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Solving Non-linear Fredholm Integro-differential Equations

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Abstract: In this paper, Semi-orthogonal (SO) B-spline scaling functions and wavelets and their dual functions are presented to approximate the solutions of linear and non-linear second order Fredholm integro-differential equations. The B-spline scaling functions and wavelets, their properties and the operational matrices of derivative for this functions are presented to reduce the solution of linear and non-linear Fredholm integro-differential equations to the solution of algebraic equations.

Key words: Semi-orthogonal . B-spline wavelet . Non-linear integro-differential equation

INTRODUCTION

Integral equations occur naturally in many fields of mechanics and mathematical physics. They also arise as representation formulas for the solutions of differential equations. Indeed, a differential equation can be replaced by an integral equation which incorporates its boundary conditions. As such, each solution of the integral equation automatically satisfies these boundary conditions. Integral equations also form one of the most useful tools in many branches of pure analysis, such as the theories of functional analysis and stochastic processes. Many physical problems which are usually solved by differential equation methods can be solved more effectively by integral equation methods [7].

The concept of wavelet analysis has been in place in one form or another since the beginning of the twentieth century. However in its present form, wavelet theory attracted attention in the 1980s through the work of several researchers from various disciplines. In application to discrete data sets, wavelets may be considered as basis functions generated by dilations and translations of a single function. A reason for the popularity of wavelet is its effectiveness in representation of non-stationary signals [2].

In the present paper, we develop a non-orthogonal (semi-orthogonal) wavelet using B-splines specially constructed for the bounded interval, this wavelet can be represented in a closed-form. In addition, no any orthogonality is imposed on the basis. This provides a compact support [3].

These wavelets satisfy all the properties on a bounded interval that are satisfied by the usual wavelets on the real line. We have chosen the SO wavelets primarily because of the following reasons:

- Unlike most of the continuous ON wavelets, compactly supported SO spline wavelets have closed-form expressions.
- These wavelets are symmetric and hence have generalized linear phase, an important factor for reconstructing the function. It is well known that symmetric or antisymmetric, real-valued, continuous and compactly supported ON scaling functions and wavelets does not exist.
- The higher the smoothness of the wavelets, the larger are their supports in time (space). The order of vanishing moments usually increases with smoothness.
- The fast wavelet transform algorithms that are available are for the octave-scales only.
- Because of the "total positivity" properties of splines, they have certain very desirable properties from an approximation points of view [1].

Integro-differential equations have gained a lot of interest in many application fields, such as biological, physical and engineering problems. Therefore, their numerical treatment is desired. Goswami *et al*. [1] used wavelet on bounded interval to solve the integral equations, Lakestani *et al*. [6] used spline wavelets to solve the integro-differentail equations, also Nevles *et al*. [8] used orthogonal wavelets to solve the integral equations, Chrysafinos [14] used wavelet-Galerkin method or integro-differential equations, Abbasa *et al*. [15] applied multiwavelet direct method for solving integro-differential equations. Furthermore other authors used different methods for solving integro-differential equations [9-13, 16-18].

Consider the linear second-order Fredholm integro-differential of the form

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$$\sum_{i=0}^{2}\mu_i(x)y^{(i)}(x)=g(x)+\int_0^1 K(x,t)y(t)dt, \qquad 0\le x\le 1 \qquad (1)$$

and the non-linear second-order Fredholm integro-differential of the form

$$\sum_{i=0}^{2}\mu_i(x)y^{(i)}(x)=f\left(x,y(x),\int_0^1 K(x,t,y(t))dt\right), \quad 0\le x\le 1 \quad (2)$$

with

$$y(0)=y_0, \quad y(1)=y_1 \qquad (3)$$

where μ_i ($i = 0, 1, 2$), g, f and K are given functions in $L^2[0,1]$, y_0 and y_1 are given real numbers and y is the unknown function to be found [6].

B-SPLINE SCALING FUNCTIONS AND WAVELETS ON [0,1]

The proposed non-orthogonal wavelets form the basis in the space $L^2(R)$. Using these basis, an arbitrary function in $L^2(R)$ can be expressed as the wavelet series. On the other hand, for an interval, one cannot complete the wavelet series by using only these basis. This is because supports of some basis are truncated at the left or right endpoint of the interval. Hence, special basis have to be introduced into the wavelet expansion on the finite interval. These functions are referred to as the boundary scaling function and the boundary wavelet.

When semi-orthogonal wavelets are constructed from B-spline of order m, the lowest octave level $j = j_0$ is determined in [1] by

$$2^{j_0} \ge 2m-1 \qquad (4)$$

so as to give a minimum of one complete wavelet on the interval [0,1]. In this paper, we will use a wavelet generated by a quadratic B-spline. From (4), the third-order B-spline lowest level, which must be an integer, is determined to $j_0 = 3$. This constrains all octave levels to $j_0 \ge 3$.

As in the case with all semi-orthogonal wavelets, the third-order B-spline also serve as scaling functions. The third-order B-spline scaling functions are given by

$$\phi_{j,k}(x)=\begin{cases}\frac{1}{2}(x_j-k)^2, & k\le x_j\le k+1;\\ \frac{3}{4}-((x_j-k)-\frac{3}{2})^2, & k+1\le x_j\le k+2;\\ \frac{1}{2}((x_j-k)-3)^2, & k+2\le x_j<k+3,\ k=0,\ldots,2^j-3\\ 0, & \text{otherwise}\end{cases} \qquad (5)$$

with the respective left and right hand side boundary scaling functions

$$\phi_{j,k}(x)=\begin{cases}\frac{1}{2}(x_j-k)^2, & 0\le x_j\le 1;\quad k=-2\\ 0, & \text{otherwise}\end{cases} \qquad (6)$$

$$\phi_{j,k}(x)=\begin{cases}\frac{3}{4}-((x_j-k)-\frac{3}{2})^2, & k+1\le x_j\le k+2;\\ \frac{1}{2}((x_j-k)-3)^2, & k+2\le x_j\le k+3,\ k=-1\\ 0, & \text{otherwise}\end{cases} \qquad (7)$$

$$\phi_{j,k}(x)=\begin{cases}\frac{1}{2}(x_j-k)^2, & k\le x_j\le k+1;\\ \frac{3}{4}-((x_j-k)-\frac{3}{2})^2, & k+1\le x_j\le k+2;\quad k=2^j-2\\ 0, & \text{otherwise}\end{cases} \qquad (8)$$

$$\phi_{j,k}(x)=\begin{cases}\frac{1}{2}((x_j-k)-3)^2, & k+2\le x_j\le k+3,\ k=2^j-1\\ 0, & \text{otherwise.}\end{cases} \qquad (9)$$

The actual coordinate position x is related to x_j, according to $x_j = 2^j x$. The third-order B-spline wavelets are given by

$$\psi_{j,k}(x)=\frac{1}{480}\begin{cases}2(x_j-k)^2, & k\le x_j\le k+\frac{1}{2};\\ -16+64(x_j-k)-62(x_j-k)^2, & k+\frac{1}{2}\le x_j\le k+1;\\ 458-884(x_j-k)+412(x_j-k)^2, & k+1\le x_j\le k+\frac{3}{2};\\ -3286+4108(x_j-k)-1252(x_j-k)^2, & k+\frac{3}{2}\le x_j\le k+2;\\ 6(1695-1558(x_j-k)+352(x_j-k)^2), & k+2\le x_j\le k+\frac{5}{2};\\ -6(2705-1962(x_j-k)+352(x_j-k)^2), & k+\frac{5}{2}\le x_j\le k+3;\\ 2(7023-4206(x_j-k)+626(x_j-k)^2), & k+3\le x_j\le k+\frac{7}{2};\\ -6338+3236(x_j-k)-412(x_j-k)^2, & k+\frac{7}{2}\le x_j\le k+4;\\ 1246-556(x_j-k)+62(x_j-k)^2, & k+4\le x_j\le k+\frac{9}{2};\\ -2((x_j-k)-5)^2, & k+\frac{9}{2}\le x_j\le k+5.\end{cases} \qquad (10)$$

for $k = 0,1,\ldots,2^j-5$ and the respective left and right hand side boundary wavelets are:

$$\psi_{j,k}(x)=\frac{1}{480}\begin{cases} 6(1695-1558(x_j-k)+352(x_j-k)^2), & k+2\le x_j\le k+\frac{5}{2};\\ -6(2705-1962(x_j-k)+352(x_j-k)^2), & k+\frac{5}{2}\le x_j\le k+3;\\ 2(7023-4206(x_j-k)+626(x_j-k)^2), & k+3\le x_j\le k+\frac{7}{2};\\ -6338+3236(x_j-k)-412(x_j-k)^2, & k+\frac{7}{2}\le x_j\le k+4;\\ 1246-556(x_j-k)+62(x_j-k)^2, & k+4\le x_j\le k+\frac{9}{2};\\ -2((x_j-k)-5)^2, & k+\frac{9}{2}\le x_j\le k+5. \end{cases} \tag{11}$$

for k = -2,

$$\psi_{j,k}(x)=\frac{1}{480}\begin{cases} 458-884(x_j-k)+412(x_j-k)^2, & k+1\le x_j\le k+\frac{3}{2};\\ -3286+4108(x_j-k)-1252(x_j-k)^2, & k+\frac{3}{2}\le x_j\le k+2;\\ 6(1695-1558(x_j-k)+352(x_j-k)^2), & k+2\le x_j\le k+\frac{5}{2};\\ -6(2705-1962(x_j-k)+352(x_j-k)^2), & k+\frac{5}{2}\le x_j\le k+3;\\ 2(7023-4206(x_j-k)+626(x_j-k)^2), & k+3\le x_j\le k+\frac{7}{2};\\ -6338+3236(x_j-k)-412(x_j-k)^2, & k+\frac{7}{2}\le x_j\le k+4;\\ 1246-556(x_j-k)+62(x_j-k)^2, & k+4\le x_j\le k+\frac{9}{2};\\ -2((x_j-k)-5)^2, & k+\frac{9}{2}\le x_j\le k+5. \end{cases} \tag{12}$$

for k = -1,

$$\psi_{j,k}(x)=\frac{1}{480}\begin{cases} 2(x_j-k)^2, & k\le x_j\le k+\frac{1}{2};\\ -16+64(x_j-k)-62(x_j-k)^2, & k+\frac{1}{2}\le x_j\le k+1;\\ 458-884(x_j-k)+412(x_j-k)^2, & k+1\le x_j\le k+\frac{3}{2};\\ -3286+4108(x_j-k)-1252(x_j-k)^2, & k+\frac{3}{2}\le x_j\le k+2;\\ 6(1695-1558(x_j-k)+352(x_j-k)^2), & k+2\le x_j\le k+\frac{5}{2};\\ -6(2705-1962(x_j-k)+352(x_j-k)^2), & k+\frac{5}{2}\le x_j\le k+3;\\ 2(7023-4206(x_j-k)+626(x_j-k)^2), & k+3\le x_j\le k+\frac{7}{2};\\ -6338+3236(x_j-k)-412(x_j-k)^2, & k+\frac{7}{2}\le x_j\le k+4; \end{cases} \tag{13}$$

for $k = 2^j-4$ and

$$\psi_{j,k}(x)=\frac{1}{480}\begin{cases} 2(x_j-k)^2, & k\le x_j\le k+\frac{1}{2}; \\ -16+64(x_j-k)-62(x_j-k)^2, & k+\frac{1}{2}\le x_j\le k+1; \\ 458-884(x_j-k)+412(x_j-k)^2, & k+1\le x_j\le k+\frac{3}{2}; \\ -3286+4108(x_j-k)-1252(x_j-k)^2, & k+\frac{3}{2}\le x_j\le k+2; \\ 6(1695-1558(x_j-k)+352(x_j-k)^2), & k+2\le x_j\le k+\frac{5}{2}; \\ -6(2705-1962(x_j-k)+352(x_j-k)^2), & k+\frac{5}{2}\le x_j\le k+3; \end{cases} \tag{14}$$

for $k = 2^j-3$.

The two-scale relation for quadratic B-spline $\phi(x)$ and $\psi(x)$ are

$$\phi(x)=\frac{1}{4}\phi(2x)+\frac{3}{4}\phi(2x-1)+\frac{3}{4}\phi(2x-2)+\frac{1}{4}\phi(2x-3) \tag{15}$$

$$\begin{aligned}\psi(x)=&\frac{1}{480}\phi(2x)-\frac{29}{480}\phi(2x-1)+\frac{147}{480}\phi(2x-2)-\frac{303}{480}\phi(2x-3)\\ &+\frac{303}{480}\phi(2x-4)-\frac{147}{480}\phi(2x-5)+\frac{29}{480}\phi(2x-6)-\frac{1}{480}\phi(2x-7)\end{aligned} \tag{16}$$

respectively [4].

Function approximation using scaling functions: For any fixed positive integer M, a function f (x) defined over [0,1] may be presented by B-spline scaling functions as

$$f(x)=\sum_{k=-2}^{2^M-1} s_k\phi_{M,k}=S^T\Phi_M \tag{17}$$

where

$$\begin{aligned} S &= \left[s_{-2},s_{-1},\ldots,s_{2^M-1}\right] \\ \Phi_M &= \left[\phi_{M,-2},\phi_{M,-1},\ldots,\phi_{M,2^M-1}\right] \end{aligned} \tag{18}$$

with

$$s_k=\int_0^1 f(x)\tilde{\phi}_{M,k}(x)dx,\quad k=-2,-1,\ldots,2^M-1 \tag{19}$$

where $\tilde{\phi}_{M,k}(x)$ are dual functions of $\phi_{M,k}(x)$. These can be obtained by linear combinations of $\phi_{M,k}(x), k=-2,-1,\ldots,2^M-1$ as follows. Let $\tilde{\Phi}_M$ be the dual functions of Φ_M given by

$$\tilde{\Phi}_M=\left[\tilde{\phi}_{M,-2},\tilde{\phi}_{M,-1},\ldots,\tilde{\phi}_{M,2^M-1}\right] \tag{20}$$

Using (5)-(9),(19) and (20), we get

$$\int_0^1\tilde{\Phi}_M\Phi_M^T dx = I_1 \tag{21}$$

where I_1 is $(2^M+2)\times(2^M+2)$ identity matrix. Let

$$P_M=\int_0^1\Phi_M\Phi_M^T dx \tag{22}$$

The entry $(P_M)_{i,j}$ of the matrix P_M in (22) is calculated from

$$\int_0^1\phi_{M,i}(x)\phi_{M,j}(x)dx \tag{23}$$

From (21) and (22), we get

$$\tilde{\Phi}_M=(P_M)^{-1}\Phi_M \tag{24}$$

The operational matrix of derivative: The differentiation of the vector Φ_M in (19) can be expressed as

$$\Phi'_M=D_\Phi\Phi_M \tag{25}$$

where D_Φ is $(2^M+2)\times(2^M+2)$ operational matrix for derivative of B-spline scaling functions. The matrix D_Φ can be obtained by considering

$$D_\Phi = \int_0^1 \Phi'_M(t)\tilde{\Phi}_M^T(t)dt = \left(\int_0^1 \Phi'_M(t)\Phi_M^T(t)dt\right)(P_M)^{-1} = E(P_M)^{-1} \tag{26}$$

where

$$E = \int_0^1 \Phi'_M(t)\Phi_M^T(t)dt \tag{27}$$

E is $(2^M+2)\times(2^M+2)$ matrix given by

$$E = \begin{bmatrix} \int_0^1 \phi'_{M,-2}(t)\phi_{M,-2}(t)dt & \cdots & \int_0^1 \phi'_{M,-2}(t)\phi_{M,2^M-1}(t)dt \\ \vdots & \ddots & \vdots \\ \int_0^1 \phi'_{M,2^M-1}(t)\phi_{M,-2}(t)dt & \cdots & \int_0^1 \phi'_{M,2^M-1}(t)\phi_{M,2^M-1}(t)dt \end{bmatrix} \tag{28}$$

for any entries of $E_{j,k}$ we have

$$E_{j,k} = \int_0^1 \phi'_{M,j}(t)\phi_{M,k}(t)dt = \int_{\frac{k}{2^M}}^{\frac{k+3}{2^M}} \phi'_{M,j}(t)\phi_{M,k}(t)dt \tag{29}$$

For example for M = 3, from (29) we get

$$E = \begin{bmatrix} -3 & -8 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -3 & -10 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 10 & 0 & -10 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 & 0 & -10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & -10 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 & -10 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 10 & 0 & -10 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 10 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 8 & 3 \end{bmatrix} \tag{30}$$

Function approximation using wavelets: Also, a function f (x) defined over [0,1] may be presented by B-spline wavelets as

$$f(x) = \sum_{k=-2}^{7} c_k\phi_{3,k}(x) + \sum_{i=3}^{\infty}\sum_{j=-2}^{2^i-3} d_{i,j}\psi_{i,j}(x) \tag{31}$$

If the infinite series in (31) is truncated at M, then (31) can be written as

$$f(x) \simeq \sum_{k=-2}^{7} c_k\phi_{3,k}(x) + \sum_{i=3}^{M}\sum_{j=-2}^{2^i-3} d_{i,j}\psi_{i,j}(x) = C^T\Psi(x) \tag{32}$$

where $\phi_{3,k}$ (x) and $\psi_{i,j}$ (x) are scaling functions and wavelets, respectively and C and Ψ are $(2^{M+1}+2)\times 1$ vectors given by

$$C = \begin{bmatrix} c_{-2}, c_{-1}, \ldots, c_7, d_{3,-2}, d_{3,-1}, \ldots, \\ d_{3,5}, \ldots, d_{M,-M+1}, \ldots, d_{M,2^M-3} \end{bmatrix}^T \tag{33}$$

$$\Psi = \begin{bmatrix} \phi_{3,-2}, \phi_{3,-1}, \ldots, \phi_{3,7}, \psi_{3,-2}, \psi_{3,-1}, \ldots, \\ \psi_{3,5}, \ldots, \psi_{M,-M+1}, \ldots, \psi_{M,2^M-3} \end{bmatrix}^T \tag{34}$$

with

$$c_k = \int_0^1 f(x)\tilde{\phi}_{3,k}(x)dx,\ k = -2, -1, \ldots, 7, \tag{35}$$

$$d_{i,j} = \int_0^1 f(x)\tilde{\psi}_{i,j}(x)dx, i = 3, 4, \ldots, M, j = -2, -1, \ldots, 2^i - 3 \tag{36}$$

where $\tilde{\phi}_{3,k}(x)$ defined before and $\tilde{\psi}_{i,j}(x)$ are dual functions of $\psi_{i,j}$ (x). These can be obtained by linear combinations of

$$\psi_{i,j}(x), i = 3, 4, \ldots, M,\ j = -2, -1, \ldots, 2^i - 3$$

as follows. Let

$$\bar{\Psi} = \left[\psi_{3,-2}(x), \psi_{3,-1}(x), \ldots, \psi_{M,2^M-3}(x)\right]^T \tag{37}$$

Using (10)-(14) and (37) we have

$$\int_0^1 \bar{\Psi}\bar{\Psi}^T dx = Q_M \tag{38}$$

where Q_M is $2^M\times 2^M$ matrix. Suppose $\tilde{\bar{\Psi}}$ is the dual function of $\bar{\Psi}$, given by

$$\tilde{\bar{\Psi}} = \left[\tilde{\psi}_{3,-2}(x), \tilde{\psi}_{3,-1}(x), \ldots, \tilde{\psi}_{M,2^M-3}(x)\right]^T \tag{39}$$

Using (10)-(14), (38) and (39) we have

$$\int_0^1 \tilde{\bar{\Psi}}\bar{\Psi}^T dx = I_2 \tag{40}$$

where I_2 is $2^M\times 2^M$ identity matrix. Then (38), (40) and (40) give

$$\tilde{\bar{\Psi}} = Q_M^{-1}\bar{\Psi} \tag{41}$$

The operational matrix of derivative using wavelets: The differentiation of the vector Ψ in (35) and can be expressed as

$$\Psi' = D_\Psi\Psi \tag{42}$$

where D_Ψ is $(2^{M+1}+2)\times(2^{M+1}+2)$ operational matrices for derivative of B-spline wavelets. The matrix D_Ψ can be obtained by considering

$$\Psi = G\Phi_{M+1} \tag{43}$$

where G is a $(2^{M+1}+2)\times(2^{M+1}+2)$ matrix, which can be calculated as follows. Let

$$\begin{aligned}\Phi_j &= [\phi_{j,-2},\phi_{j,-1},\ldots,\phi_{j,2^j-1}]^T\\ \Psi_j &= [\psi_{j,-2},\psi_{j,-1},\ldots,\psi_{j,2^j-3}]^T\end{aligned} \tag{44}$$

Using (15) and (45) we get

$$\Phi_j = \beta_j\Phi_{j+1} \tag{45}$$

where β_j, j = 3,… is $(2^j+2)\times(2^{j+1}+2)$ matrix and can be calculated from (5)-(9) and (15). From (16) and (46) we have

$$\Psi_j = L_j\Phi_{j+1} \tag{46}$$

where L_j, j = 2,3,… is $2^j\times(2^{j+1}+2)$ matrix and can be calculated from (10)-(14) and (16). Using (35) and (48) we get

$$G = \begin{bmatrix} \beta_3\times\beta_4\times\ldots\times\beta_M \\ \hline L_3\times\beta_4\times\ldots\times\beta_M \\ \hline \vdots \\ L_{M-2}\times\beta_{M-1}\times\beta_M \\ \hline L_{M-1}\beta_M \\ \hline L_M \end{bmatrix} \tag{47}$$

Using (26), (27), (43) and (49) we get

$$\Psi' = G\Phi'_{M+1} = GD_\Phi\Phi_{M+1} = GE(P_{M+1})^{-1}\Phi_{M+1} = D_\Psi\Psi \tag{48}$$

where

$$D_\Psi = GE(P_{M+1})^{-1}G^{-1} \tag{49}$$

NON-LINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

In this section we solve non-linear second order Fredholm integro-differential equations of the form in (2) with (3) by using quadratic B-spline wavelets. We first write (2) as

$$\begin{aligned}&\sum_{i=0}^{2}\mu_i(x)y^{(i)}(x)=f(x,y(x),z(x)),\\ &y(0)=y_0,\quad y(1)=y_1\quad 0\le x\le 1\end{aligned} \tag{50}$$

where

$$z(x)=\int_0^1 K(x,t,y(t))dt \tag{51}$$

We now use (32) to approximate y (x) and z (x) as

$$\begin{aligned}y(x)&=C^T\Psi(x)\\ z(x)&=\int_0^1 K(x,t,C^T\Psi(t))dt\end{aligned} \tag{52}$$

where Ψ (x) is defined in (35) and C is $(2^{M+1}+2)\times 1$ unknown vector defined similarly to C in (34). We can approximate (55), using Newton-Cotes integration techniques as:

$$\begin{aligned}z(x)&=\int_0^1 K(x,t,C^T\Psi(t))dt\\ &=\sum_{i=1}^{n}\omega_i K(x,t_i,C^T\Psi(t_i))=F(x,C)\end{aligned} \tag{53}$$

where ω_i and t_i are weight and nodes of Newton-Cotes integration techniques. Using (50) and (54) we get

$$\begin{aligned}y'(x)&=C^T\Psi'(x)=C^TD_\Psi\Psi(x)\\ y''(x)&=C^TD_\Psi^2\Psi(x)\end{aligned} \tag{54}$$

From (52), (56), (57) and (58) we get

$$\begin{aligned}&\mu_0(x)C^T\Psi(x)+\mu_1(x)C^TD_w\Psi(x)\\ &+\mu_2(x)C^TD_w^2\Psi(x)=f\left(x,C^T\Psi(x),F(x,C)\right)\end{aligned} \tag{55}$$

Also using boundary values in (3) we have

$$C^T\Psi(0)=y_0,\quad C^T\Psi(1)=y_1 \tag{56}$$

To find the solution y (x) in (52), we first collocate (59) in $x_i=(2i-1)/(2^{M+2}-1), i=1,2,\ldots,2^{M+1}$

the resulting equation generates 2^{M+1} algebraic equations. The total unknowns for vector C in (54) is $2^{M+1}+2$. These can be obtained by using (59)-(60).

Example 1:1 Consider first-order Fredholm integro-differential equation

Table 1: Absolute values of error

x_i	M = 4	M = 5	M = 6
0.125	9.4×10^{-6}	2.6×10^{-6}	7.5×10^{-7}
0.250	5.1×10^{-6}	1.0×10^{-6}	2.0×10^{-7}
0.375	3.0×10^{-5}	7.3×10^{-6}	1.8×10^{-6}
0.500	4.9×10^{-5}	1.2×10^{-5}	3.0×10^{-6}
0.625	5.5×10^{-5}	1.3×10^{-5}	3.4×10^{-6}
0.750	4.5×10^{-5}	1.1×10^{-5}	3.0×10^{-6}
0.875	2.1×10^{-5}	5.7×10^{-6}	1.6×10^{-6}

$$y'(x) = -y(x) + \int_0^1 y(t)^2 dt + \frac{1}{2}(e^{(-2)} - 1),\ 0 \le x \le 1,\ \ y(0) = 1$$

exact solution of this problem is e^{-x}.

We solved this problem with the defined method. Table 1 shows the results.

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