World Applied Sciences Journal 7 (Special Issue of Computer & IT): 161-167, 2009
ISSN 1818.4952

Robust Control of Electrical Manipulators by Reducing the Effects of Uncertainties

Mohammad Mehdi Fateh

Department of Electrical and Robotic Engineering, Shahrood University of Technology, Shahrood, Iran

Abstract: This paper presents a novel robust voltage-based control approach for tracking control of electrical manipulators by reducing the effects of uncertainties. The proposed control law is simple, fast response and robust with acceptable tracking error and without chattering problem. This approach is superior to torque-based control approaches due to considering the role of actuators. The stability of control system is analyzed. Simulations are presented to confirm the performance of control approaches applied on a two-link manipulator driven by permanent magnet dc motors.

Key words: Chattering . electrical manipulators . robust control . stability . tracking . uncertainties

INTRODUCTION

Perfect tracking control is a desired goal in many robotic applications while robot manipulators are nonlinear, highly coupled, multivariable, multi-input/multi-output with uncertainties. The uncertainties are due to parametric errors, un-modeled dynamics and external disturbances. We can see the art of control in literature to overcome uncertainties, nonlinearities and couplings from different aspects in the robust control of robot manipulators as surveyed in [1-4]. Majority of developed approaches deserve advantages such as analytic presentation due to applying Lyapunov method [5] to propose the control laws for stability purposes in the presence of uncertainties. Despite their achievement of high grade in theory, however, there are some practical problems.

The control laws may involve problems such as system limits, sensing requirements, actuator saturation, chattering and long processing time. Moreover, the control laws were frequently presented as torque control commands while we cannot apply them directly to the inputs of electrical manipulators, which are driven by electrical motors. The torque-based approaches ignore dynamics of actuators and drives while actuators show uncertainties. Consequently, the voltage control laws are preferred for electrically-driven manipulators [6, 7].

Robust control approaches in the case of wide range of uncertainties can present the uniform bounded error convergence. This is a result of uniform ultimate boundedness (u.u.b.) of the tracking error using the Lyapunov based theory of guaranteed stability of uncertain system [8, 9]. The u.u.b. of tracking error will not result in a perfect tracking performance as compared with asymptotically stability. However, a zero tracking error cannot be achievable in practice due to the mechanical resolution of system. Therefore, the u.u.b. of tracking error may provide a desired tracking error in the operating range of manipulator.

Chattering is a side effect of using the switching control laws, which may degrade the performance of control system by exciting un-modeled dynamics. In practice, a chattering control signal results due to non-zero switching delays for providing solutions to the closed loop differential equations on tracking error [10]. The Lyapunov based theory of guaranteed stability of uncertain system is then used for avoiding chattering and thus, providing a continuous control law. This yields u.u.b. of the tracking error.

In this paper, a robust control approach is developed based on reducing the effects of uncertainties in the closed loop control system. The proposed robust control is applied on an electrically-driven manipulator. The stability is analyzed and simulations are presented to confirm the results.

This paper is organized as follows. Next section presents the dynamics of an electrically-driven manipulator. Then the robust control approach is formulated. After that, the paper introduces a robust control approach by reducing the effects of uncertainties. Next section presents the simulation results and the final section concludes the paper.

MANIPULATOR DYNAMICS

An electrical manipulator is driven by electrical motors and thus the control laws are applied on motors

Corresponding Author: Dr. Mohammad Mehdi Fateh, Department of Electrical and Robotic Engineering, Shahrood University of Technology, Shahrood, Iran

to control the manipulator. Therefore, it is more realistic to consider the manipulator dynamics including the motors. The dynamical model of dc motors was included in the dynamics of an electrically driven rigid manipulator for tracking control [11]. The manipulator dynamics [12] is given by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \quad (1)$$

where $q \in R^n$ is a vector of generalized joint positions, $D(q) \in R^{n\times n}$ is the inertia matrix, $C(q,\dot{q})\dot{q} \in R^n$ is a vector of centripetal and Coriolis generalized forces, $g(q) \in R^n$ is a vector of generalized gravitational forces and $\tau \in R^n$ is a vector of generalized joint forces. In order to deserve a high controllability for tracking purpose, a manipulator can be driven by dc motors. Based on the motion equation of permanent magnet dc motor, the matrix form of dynamic equation for the motors maybe expressed as

$$J_m\ddot{\theta}_m + (B_m + R^{-1}K_bK_m)\dot{\theta}_m = R^{-1}K_m V - r\tau \quad (2)$$

where $V \in R^n$ is a vector of motor voltages, $\theta_m \in R^n$ is a vector of motor angles and $K_m, K_b, R, J_m, B_m, r \in R^{n\times n}$ are constant diagonal matrices of torque constant, back emf constant, resistance, inertia, damping and reduction gear ratio of motors, respectively. In this kind of motor $K_m = K_b$. The joint angle vector q is related to the motor angle vector θ through the gear reduction ratio as

$$q = r\theta_m \quad (3)$$

Substituting (3) and (2) into (1) yields dynamics of robot including the actuators as follows:

$$\begin{aligned} &RK_m^{-1}\left(J_m r^{-1} + rD(q)\right)\ddot{q} + RK_m^{-1}rg(q) + \\ &\left(RK_m^{-1}B_m r^{-1} + RK_m^{-1}rC(q,\dot{q}) + K_b r^{-1}\right)\dot{q} = V \end{aligned} \quad (4)$$

A simple presentation of (4) is

$$M(q)\ddot{q} + N(q,\dot{q})\dot{q} + W(q) = V \quad (5)$$

$$M(q) = RK_m^{-1}\left(J_m r^{-1} + rD(q)\right) \quad (6)$$

$$N(q,\dot{q}) = RK_m^{-1}B_m r^{-1} + RK_m^{-1}rC(q,\dot{q}) + K_b r^{-1} \quad (7)$$

$$W(q) = RK_m^{-1}rg(q) \quad (8)$$

In order to include the unmodeled dynamics and external disturbances, we may write the electrical manipulator dynamics in the form of

$$M(q)\ddot{q} + H(q,\dot{q}) = V \quad (9)$$

$$H(q,\dot{q}) = N(q,\dot{q})\dot{q} + W(q) + d \quad (10)$$

d stands for unmodeled dynamics and external disturbances. Based on the nominal model of electrical manipulator and due to presence of uncertainties, we can write the dynamics in the form of

$$\hat{M}(q)\ddot{q} + \hat{H}(q,\dot{q}) + V_d = V \quad (11)$$

where $\hat{M}(q)$ and $\hat{H}(q,\dot{q})$ are the nominal terms to show $M(q)$ and $H(q,\dot{q})$, respectively as in which $\hat{H}(q,\dot{q}) = \hat{N}(q,\dot{q})\dot{q} + \hat{W}(q)$. The time variant unknown function V_d includes all uncertainties to satisfy (11). It is noted that the nominal terms $\hat{M}(q)$, $\hat{N}(q,\dot{q})$ and $\hat{W}(q)$ involve parametric errors but they are in the same structure as M (q), $N(q,\dot{q})$ and W (q). Substituting (11) into (9) yields

$$(M(q) - \hat{M}(q))\ddot{q} + (H(q,\dot{q}) - \hat{H}(q,\dot{q})) = V_d \quad (12)$$

The uncertainties V_d is a function of robot dynamics as expressed by (12).

ROBUST CONTROL

To cancel the known nonlinear terms, a nonlinear control law can be proposed as

$$\hat{M}(q)u + \hat{H}(q,\dot{q}) = V \quad (13)$$

Substituting (13) into (11), yields

$$\ddot{q} + \hat{M}(q)^{-1}V_d = u \quad (14)$$

Equation (14) shows a new system with input u. For tracking purposes, a control law is then proposed as

$$\ddot{q}_d + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) + u_r = u \quad (15)$$

where q_d is the desired trajectory in the joint space and u_r is designed to overcome uncertainties. The positive diagonal matrixes K_p and K_d are selected in order to regulate the system responses. Figure 1 shows a block diagram of control system.

Substituting (15) into (14) obtains the closed loop control system as follows:

$$\ddot{q}_d - \ddot{q} + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q) = \lambda - u_r \quad (16)$$

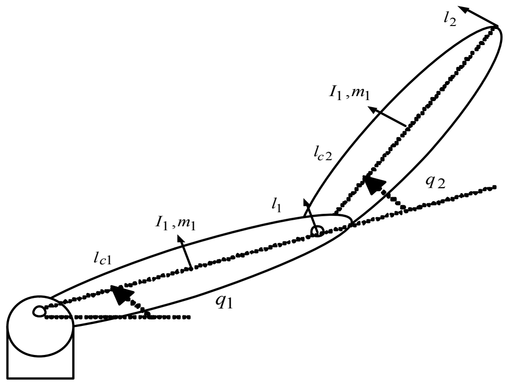

Fig. 1: Two-link elbow manipulator

We present tracking error by $e = q_d - q$, then the dynamics of tracking error expressed by (16) is formed as

$$\ddot{e} + K_d\dot{e} + K_p e = \lambda - u_r \quad (17)$$

The role of robust control law given by u_r is presented by (17). If we remove the robust controller, then uncertainties influence the dynamics of tracking error as follows:

$$\ddot{e} + K_d\dot{e} + K_p e = \lambda \quad (18)$$

The direct method of Lyapunov is then applied to propose a robust control law. The state space form of (17) is

$$\dot{x} = Ax + Bu_s \quad (19)$$

where x is the state vector, u_s is the input vector, A is the state matrix and B is the coefficient matrix of inputs. The details are:

$$x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}, A = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (20)$$

$$u_s = \lambda - u_r \quad (21)$$

A Lyapunov candidate is suggested as

$$F(x) = x^T P x \quad \text{for} \quad x \neq o, \quad F(0) = 0 \quad (22)$$

where F is the Lyapunov candidate and P is a positive definite symmetric matrix. Then the time derivative of F is given by

$$\begin{aligned}\dot{F} &= x^T(A^TP + PA)x + 2x^TPBu_s \\ &= -x^TQx + 2x^TPBu_s\end{aligned} \quad (23)$$

with A is Hurwitz and given a positive definite symmetric matrix Q, the positive definite symmetric matrix P is calculated from the Riccati equation as follows:

$$A^TP + PA + Q = 0 \quad (24)$$

The system is asymptotically stable if the time derivative of Lyapunov candidate $\dot{F}$ is less than zero. Thus, it is required that

$$x^TPBu_s \leq 0 \quad (25)$$

Substituting (21) into (25) yields

$$x^TPB\lambda \leq x^TPBu_r \quad (26)$$

Since

$$x^TPB\lambda \leq \|x^TPB\|.\|\lambda\| \quad (27)$$

Then to satisfy (26), the robust control law u_r can be given by

$$\|\lambda\| B^TPx/\|x^TPB\| \leq u_r \quad (28)$$

We can propose

$$\phi B^TPx/\|x^TPB\| = u_r \quad (29)$$

where φ is a the upper bound of λ defined by $\|\lambda\| \leq \varphi$. However, the control law (29) is not defined at $\|x^TPB\| = 0$. Thus, chattering problem is caused on the surface of $x^TPB = 0$. Therefore, the control law is modified as

$$\begin{aligned} u_r &= \phi B^TPx/\|x^TPB\| \quad &\text{for} \quad \|x^TPB\| \geq \varepsilon \\ u_r &= \phi B^TPx/\varepsilon \quad &\text{for} \quad \|x^TPB\| < \varepsilon \end{aligned} \quad (30)$$

where ε is a small positive constant value. Then, the stability is uniform ultimate boundedness [3, 10]. However, finding a suitable scalar function for φ is a hard task. It is derived based on applying bounding functions with a prior knowledge about the model of manipulator. The difficulty will arise when the manipulator has multi degrees of freedom. Moreover, it is required to protect motors from over voltages. Therefore, the robust control law should be limited and thus, may cause the instability.

REDUCING EFFECTS OF UNCERTAINTIES

By considering (17), a simple robust control law can be proposed as:

$$u_r = k_r(\ddot{e} + K_d\dot{e} + K_p e) \quad (31)$$

where u_r is the new control law and k_r is a positive constant. Substituting (31) into (30), yields

$$\ddot{e} + K_d\dot{e} + K_p e = (k_r + 1)^{-1}\lambda \quad (32)$$

Influence of uncertainties on tracking error is reduced significantly by factor $(k_r+1)^{-1}$ in (32) such that $(k_r+1)^{-1}$ becomes very small by selecting a large k_r. The effect of uncertainties on the closed loop system has been reduced if we compare the dynamics of tracking error in (18) with (32). If λ is a bounded function then $(k_r+1)^{-1}\lambda$ can be assumed as a limited input to the system (32). By considering the fact that attenuating the input to a stable system attenuates the output, the tracking error e as the output is reduced due to given $(k_r+1)^{-1}\lambda$ as the input to equation (32). As a result, selecting a large value of k_r can reduce the input $(k_r+1)^{-1}\lambda$ significantly.

The direct method of Lyapunov is applied to study the system stability. The algorithm of studying stability is the same as before via (19) to (25) where the state equation is

$$\dot{x} = Ax + Bu_d \quad (33)$$

$$u_d = (k_r + 1)^{-1}\lambda \quad (34)$$

Thus, asymptotically stability is provided if

$$2(k_r + 1)^{-1} x^T PB\lambda < x^T Qx \quad (35)$$

For Q = I where I is the identity matrix, we have

$$\|x^T Qx\| = \|x\|^2 \quad (36)$$

With $\|B\| = 1$ from (35), we may write

$$2(k_r + 1)^{-1}\|x\|.\|P\|.\|\lambda\| < \|x\|^2 \quad (37)$$

Therefore, we should select k_r as follows:

$$2\|P\|.\|\lambda\|/\|x\| - 1 < k_r \quad \text{for} \quad \|x\| \neq 0 \quad (38)$$

with $\|\lambda\| \le \phi$, in order to establish asymptotically stability, k_r can be selected as

$$2\phi\|P\|/\|x\| = k_r \quad \text{for} \quad \|x\| \neq 0 \quad (39)$$

In order to avoid chattering by selecting a proper small value of σ then (39) is modified as

$$\begin{aligned} 2\phi\|P\|/\|x\| &= k_r \quad \text{for} \quad \|x\| \ge \sigma \\ 2\phi\|P\|/\sigma &= k_r \quad \text{for} \quad \|x\| < \sigma \end{aligned} \quad (40)$$

The tracking error becomes u.u.b. [8, 9]. A prior consideration on $2\phi\|P\|$ will simplify the control law as

$$\begin{aligned} \beta/\|x\| &= k_r \quad \text{for} \quad \|x\| \ge \sigma \\ \beta/\sigma &= k_r \quad \text{for} \quad \|x\| < \sigma \end{aligned} \quad (41)$$

where β = Sup ($2\phi\|P\|$). The proposed robust approach has these advantages:

- It is simple, fast response, robust and easy to implement.
- It yields desired norm of tracking error without chattering problem.

Alternatively, simply we can choose a large constant value for k_r. Since system is asymptotically stable using (38), $\|x\|$ reduces as time goes to infinity. Consequently the value of $2\|P\|.\|\lambda\|/\|x\|$ will be large enough to obtain $2\|P\|.\|\lambda\|/\|x\| - 1 > k_r$. Thus, (38) is failed and system will be unstable. Then $\|x\|$ becomes larger, which yields $2\|P\|.\|\lambda\|/\|x\| - 1 < k_r$. This routine obtains

$$2\|P\|.\|\lambda\|/\|x\| - 1 = k_r \quad \text{for} \quad \|x\| \neq 0 \quad (42)$$

Thus, system stays on the boundary of stability circled with a limited value of $\|x\|$ given by

$$2\|P\|.\|\lambda\|/(k_r + 1) = \|x\| \quad (43)$$

Therefore, given a large value to k_r in (43) yields a small value of $\|x\|$. Alternatively, k_r is determined from (43) by predetermined value for $\|x\|$.

SIMULATIONS

We simulate the proposed control laws applied on a two-link elbow manipulator operating in a vertical surface. The details are [12]:

$$D(q) = \begin{bmatrix} m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)) + I_1 + I_2 & m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2 \\ m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_{c2}\dot{q}_2 \sin(q_2) & -m_2 l_1 l_{c2}(\dot{q}_1 + \dot{q}_2)\sin(q_2) \\ m_2 l_1 l_{c2}\dot{q}_1 \sin(q_2) & 0 \end{bmatrix}$$

Table 1: Parameters of links

Link	l	l_c	m	,
1	1	0.5	15	5
2	1	0.5	6	2

Table 2: Parameters of dc servo motors

Motor	R	K_m	J_m	B_m	1/r
1	1.6	0.26	0.0002	0.000817	107.8200
2	1.6	0.26	0.0002	0.00138	53.7063

Motors are 40V

$$G(q)=\begin{bmatrix}(m_1 l_{c1}+m_2 l_1)g\cos(q)+m_2 l_{c2} g\cos(q_1+q_2)\\ m_2 l_{c2} g\cos(q_1+q_2)\end{bmatrix} \quad (44)$$

where q_i for i = 1, 2 denotes the joint angle, l_i is the link length, m_i is the link mass, I_i is the link's moment of inertia given in center of mass, l_{ci} is the distance between the center of mass of link and the ith joint as shown in Fig. 2. The details are:

$$M=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}$$

$$M_{11}=\frac{R_1}{K_{m1}}\left(\frac{J_{m1}}{r_1}+r_1(m_1 l_{c1}^2+m_2(l_1^2+l_{c2}^2+2l_1 l_{c2}\cos(q_2))+I_1+I_2)\right)$$

$$M_{12}=M_{21}=\frac{R_1 r_1}{K_{m1}}(m_2(l_{c2}^2+l_1 l_{c2}\cos(q_2))+I_2) \quad (45)$$

$$M_{22}=\frac{R_2}{K_{m2}}\left(\frac{J_{m2}}{r_2}+r_2(m_2 l_{c2}^2+I_2)\right)$$

$$N=\begin{bmatrix}N_{11} & N_{12}\\ N_{21} & N_{22}\end{bmatrix}$$

$$N_{11}=\frac{R_1 B_{m1}}{r_1 K_{m1}}-\frac{R_1 r_1 m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2}{K_{m1}}+\frac{K_{m1}}{r_1}$$

$$N_{12}=\frac{R_1 r_1 m_2 l_1 l_{c2}\sin(q_2)(\dot{q}_1+\dot{q}_2)}{K_{m1}} \quad 46)$$

$$N_{21}=\frac{R_2 r_2 m_2 l_1 l_{c2}\sin(q_2)\dot{q}_1}{K_{m2}}$$

$$N_{22}=\frac{R_2 B_{m2}}{r_2 K_{m2}}+\frac{K_{m2}}{r_2}$$

$$W=\begin{bmatrix}\dfrac{R_1 r_1 g((m_1 l_{c1}+m_2 l_1)\cos(q)+m_2 l_{c2}\cos(q_1+q_2))}{K_{m1}}\\ \dfrac{R_2 r_2 m_2 l_{c2} g\cos(q_1+q_2)}{K_{m2}}\end{bmatrix} \quad (47)$$

The desired trajectory of joint angles is shown in Fig. 2. The initial tracking error is given zero. Parameters of manipulator and motors are given in Table 1 and 2, respectively. The nominal terms set as

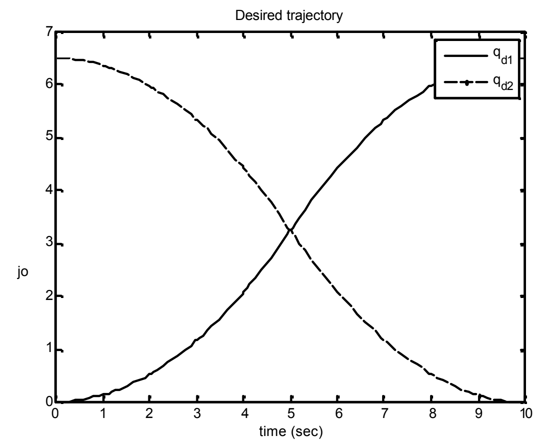

Fig. 2: The desired trajectory

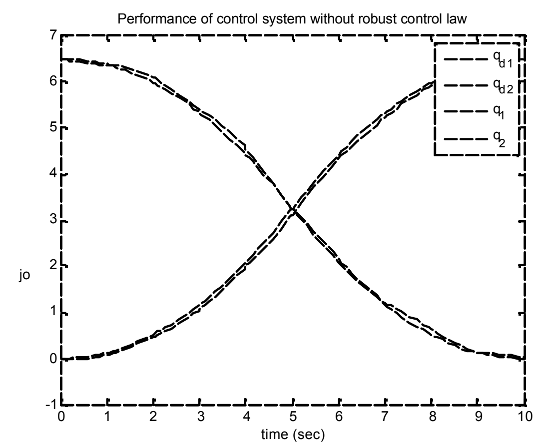

Fig. 3: Performance of controller

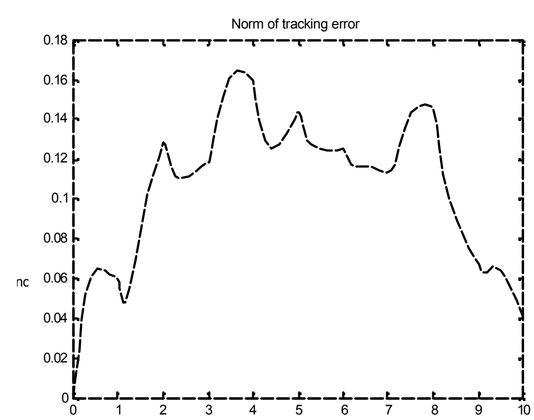

Fig. 4: Norm of tracking error

$\hat{M}=0.8M$ and $\hat{H}=0.8(N(q,\dot{q})+W(q))$. A pulse function is given by extemal disturbance d with a period of 2sec and amplitude 4V. Controller coefficients are selected $K_p = 100I$ and $K_d = 20I$.

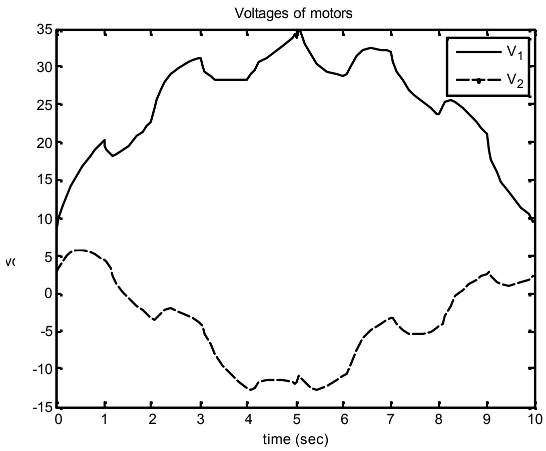

Fig. 5: Voltages of motors

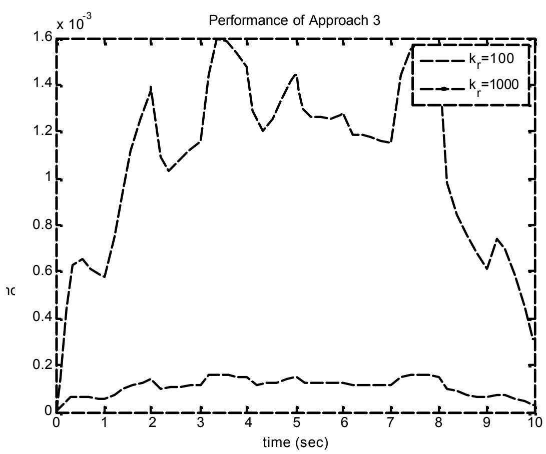

Fig. 6: Performance of novel approach

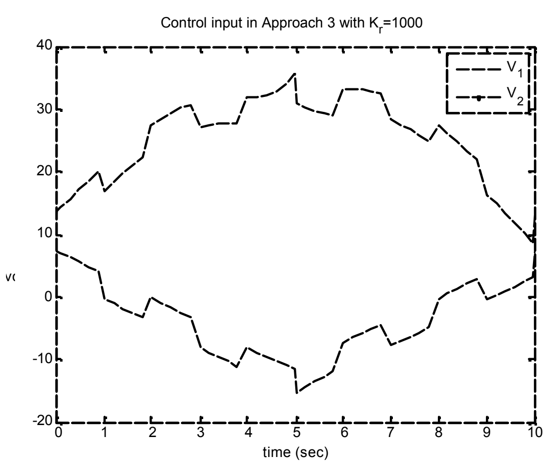

Fig. 7: Control input in novel approach

Simulation 1: This simulation shows the effect of uncertainties on system responses. The robust control is removed from the control system. The control law is given by (13) and (15) with the robust control law $u_r = 0$. The performance of tracking control is shown in Fig. 3 with a maximum norm of tracking error 0.164 rad as shown in Fig. 4. Voltages of motors are under the permitted values as shown in Fig. 6. In this case, (18) describes the dynamics of error.

Simulation 2: Robust control by reducing the effects of uncertainties is simulated using (13), (15), (31) and a constant k_r. First of all, we consider the role of k_r on the performance of control system as shown in Fig. 7. k_r is given 100 and 1000. It is confirmed that a larger value of k_r results in a smaller norm of tracking error, as its maximum value is decreased from 1.6E – 3 rad to 1.6E – 4 rad. The control input or voltages of motors are in the permitted values without chattering problem as shown in Fig. 8.

CONCLUSION

A novel robust control approach was introduced for tracking control of electrically driven manipulators by reducing the effects of uncertainties. The stability of proposed control system was analyzed with presence of both structured and unstructured uncertainties and results were confirmed by simulations. It can be concluded that the proposed control law is simple, fast response and robust with acceptable tracking error and without chattering problem.

REFERENCES

1. Abdallah, C., D. Dawson, P. Dorato and M. Jamshidi, 1991. Survey of robust control for rigid robots. IEEE Control Systems Magazine, 11: 24-30.
2. Corless, M.J., 1993. Control of uncertain nonlinear systems. ASME Transactions Journal of Dynamic Systems and Measurement Control, 115 (2B): 362-372.
3. Qu, Z. and D.M. Dawson, 1996. Robust tracking control of robot manipulators, IEEE Press, Inc., New York.
4. Sage, H.G., M.F. De Mathelin and E. Ostertag, 1999. Robust control of robot manipulators: A survey, International Journal of Control, 72 (16): 1498-1522.
5. Slotine, J.J.E. and W. Li, 1991. Applied nonlinear control, Perints Hall International, Inc.
6. Liu, C., C.C. Cheah and J.J.E. Slotine, 2006. Adaptive Jacobian tracking control of rigid-link electrically driven robots based on visual task-space information, Automatica, 42: 491-1501.

7. Fateh, M.M., 2008. On the Voltage-Based Control of Robot Manipulators. International Journal of Control, Automation and system, 6 (5): 702-712.
8. Corless, M. and G. Leitmann, 1981. Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamics systems. IEEE Transactions on Automatic Control, 26: 1139-1144.
9. Leitmann, G., 1981. On the efficacy of nonlinear control in uncertain linear systems. ASME Transactions Journal of Dynamic Systems and Measurement Control, pp: 95-102.
10. Spong, M.W., 1996. Motion control of robot manipulators. In Handbook of Control, CRC Press, pp: 1339-1350.
11. Dawson, D.M., Z. Qu and J.J. Carroll, 1992. Tracking control of rigid link electrically-driven robot manipulators. International Journal of Control, 56: 991-1006.
12. Spong, M.W., S. Hutchinson and M. Vidyasagar, 2006. Robot Modelling and Control, John Wiley and Sons, Inc.