# Boundary-Layer Flow and Heat Transfer of a Power Law Non-Newtonian Nanofluid over a Vertical Stretching Sheet

<sup>1</sup>M.A.A. Hamad and <sup>2</sup>Muatazz Abdulhadi Bashir

<sup>1</sup>Department of Mathematics Faculty of Science, Assiut University, Assiut 71516, Egypt <sup>2</sup>School of Mathematical Sciences, Universiti Sains Malaysia, Pinang 11800, Malaysia

Abstract: The forced convection heat transfer to the power law non-Newtonian *nanofluid* from the stretching surface has been investigated numerically. This is the first paper on non-Newtonian with stretching sheet in nanofluids. The stretching of the surface velocity and the prescribed surface temperature is assumed to vary as a linear function of the distance from the origin. A similarity solution is presented which depends on nanoparticle volume fraction  $\phi$  power law index n, buoyancy convection parameter  $\lambda$  and the modified Prandtl number NPr. Velocity and Temperature variations as well as the variation of skin friction and Nusselt number with the parameters govern the problem is presented and discussed. Different types of nanoparticles are tested.

Key words: Nanofluid · Boundary layer flow · Stretching Sheet · Non-Newtonian

## INTRODUCTION

Nanofluid is a colloidal solution of nanosized solid particles in liquids. Nanofluids show anomalously high thermal conductivity in comparison to the base fluid, a fact that has drawn the interest of lots of research groups. Thermal conductivity of nanofluids depends on factors such as the nature of base fluid and nanoparticle, particle concentration, temperature of the fluid and size of the particles. Also, the nanofluids show significant change in properties such as viscosity and specific heat in comparison to the base fluid. Therefore numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro sized particle materials in liquids.

Polidori et al. [1] investigated the natural convection heat transfer of Newtonian nanofluids in a laminar external boundary-layer via the integral formalism approach. Moghadassi et al. [2] presented a model of nanofluids effective thermal conductivity, According to this model, thermal conductivity changes nonlinearly with nanoparticle loading. Jain et al. [3] used the technique of Brownian dynamic simulation coupled with the Green Kubo model has been used in order to compute the thermal conductivity of nanofluids. There have been published several recent numerical studies on the modeling of natural convection heat transfer in nanofluids.

Congedo et al. [4], Ghasemi and Aminossadati [5], Ho et al. [6, 7], Oztop and Abu-Nada [8], etc. These studies have used traditional finite difference and finite volume techniques with the tremendous call on computational resources that these techniques necessitate. A very good collection of the published papers on nanofluids can be found in the book by Das et al. [9] and in the review papers by Wang and Mujumdar [10-12] and Kakaç and Pramuanjaroenkij [13].

A number of industrially important fluids such as molten plastics, polymers, pulps, foods, etc. exhibit non-Newtonian fluid behavior. Due to the growing use of these non-Newtonian substances in various manufacturing and processing industries, considerable efforts have been directed towards understanding nanofluids heat transfer characteristic. Many of the inelastic non-Newtonian fluids encountered in chemical engineering processes, are known to follow the empirical Ostwald-de Waele model [14], or the so-called "power-law model" in which the shear stress varies according to a power function of the strain rate. Bizzele and Slattery [15] extended a simple integral method originally suggested for an isothermal flat plate by Acrivos et al. [16] to examine the momentum equation for a pseudoplastic fluid flow past an axisymmetric body. Nakayama et al. [17] suggested a general integral solution procedure for the analysis of the mixed convective heat transfer to the power-law-non-Newtonian fluids from bodies of arbitrary geometrical configuration.

Cobble [18] studied the problem of an incompressible, non-Newtonian power-law fluid lowing over flat plate under the influence of a magnetic field and a pressure gradient and with or without fluid injection or ejection. Tashtoush et al. [19] investigated heat transfer characteristics of a non-Newtonian fluid on a power-law stretching surface of variable temperature with suction or injection. Uniform surface temperature and variable surface temperature considered. Andersson Dandapat [20] extended the work of Crane [21] to non-Newtonian power law fluid over a linear stretching sheet and further extended by Hassanien et al. [22] to include heat transfer analysis. Sahu et al. [23] considered the momentum and heat transfer in a power law fluid from a continuous moving plate. A rigorous derivation and subsequent analysis of the boundary layer equations, for power law fluids provided by Denier and Dabrowski [24]. They focused on boundary layer flow driven by free stream  $U(x) = x^m$ , i.e., Falkner–Skan type.

The heat, mass and momentum transfer in the laminar boundary layer flow on a stretching sheet are important from theoretical as well as practical point of view because of their wider applications to polymer technology and metallurgy. Elbashbeshy and Bazid [25] studied flow and heat transfer in a porous medium over a stretching surface with internal heat generation and suction/blowing when the surface is kept at constant temperature.

Nazar et al. [26] studied the unsteady boundary layer flow in the region of stagnation-point on a stretching sheet. Layek et al. [27] investigated the steady twodimensional stagnation-point flow of an incompressible viscous fluid towards a porous stretching surface embedded in a porous medium subject to suction/blowing with internal heat generation or absorption. Ishak et al. [28] studied the effect of a uniform transverse magnetic field on the stagnation point flow toward a vertical stretching sheet. Raptis and Perdikis [29] investigated the effect of a chemical reaction of an electrically conducting viscous fluid on the flow over a non-linearly (quadratic) semi-infinite stretching sheet in the presence of a constant magnetic field which is normal to the sheet. Abel et al. [30] presented a mathematical analysis for the momentum and heat transfer characteristics of the boundary layer flow of an incompressible and electrically conducting viscoelastic fluid over a linear stretching sheet. The momentum boundary layer equation includes both the effect of transverse magnetic and electric fields.

In the present study, the similarity solution of two-dimensional flow and heat transfer of an incompressible viscous power-law non-Newtonian nanofluid past a vertical stretching surface is presented. The resulting ordinary differential equations are then solved numerically. The aim is to investigate the influence of various nanofluid parameter (the solid volume fraction of the nanoparticles  $\phi$ ) and the effect of power law index n on flow and heat-transfer characteristics.

#### MATERIALS AND METHODS

Problem Formulation: Consider the steady two dimensional state, mixed convection boundary layer flow of an incompressible viscous nanofluid obeying power law model. Two equal and opposite forces are introduced along the sheet so that the wall is stretched linearly whilst keeping the position of the origin fixed. The positive xcoordinate is measured along the direction of the motion, with the slot as the origin and the positive y-coordinate is measured normal to the surface of the sheet and is positive from the sheet to the fluid. Assume the velocity  $u_{\nu}(x)$  and the temperature  $T(x)_{\nu}$  at the continuous stretching surface are linear function of x, x is the distance from the slit (see eqn. 5). The temperature at ambient fluid has constant value  $T_{\infty}$ . It is further assumed that the regular fluid and the suspended nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of the nanofluid are given in Oztop and Abu-Nada [8]. Under the above assumptions, the boundary layer equations governing the flow and temperature in the presence of heat source or heat sink are (using the boundary layer approximations and neglecting viscous dissipation).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}} \left[ -\mu_{nf} \frac{\partial}{\partial y} \left( -\frac{\partial u}{\partial y} \right)^n \pm g \left( \rho \beta \right)_{nf} (T - T_{\infty}) \right]$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}$$
 (3)

Where u, v are the velocity components in the x and y directions, respectively, T is the local temperature of the fluid, n is the power law index,  $\beta$  is the heat source/sink parameter, g is the acceleration due to gravity. Further,  $\rho_{nf}$  is the effective density,  $\mu_{nf}$  is the effective dynamic viscosity,  $(\rho C_p)$  is the heat capacitance,  $(\rho \beta)_{nf}$  the thermal expansion coefficient,  $\alpha_{nf}$  is the effective thermal diffusivity and  $\kappa_{nf}$  is the effective thermal conductivity of the nanofluid, which are defined as (see Oztop and Abu-Nada [8]; Aminossadati and Ghasemi [31])

World Appl. Sci. J., 7 (Special Issue for Applied Math): 172-178, 2009

$$\rho_{nf} = (1 - \varphi) \rho_{f} + \varphi \rho_{s}, \quad \mu_{nf} = \frac{\mu_{f}}{(1 - \varphi)^{2.5}}, \quad (\rho \beta)_{nf} = (1 - \varphi)(\rho \beta)_{f} + \varphi(\rho \beta)_{s} 
(\rho C_{p})_{nf} = (1 - \varphi)(\rho C_{p})_{f} + \varphi(\rho C_{p})_{s}, \quad \alpha_{nf} = \frac{\kappa_{nf}}{(\rho C_{p})_{nf}}, 
\kappa_{nf} = \kappa_{f} \{ \frac{\kappa_{s} + 2k_{f} - 2\varphi(\kappa_{f} - \kappa_{s})}{\kappa_{s} + 2\kappa_{f} + 2\varphi(\kappa_{f} - \kappa_{s})} \}$$
(4)

Where  $\phi$  is the solid volume fraction of the nanoparticles.

The appropriate boundary conditions for the problem are given by

$$u = u_{w}(x) = b x, \quad v = 0, \quad T = T_{w}(x) = T_{\infty} + A(\frac{x}{l}) \quad \text{at} \quad y = 0$$

$$u \to 0, \quad T \to T_{\infty} \quad \text{as} \quad y \to \infty$$

$$(5)$$

Where b is the linear stretching constant, l is the characteristic length and A is a constant whose value depends on the properties of the fluid.

By introducing the following non-dimensional variables

$$\eta = \frac{y}{x} (\text{Re}_x)^{\frac{1}{n+1}}, \quad \psi = x u_w (\text{Re}_x)^{\frac{-1}{n+1}} F(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}$$
 (6)

Where  $\Psi(x, y)$  is the stream function,  $\eta$  is the similarity variable, F and  $\theta$  are the dimensionless similarity functions,  $\text{Re}_x = x_n(u_w)$  is the local Reynolds number and  $u = \partial \Psi/\partial y$ ,  $v = -\partial \Psi/\partial x$ .

Using (6), equations (1) to (3) leads to the following non-dimensional ODEs

$$\frac{n}{(1-\phi)^{2.5}} (-F'')^{n-1} F''' + \left[1 - \phi + \phi(\rho_s/\rho_f)\right] (\frac{2n}{n+1} F F'' - F'^2) 
+ \lambda \left[1 - \phi + \phi((\rho\beta)_s/(\rho\beta)_f)\right] \theta = 0$$
(7)

$$\frac{1}{\text{NPr}} (\frac{k_{nf}}{k_f}) \theta'' + [1 - \phi + \phi(\rho C_p)_s / (\rho C_p)_f] (\frac{2n}{n+1} F \theta' - F' \theta) = 0$$
(8)

Where NPT =  $\frac{b x^2}{\alpha_f} (Re_x)^{-2/(n+1)}$  is the modified Prandtl number (for power law fluids),  $\lambda = \pm Gr_x/Re_x$  is the buoyancy

convection parameter and  $Gr_x = g\beta(T_w - T_\infty) xb^{-n} / v_f$  is Grashof number. It is worth mentioning that  $\lambda > 0$  and  $\lambda < 0$  correspond to the assisting and opposing flows, respectively, while  $\lambda = 0$ , i.e.  $(T_w = T_\infty)$  represent the case when the buoyancy force is absent.

The boundary conditions (5) become

$$F = 0, F' = 1, \quad \boldsymbol{\theta} = 1 \text{ at } \eta = 0$$
  
$$F' \to 0, \quad \boldsymbol{\theta} \to 0 \text{ as } \eta \to \infty$$
 (9)

We noticed that the absence of nanoparticles volume fraction and the buoyancy parameter, Eqs. (8) and (9) reduce to those of Hassanien *et al.* [22].

The physical quantities of practical interest in this work are the skin friction coefficient  $C_f$  and the local Nusselt number  $Nu_{\infty}$  which are defined as

$$C_{f} = \frac{2\mu_{nf}}{\rho_{f} u_{w}^{2}} \left( -\frac{\partial u}{\partial y} \Big|_{y=0} \right)^{n}, \quad Nu_{x} = -\frac{x \kappa_{nf}}{\kappa_{f} (T_{w} - T_{\infty})} \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

$$(10)$$

Using equations (6) and (11), the skin friction coefficient  $C_t$  and the local Nusselt number  $Nu_x$ , can be expressed as

$$(\operatorname{Re}_{x})^{\frac{1}{n+1}} C_{f} = \frac{2}{(1-\phi)^{2.5}} (-F''(0))^{n}, \quad (\operatorname{Re}_{x})^{\frac{-1}{n+1}} N u_{x} = -\frac{\kappa_{nf}}{\kappa_{f}} \theta'(0)$$
 (11)

## RESULTS AND DISCUSSION

The system (7) and (8) with the boundary conditions (9) have been solved numerically for some values of the governing parameters  $\phi$ , Npr,  $\lambda$  and n using finite difference method. In order to bring out the salient features of the flow and the heat transfer characteristics the numerical values are plotted in Figures 1-8. These figures depict the velocity profiles (Figures 1, 3, 6), the temperature profiles (Figures 2, 4, 5, 6), the variation of the skin friction at the wall (Fig. 7) and the variation of the Sherwood number (Figures 8) at the wall for different values of the physical parameters. It worth to mention that for n=1 the problem transforms to Newtonian fluid.

Figures 1 and 2 are the graphical representations of the velocity  $F'(\eta)$  and the temperature  $\theta(\eta)$  for various values of the Cu nanoparticles volume fraction for two values of power law index  $n = (=0.4 \text{ and } 1) \text{ when } \lambda = 1 \text{ and}$ Npr = 6.8 (water). It is found from Figure 1 that the momentum boundary layer thickness decreases with the increase in  $\phi$ . Then, the existence of nanfluids leads to more thinning of the boundary layer. While, from figure 2 the thermal boundary layer thickness increases with the increase in the nanoparticle volume  $\phi$ . Also it is the thermal boundary layer for Cu-water is greater mare than for pure water ( $\phi = 0$ ), this because the cupper has high thermal conductivity and the addition of it increase the thermal conductivity for the fluid, so the thickness of the thermal boundary layer increases. Furthermore, we notice both momentum and thermal boundary layers are decreased as the power law parameter increases.

Figures 3 and 4 depicts the effect mixed convection parameter  $\lambda$  for Cu/pure-water on the velocity and the temperature, respectively, when n=0.4. It is seen that the momentum boundary layer increases while the thermal boundary layer decreases when  $\lambda$  increases and at each value of  $\lambda$  the thickness of the momentum boundary layer in Cu-water is smaller than for pure water while the thermal boundary layer thickness in Cu-water is greater than for pure water. It is also seen the momentum boundary layer thickness for the case of assisting flow is greater than for opposing flow, while the thermal boundary layer thickness has opposite behavior. This is due to the fact that  $\lambda > 0$  physically means heating of the fluid or cooling of the surface while  $\lambda < 0$  means cooling of the fluid or heating of the surface.

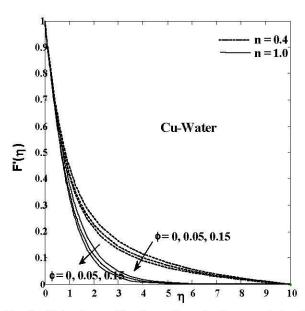


Fig. 1: Velocity profiles for various  $\phi$  when n = 0.4 and n = 1 (Newtonian fluid) with  $\lambda = 1$ .

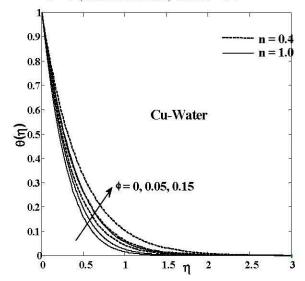


Fig. 2: Temperature profiles for various  $\phi$  when n = 0.4 and n = 1 (Newtonian fluid) with  $\lambda = 1$ .

Figure 5 shows the behavior of the temperature for various values of NPr for Cu/pure-water when n = 0.4 and  $\lambda = 1$ . It is seen that increase in the values of NPr leads to decrease the thickness of the thermal layer and at each value of NPr the thickness of the thermal boundary layer in Cu-water is greater than for pure water.

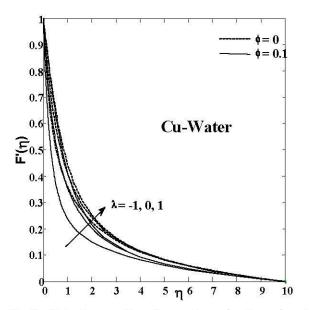


Fig. 3: Velocity profiles for various  $\lambda$  when  $\phi = 0$  (pure water),  $\phi = 0.1$  with n = 0.4.

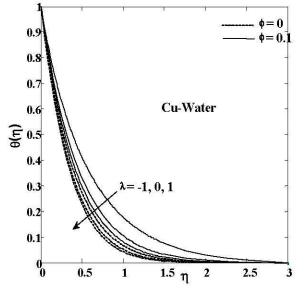


Fig. 4: Temperature profiles for various  $\lambda$  when  $\phi = 0$  (pure water),  $\phi = 0.1$  with n = 0.4.

Figure 6 shows the behavior of the velocity and the temperature for different types of nanofluids when  $\phi = 0.1$ , n = 0.4, Npr = 6.8 and  $\lambda = 1$ . It is seen that both momentum and thermal boundary layer thicknesses change with the change of the type of nanoparticles, this means the addition of nanoparticles in regular fluid will play an important rule to develop the industry.

Figures 7 and 8 show the variation in shear stress and heat transfer rates versus the power law parameter n for Cu-water. Figures 7 shows the effect of the nanoparticles

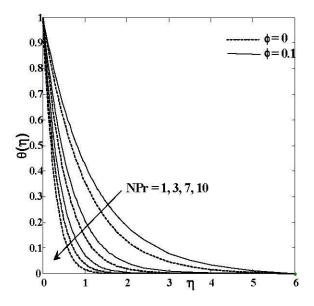


Fig. 5: Temperature profiles for various Npr when  $\phi = 0$  (pure water),  $\phi = 0.1$  with n = 0.4 and  $\lambda = 1$ .

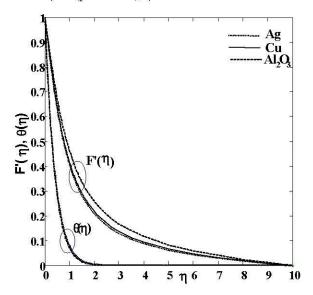


Fig. 5: Velocity and temperature profiles for different types of nanofluids when  $\phi$ =0.1, n=0.4, NPr=6.8 and  $\lambda$ =1.

volume fraction  $\phi$  on the shear stress, while Figures 8 depict the effect  $\phi$  on the heat transfer rates for the assisting flow  $(\lambda = 1)$  and opposing flow  $\lambda = -1$ . It is seen from figure 5 that the shear stress increases with the increase  $\phi$  also one can see that the change in the shear stress decreases with the increase n. Also from Fig. 8 it is noted that the heat transfer rates increases with the increase  $\phi$  and the change in the heat transfer rates increases with the increase n.

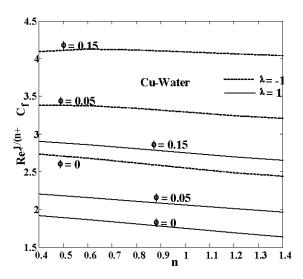


Fig. 7: Variation of the skin friction with *n* for various  $\phi$  when  $\lambda = -1, 1$ .

Also, for fixed values of n and  $\phi$  it observed that the value of the shear stress for opposing flow is greater than for assisting flow, while the heat transfer rates for assisting flow is greater than for opposing flow.

## CONCLUSION

The problem of two-dimensional laminar mixed convection flow of a power-law non-Newtonian nanofluid over a vertical stretching surface has been studied. A similarity solution is presented and the numerical solutions are discussed. This depends on the nanoparticle volume fraction  $\phi$ , power law parametern, mixed convection parameter  $\lambda$  and the modified Prandtl number NPr. The working fluid is water with the Prandtl number of Pr = 6.8. We have explored the way in which the velocity and temperature profiles as well as the surface skin friction and the surface heat flux depend on these parameters. It is shown that the inclusion of nanoparticles into the base fluid of this problem is capable to change the flow pattern. The study of nanofluids is still at its stage so that it seems difficult to have a precise idea on the way the use of nanoparticles to understand the flow and heat transfer characteristics of nanofluids and identify new and unique applications for these fluids.

#### ACKNOWLEDGMENTS

Msc. Muatazz Abdulhadi Bashir gratefully acknowledges, University of Garyounis, Benghazi, (Libya) for a demonstrator (Teacher Assistant)

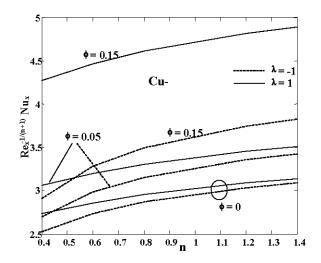


Fig. 8: Variation of the local Nusselt number with n for various  $\phi$  when  $\lambda = -1, 1$ .

opportunity and School of Mathematical Sciences Universiti Sains Malaysia, Pinang, (Malaysia), for the scientific hospitality.

### REFERENCES

- Polidori, G., S. Fohanno and C.T. Nguyen, 2007.
   A note on heat transfer modelling of Newtonian nanofluids in laminar free convection. Int. J. Thermal. Sci., 46: 739-744.
- Moghadassi, A.R., S.M. Hosseini, D. Henneke and A. Elkamel, 2009. A Model of nanofluids effective thermal conductivity based on dimensionless group. J. Thermal Analysis and Calorimetry, 96(1): 81-84.
- 3. Jain, S., H.E. Patel and S.K. Das, 2009. Brownian dynamic simulation for the prediction of effective Thermal Conductivity of Nanofluid. J. Nanopart Res., 11: 767-773.
- Congedo, P.M., S. Collura and P.M. Congedo, 2009. Modeling and analysis of natural convection heat transfer in nanofluids. In: Proc. ASME Summer Heat Transfer Conf., 3: 569-579.
- Ghasemi, B. and S.M. Aminossadati, 2009. Natural convection heat transfer in an inclined enclosure filled with a water-Cuo nanofluid. Numer. Heat Transfer; Part A: Applications, 55: 807-823.
- Ho, C.J., M.W. Chen and Z.W. Li, 2008. Numerical simulation of natural convection of nanofluid in a square enclosure: Effects due to uncertainties of viscosity and thermal conductivity. Int. J. Heat Mss Transfer., 51: 4506-4516.

- Ho, C.J., M.W. Chen and Z.W. Li, 2007. Effect of natural convection heat transfer of nanofluid in an enclosure due to uncertainties of viscosity and thermal conductivity. In. Proc. ASME/JSME Thermal Engng. Summer Heat Transfer Conf. HT, 1: 833-841.
- Oztop, H.F. and E. Abu-Nada, 2008. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. Int. J. of Heat and Fluid, 29: 1326-1336.
- Das, S.K., S.U.S. Choi, W. Yu and T. Pradeep, 2007. Nanofluids: Science and Technology, Wiley, New Jersey.
- Wang, X.Q. and A.S. Mujumdar, 2007. Heat transfer characteristics of nanofluids: a review. Int. J. Thermal. Sci., 46: 1-19.
- Wang, X.Q. and A.S. Mujumdar, 2008. A review on nanofluids – Part I: theoretical and numerical investigations. Brazilian J. Chem. Engng., 25: 613-630.
- Wang, X.Q. and A.S. Mujumdar, 2008. A review on nanofluids Part II: experiments and applications. Brazilian J. Chem. Engng., 25: 631-648.
- Kakaç, S. and A. Pramuanjaroenkij, 2009. Review of convective heat transfer enhancement with nanofluids. Int. J. Heat Mass Transfer, 52: 3187-3196.
- Metzner, A.B., 1965. Heat transfer in non-Newtonian fluids. Adv. Heat Transfer, 5: 357-397.
- 15. Brizzell, G.D. and J.C. Slattery, 1962. Non-Newtonian boundary layer flow. Chem. Eng. Sci., 17: 777-782.
- Acrivos, A., M.J. Shah and E.E. Petersen, 1960. Momentum and heat transfer in laminar boundary layer flows of non-Newtonian fluids past external surfaces. AIChE J., 6: 312-317.
- Nakayama, H. and H. Koyama, 1988. An analysis for friction and heat transfer characteristics of power-law non-Newtonian fluid flows past bodies of arbitrary geometrical configuration. Wäme-und stoffübertragung, 22: 29-36.
- Cobble, M.H., 1980. Magnetohydrodynamic flow for a non-Newtonian power-law fluid having a pressure gradient and fluid injection. J. Eng. Mathematics, 14: 47-55.
- Tashtoush, B., Z. Kodah and A. Al-Ghasem, 2001.
   On thermal boundary layer of a non-Newtonian fluid on a power-law stretching surface of variable temperature with suction or injection. Heat and Mass Transfer, 37: 459-465.

- Andersson, H.I. and B.S. Dandapat, 1991. Flow of a power law fluid over a stretching sheet, Stability Appl. Anal. Continuous Media, 1: 339-347.
- Crane, L.J., 1970. Flow past a stretching plate. ZAMP, 21: 645-647.
- Hassanien, I.A., A.A. Abdullah and R.S.R. Gorla, 1998. Flow and heat transfer in a power law fluid over a non-isothermal stretching sheet. Math. Comput. Model., 28: 105-116.
- Sahu, A.K., M.N. Mathur, P. Chaturani and S.B. Saxena, 2000. Momentum and heat transfer from a continuous moving surface to a power law fluid. Acta Mech., 142: 119-131.
- Denier, J.P. and P.P. Dabrowski, 2004. On the boundary layer equations for power law fluids. Proc. R. Soc. Lond. Ser. A, 460: 3143-3158.
- Elbashbeshy, E.M.A. and M.A.A. Basid, 2004: Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. Appl. Math. Comput., 158: 799-807.
- Nazar, R., N. Amin, D. Filip and I. Pop, 2004. Unsteady boundary layer flow in the region of the stagnation-point on a stretching sheet. Int. J. Eng. Sci., 42: 1241-1253.
- Layek, G.C., S. Mukhopadhyay and Sk.A. Samad, 2007. Heat and mass transfer analysis for boundary layer stagnation-point flow towards a heated porous stretching sheet with heat absorption/generation and suction/blowing. Int. Comm. Heat Mass Transfer, 34: 347-356.
- Ishak, A., R. Nazar and I. Pop, 2006.
   Magnaetohydrodynamic stagnation point flow towards a stretching vertical sheet.
   Magnaetohydrodynamic, 42: 17-30.
- Raptis, A. and C. Perdikis, 2006. Viscous flow over a non-linearly stretching sheet in the presence of chemical reaction and magnetic field. Int. J. Non-Linear Mechanics, 41: 527-529.
- Abel, M.S., E. Sanjayanand and M.M. Nandeppanavar, 2008. Viscoelastic MHD flow and heat transfer over a stretching sheet with viscous and ohmic dissipations. Comm. in Nonlinear Science and Numerical Simulation, 13: 1808-1821.
- 31. Aminossadati, S.M. and B. Ghasemi, 2009. Natural convection cooling of a localized heat source at the bottom of a nanofluid-filled enclosure, Eur. J. Mech. B/Fluids, 28: 630-640.