operator, $f(r)$ is a known analytic function; B is a boundary operator and Γ is the boundary of the domain Ω. The operator A can be generally divided into two operators, L and N, where L is a linear and N a nonlinear operator. Eq. (3) can be, therefore, written as follows:

$$L(u)+N(u)-f(r)=0 \tag{5}$$

Using the homotopy technique, we constructed a homotopy $v(r,p):\Omega\times[0,1]\rightarrow R$ which satisfies:

$$H(v,p)=(1-p)\left[L(v)-L(u_0)\right]+p\left[A(v)-f(r)\right]=0 \tag{6}$$

Or

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[N(v)-f(r)\right]=0 \tag{7}$$

Where $p\in[0,1]$, is called homotopy parameter and u_0 is an initial approximation for the solution of Eq. (3), which satisfies the boundary conditions. Obviously from Eqs. (6) and (7) we will have:

$$H(v,0)=L(v)-L(u_0)=0 \tag{8}$$

$$H(v,1)=A(v)-f(r)=0 \tag{9}$$

We can assume that the solution of (6) or (7) can be expressed as a series in p, as follows:

$$v=v_0+pv_1+p^2v_2+... \tag{10}$$

Setting $p=1$ results in the approximate solution of Eq. (3)

$$u=\lim_{p\to 1}v=v_0+v_1+v_2+... \tag{11}$$

Implementation of HPM: To solve Eq. (1) with the initial condition (2), according to the homotopy perturbation, we construct the following He's polynomials corresponding to Eq. (7):

$$L(v)=m\frac{d^2v(t)}{dt^2}+c\frac{dv(t)}{dt}+k_1v(t)$$

$$N(v)=k_2v(t)^3 \tag{12}$$

$$\begin{aligned}H(v,p)=&m\frac{d^2v(t)}{dt^2}+c\frac{dv(t)}{dt}+k_1v(t)-m\frac{d^2u_0(t)}{dt^2}-c\frac{du_0(t)}{dt}-k_1u_0(t)\\&+p\left(m\frac{d^2u_0(t)}{dt^2}+c\frac{du_0(t)}{dt}+k_1u_0(t)\right)+p\left(k_2v(t)^3\right)\end{aligned} \tag{13}$$

Substituting $v=v_0+pv_1+p^2v_2+...$ in to Eq. (13) and rearranging the resultant equation based on powers of p-terms, one has:

$$p^0: m\frac{d^2v_0(t)}{dt^2}+c\frac{dv_0(t)}{dt}+k_1v_0(t)-m\frac{d^2u_0(t)}{dt^2}-c\frac{du_0(t)}{dt}-k_1u_0(t)=0$$

$$p^1: m\frac{d^2v_1(t)}{dt^2}+c\frac{dv_1(t)}{dt}+k_1v_1(t)+m\frac{d^2u_0(t)}{dt^2}+c\frac{du_0(t)}{dt}+k_1u_0(t)+k_2v_0(t)^3=0$$

$$p^2: m\frac{d^2v_2(t)}{dt^2}+c\frac{dv_2(t)}{dt}+k_1v_2(t)+3k_2v_0(t)^2v_1(t)=0 \tag{14}$$

We assume $m = 1$, $c = 1$, $k_1 = 100$, $k_2 = 100$ then solution of Eq. (14) may be written as follow:

$$v_0(t) = 0.05(3+A)e^{-0.5t}\sin(9.9875t) + Ae^{-0.5t}\cos(9.9875t)$$

$$\begin{aligned} v_1(t) = {} & e^{-0.5t}\sin(9.9875t)\left[-0.0003-4.181A^3-0.1396A^2-0.09345A\right]+ \\ & e^{-0.5t}\cos(9.9875t)\left[0.01394+0.3798A^3+0.6292A^2+0.0209A\right]+ \\ & e^{-1.5t}\sin(29.962t)\left[0.0077A^3-0.0003A+0.0153A^2-0.0001\right]+ \\ & e^{-1.5t}\cos(29.962t)\left[-0.0027A^2-0.0023A+0.0336A^3+0.000008\right]+ \\ & e^{-1.5t}\sin(9.9875t)\left[4.1198A^3+0.0307A^2+0.0922A-0.0006\right]+ \\ & e^{-1.5t}\cos(9.9875t)\left[-0.4135A^3-0.6264A^2-0.0186A-0.0139\right] \end{aligned}$$

$$\begin{aligned} v2(t) = {} & \\ & e^{-2.5t}\sin(49.938t)\left[0.0004A^5+8.9\times10^{-8}-0.00003A^2-0.00008A^3+6.2\times10^{-7}A+0.0008A^4\right]+ \\ & e^{-2.5t}\cos(29.962t)\left[0.00003+0.0001A-0.1864A^4-0.0026A^2+0.0007A^3-0.1395A^5\right]+ \\ & e^{-2.5t}\cos(49.938t)\left[0.001A^5+1.4\times10^{-8}+0.00001A^2-0.0002A^3+2.9\times10^{-6}A-0.0003A^4\right]+ \\ & e^{-2.5t}\sin(29.962t)\left[5.9\times10^{-6}-0.0006A-0.0476A^4-0.0013A^2-0.0197A^3+0.4032A^5\right]+ \\ & e^{-2.5t}\sin(9.9875t)\left[-0.0006-0.002A-1.3281A^4-0.0583A^2-0.1492A^3-2.4671A^5\right]+ \\ & e^{-2.5t}\cos(9.9875t)\left[0.0001-0.0039A+0.0603A^4+0.0083A^2-0.3637A^3-8.2482A^5\right]+ \\ & e^{-1.5t}\cos(9.9875t)\left[0.00003+0.0087A+0.4981A^4+0.0202A^2+0.786A^3+17.0483A^5\right]+ \\ & e^{-1.5t}\cos(29.962t)\left[-0.00003-0.0001A+0.1939A^4+0.0027A^2+0.004A^3+0.114A^5\right]+ \\ & e^{-1.5t}\sin(9.9875t)\left[0.0013+0.0028A+2.6065A^4+0.1178A^2+0.1613A^3+3.835A^5\right]+ \\ & e^{-1.5t}\sin(29.962t)\left[-1.6\times10^{-6}+0.0006A+0.025A^4+0.0014A^2+0.0204A^3-0.4191A^5\right]+ \\ & e^{-0.5t}\cos(9.9875t)\left[-0.0001-0.0048A-0.5656A^4-0.0287A^2-0.4268A^3-8.776A^5\right]+ \\ & e^{-0.5t}\sin(9.9875t)\left[-0.0006-0.0006A-1.1708A^4-0.056A^2-0.0073A^3-1.2835A^5\right] \end{aligned} \quad (15)$$

In the same manner, the rest of components were obtained using the Maple package. According to the HPM, we can conclude that:

$$x(t) = v_0(t) + v_1(t) + v_2(t) \quad (16)$$

Fundamental of Differential Transformation Method: Let $x(t)$ be analytic in a domain D and let $t = t_i$ represent any point in D. The function $x(t)$ is then represented by one power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is in form of:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \qquad \forall t \in D \quad (17)$$

The particular case of Eq. (17) when $t_i = 0$ is referred to as the Maclaurin series of $x(t)$ and is expressed as:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \qquad \forall t \in D \quad (18)$$

As explained in [15] the differential transformation of the function $x(t)$ is defined as follows:

$$X(k) = \sum_{k=0}^{\infty} \frac{(H)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \quad (19)$$

Where $x(t)$ is the original function and $X(k)$ is the transformed function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where H is a constant. The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H}\right)^k X(k) \quad (20)$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function $X(k)$ at values of argument k are referred to as discrete, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete, etc. the more discrete available, the more precise it is possible to restore the unknown function. The function $x(t)$ consists of T-function $X(k)$ and its value is given by the sum of the T-function with $\left(\frac{t}{H}\right)^k$ as its coefficient. In real applications, at the right choice of constant H, the larger values of argument k the discrete of spectrum reduce rapidly. The function $x(t)$ is expressed by a finite series and Eq. (20) can be written as:

$$x(t) = \sum_{k=0}^{n} \left(\frac{t}{H}\right)^k X(k) \quad (21)$$

Mathematical operations performed by differential transform method are listed in Table 1.

Application of DTM: We applied the DTM for the Eq. (1) and taking the differential transform Eq.(1) with respect to t for $H = 1$ gives:

Table 1: The fundamental operations of differential transform method

Original function	Transformed function
$x(t)=\alpha f(t)\pm\beta g(t)$	$X(k)=\alpha F(k)\pm\beta G(k)$
$x(t)=\frac{df(t)}{dt}$	$X(k)=(k+1)F(k+1)$
$x(t)=\frac{d^2 f(t)}{dt^2}$	$X(k)=(k+1)(k+2)F(k+2)$
$x(t)=t^m$	$X(k)=\delta(k-m)=\begin{cases}1 & k=m\\ 0 & k\neq m\end{cases}$
$x(t)=\exp(\lambda t)$	$X(k)=\frac{\lambda^k}{k!}$
$x(t)=f(t)g(t)$	$X(k)=\sum_{l=0}^{k}F(l)G(k-l)$

$$m(k+2)(k+1)X(k+2)+c(k+1)X(k+1)+k_1X(k)+k_2\left(\sum_{w=0}^{k}X(k-w)\left(\sum_{j=0}^{w}X(w-j)X(j)\right)\right)=0 \quad (22)$$

From initial condition in Eq. (2), then we can obtain:

$$X(0)=A, X(1)=1.5 \quad (23)$$

From a process of inverse differential transformation, it can be shown that the solutions of each sub-domain take $n+1$ term for the power series like Eq. (22), we can write:

$$x_i(t)=\sum_{k=0}^{n}\left(\frac{t}{H_i}\right)^k X_i(k) \quad 0\le t\le H_i \quad (24)$$

Where $k = 0,1,2,...,n$ represents the number of term of the power series, $I = 0,1,2,...$ expresses the i^{th} sub-domain and H_i is the sub-domain interval. In this example we consider ten sub-domains (i=10) and distance of each interval is 0.2 for the case of $m = 1, c = 1, k_1 = 100, k_2 = 110$, we calculated $X(k + 2)$ from Eq. (22) for $A = 0.1$ as following:

$X(2)=-5.805$
$X(3)=-23.89$
$X(4)=-49.7563$
$X(5)=-123.6128$
$\vdots$

$$X(20) = 3.29 \times 10^8 \quad (25)$$

Substituting Eq. (25) into the main equation based on DTM, it can be obtained that the closed form of the solutions is:

$$x_1(t)=\sum_{k=0}^{20}\left(\frac{t}{H_1}\right)^k X_1(k)$$

$$x_1(t)=0.1+1.5t-5.805t^2-23.89t^3+49.7563t^4+123.6128t^5-6.4869t^6-613.127t^7-2852.9918t^8+5415.7281t^9+31660.16t^{10}-3234.58t^{11}-2.23\times10^5t^{12}-79598.9t^{13}+1.28\times10^6t^{14}+3.88\times10^6t^{15}-6.49\times10^6t^{16}-4.58\times10^7t^{17}+1.37\times10^7t^{18}+3.55\times10^8t^{19}+3.29\times10^8t^{20} \quad (26)$$

In the similar manner, we will obtain another sub-domain's series solution and we can present the solution of Eq. (1) accurately.

RESULT AND DISCUSSION

In this study, the DTM was applied successfully to find the displacement of the nonlinear oscillator problem. For the different value of A (initial amplitude, 0, 0.1, 0.2, 0.3, 0.4), results of the present analysis are tabulated in Tables (2-6).The errors for small value of A parameter are very small but for large value of A parameter, errors are third order $O(h^3)$.The comparison of the solutions between DTM, HPM and exact solution is shown in Figs. (1-3).a very interesting agreement between the results is observed, which confirms the validity of the DTM. By comparison the results, the advantages and

Table 2: Compare results of Eq. (1) between HPM, DTM and Exact solution for A=0

t	$x(t)_{HPM}$	$x(t)_{DTM}$	$x(t)_{Exact}$	$Error_{\lvert HPM-Exact \rvert}$	$Error_{\lvert DTM-Exact \rvert}$
0	0	0	0	0	0
0.2	0.1220526969	0.1220525878	0.1220526079	8.9E-08	2.01E-08
0.4	-0.0945045030	-0.0945070717	-0.0945068146	2.3116E-06	2.571E-07
0.6	-0.0272984102	-0.0273022943	-0.0272984544	4.42E-08	3.8399E-06
0.8	0.0985265028	0.0985367087	0.0985302951	3.7923E-06	6.4136E-06
1	-0.0527370105	-0.0527409434	-0.0527393777	2.3672E-06	1.5657E-06
1.2	-0.0405911425	-0.0406006304	-0.0405955781	4.4356E-06	5.0523E-06
1.4	0.0740627689	0.0740729325	0.0740673999	4.631E-06	5.5326E-06
1.6	-0.0227563914	-0.0227558993	-0.0227555713	8.201E-07	3.28E-07
1.8	-0.0434669932	-0.0434769731	-0.0434729300	5.9368E-06	4.0431E-06
2	0.0514404483	0.0514468241	0.0514435417	3.0934E-06	3.2824E-06

Table 3: Compare results of Eq. (1) between HPM, DTM and Exact solution for A=0.1

t	$x(t)_{HPM}$	$x(t)_{DTM}$	$x(t)_{Exact}$	$Error_{\lvert HPM-Exact \rvert}$	$Error_{\lvert DTM-Exact \rvert}$
0	0.1	0.1	0.1	0	0
0.2	0.08490988562	0.08490767205	0.08490653921	3.34641E-06	1.13284E-06
0.4	-0.14949981590	-0.14950000870	-0.14950904345	9.22755E-06	9.03475E-06
0.6	0.04668413322	0.04668244327	0.04669650569	1.23725E-05	1.40624E-05
0.8	0.08637066193	0.08637757134	0.08638851104	1.78491E-05	1.09397E-05
1	-0.10497172070	-0.10496766980	-0.10498865783	1.69371E-05	2.0988E-050
1.2	0.00956294369	0.00954883550	0.00955367884	9.26485E-06	4.84334E-06
1.4	0.07875395127	0.07876530966	0.07878110247	2.71512E-05	1.57928E-05
1.6	-0.06775851057	-0.06775028075	-0.06776714646	8.63589E-06	1.68657E-05
1.8	-0.01312120955	-0.01314416694	-0.01314375810	2.25485E-05	4.0884E-070
2	0.06547719741	0.06548689358	0.06550068107	2.34837E-05	1.37875E-05

Table 4: Compare results of Eq. (1) between HPM, DTM and Exact solution for A=0.2

t	$x(t)_{HPM}$	$x(t)_{DTM}$	$x(t)_{Exact}$	$Error_{\lvert HPM-Exact \rvert}$	$Error_{\lvert DTM-Exact \rvert}$
0	0.2	0.2	0.2	0	0
0.2	0.04151612425	0.04141430379	0.04147165851	4.44657E-05	5.73547E-05
0.4	-0.19977448710	-0.20030240830	-0.19984697897	7.24919E-05	0.000455429
0.6	0.12700132150	0.12758639270	0.12708783424	8.65127E-05	0.000498558
0.8	0.06237607654	0.06262260071	0.06252155614	0.000145480	0.000101045
1	-0.15418989670	-0.15487690950	-0.15433283893	0.000142942	0.000544071
1.2	0.06915738837	0.06947199766	0.06910566148	5.17269E-05	0.000366336
1.4	0.07258135499	0.07299136962	0.07283567405	0.000254319	0.000155696
1.6	-0.11301671270	-0.11353774030	-0.11310305849	8.63458E-05	0.000434682
1.8	0.02713631427	0.02717577577	0.02695491430	0.000181400	0.000220861
2	0.07186208510	0.07227325395	0.07209163180	0.000229547	0.000181622

Table 5: Compare results of Eq. (1) between HPM, DTM and Exact solution for A=0.3

t	$x(t)_{HPM}$	$x(t)_{DTM}$	$x(t)_{Exact}$	$Error_{\lvert HPM-Exact \rvert}$	$Error_{\lvert DTM-Exact \rvert}$
0	0.3	0.3	0.3	0	0
0.2	-0.01091960560	-0.0118219246	-0.0112219536	0.000302348	0.000599971
0.4	-0.23891681720	-0.2433678494	-0.2393758312	0.000459014	0.003992018
0.6	0.21383340800	0.2196407880	0.2142679299	0.000434522	0.005372858
0.8	0.01730901616	0.0153062337	0.0182393492	0.000930333	0.002933116
1	-0.19129410700	-0.1948202500	-0.1921998958	0.000905789	0.002620354
1.2	0.14058009020	0.1452666770	0.1401744334	0.000405657	0.005092244
1.4	0.04415270503	0.0438407228	0.0459336036	0.001780899	0.002092881
1.6	-0.15137429920	-0.1546046729	-0.1519032988	0.000529000	0.002701374
1.8	0.08210257304	0.0848765830	0.0808391436	0.001263429	0.004037439
2	0.06008677735	0.0607564583	0.0617481811	0.001661404	0.000991723

Table 6: Compare results of Eq. (1) between HPM, DTM and Exact solution for A=0.4

t	$x(t)_{HPM}$	$x(t)_{DTM}$	$x(t)_{Exact}$	$Error_{\|HPM-Exact\|}$	$Error_{\|DTM-Exact\|}$
0	0.4	0.4	0.4	0	0
0.2	-0.0741056719	-0.0765130673	-0.0754457233	0.001340051	0.001067344
0.4	-0.2589108464	-0.2652064714	-0.2610869723	0.002176126	0.004119499
0.6	0.3039112547	0.3149550188	0.3053700738	0.001458819	0.009584945
0.8	-0.0577034729	-0.0630450989	-0.0533878287	0.004315644	0.009657270
1	-0.2016518416	-0.2011193445	-0.2061284339	0.004476592	0.005009089
1.2	0.2216543016	0.2234969665	0.2193148261	0.002339475	0.004182140
1.4	-0.0201771329	-0.0195075738	-0.0112969728	0.008880160	0.008210601
1.6	-0.1689098584	-0.1681384593	-0.1716125134	0.002702655	0.003474054
1.8	0.1534984046	0.1512846698	0.1471669879	0.006331417	0.004117682
2	0.0155985076	0.0180957999	0.0242377943	0.008639287	0.006141994

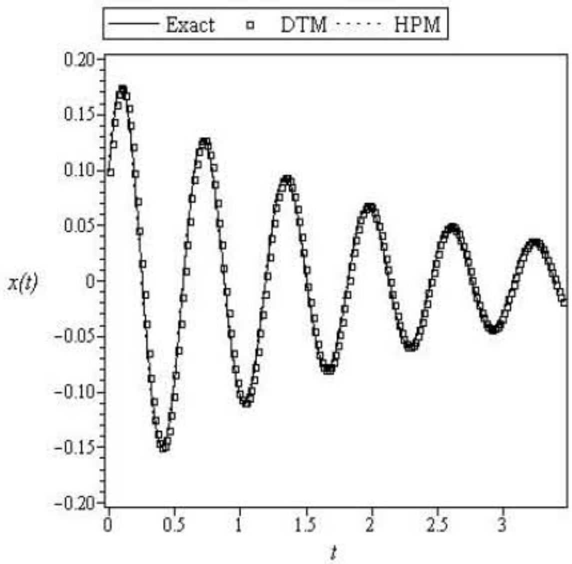

Fig. 1: The comparison of the results of the HPM, DTM and Exact solution for A=0.1

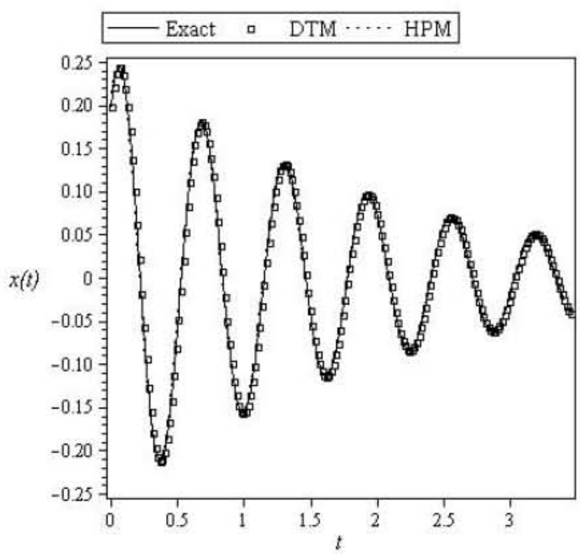

Fig. 2: The comparison of the results of the HPM, DTM and Exact solution for A=0.2

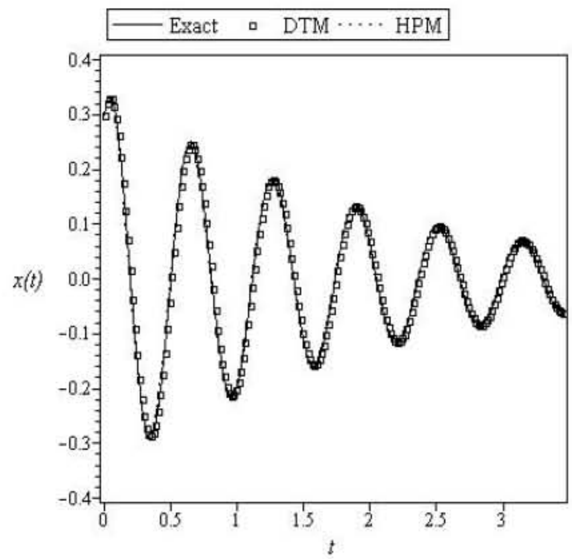

Fig. 3: The comparison of the results of the HPM, DTM and Exact solution for A=0.3

features of the DTM can be summarized as the present method reduces the computational difficulties of the other methods and all the calculations can be made simple. By using sub domain technique the accuracy of the method is very good. DTM can be used for both linear and nonlinear differential equations. It is observed that DTM is a robust and powerful tool for solving the nonlinear equations with high nonlinearity.

CONCLUSION

In this paper, we presented the definition and operation of one dimensional Differential Transformation Method (DTM) and Homotopy Perturbation Method (HPM). We had shown that the homotopy perturbation method and differential transformation method can be used successfully for finding the displacement of nonlinear oscillator problem. The comparison of the results between DTM, HPM and exact solution were shown a very interesting agreement which confirms the validity and high accuracy of the DTM. It revealed that these techniques are very powerful and efficient in finding the numerical and analytical solutions for predicting the solution of such problem.

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