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A Note on One-step Iteration Methods for Solving Nonlinear Equations

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Abstract: In this paper, two one-step iteration methods are introduced to solve nonlinear equations. The convergence criteria for these methods are also discussed. Several examples are presented and compared to other well known methods, showing the accuracy and fast convergence of the proposed methods. An important property of the proposed methods is that these are often convergent from both sides of the root whereas the other well known methods are often convergent from one side only.

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INTRODUCTION

One of the oldest and most basic problems in mathematics is that of solving an nonlinear equation f (x) = 0. This problem has motivated many theoretical developments including the fact that solution formulas do not in general exist. Thus, the development of algorithms for finding solutions has historically been an important enterprise. Newton-Raphson method [3] is the most popular technique for solving nonlinear equations. Many topics related to Newton's method still attract attention from researchers. As is well known, a disadvantage of the method is that the initial approximation x_0, must be chosen sufficiently close to a true solution in order to guarantee their convergence. Finding a criterion for choosing x_0 is quite difficult and therefore effective and globally convergent algorithms are needed [30]. In recent years, several methods have been developed to solve the nonlinear equation f (x) = 0, by using Newton method, decomposition method, iterative methods, Homotopy analysis method, Variational iteration method and their modifications [1-31]. For example, Newton method is based on Taylor's series expansion and has at least second-order convergence for simple roots. In this work, a cubic and a quartic iterative method based on Taylor's series expansion are introduced.

Consider a nonlinear equation f(x)=0. The Taylor's series expansion around a given initial point $x = \gamma$, assuming γ being close enough to the simple root $x = \alpha$, is given as follows:

$$f(x) = f(\gamma) + f'(\gamma)(x-\gamma) + HOT = 0 \tag{1}$$

where HOT denotes the higher order terms. Then the nonlinear equation becomes,

$$f(x) = f(\gamma) + f'(\gamma)(x-\gamma) + HOT \tag{2}$$

When γ is close enough to α, equation (2) can be approximated as

$$f(\gamma) + f'(\gamma)(x-\gamma) \approx 0 \tag{3}$$

Thus we have

$$(x-\gamma) \approx -\frac{f(\gamma)}{f'(\gamma)} \tag{4}$$

by assuming $f'(\gamma) \neq 0$, which yields the one-step iteration method called Newton method [3] with second-order convergence

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{5}$$

ased on above relations, a one-step iteration method can be constructed with third-order convergence. For this reason, we rewrite equation (2) as

$$f(x) = f(\gamma) + f'(\gamma)(x-\gamma) + \frac{f''(\gamma)}{2}(x-\gamma)^2 + HOT \tag{6}$$

Equation (6) can be approximated as

$$f(\gamma) + f'(\gamma)(x-\gamma) + [(x-\gamma)]\frac{f''(\gamma)}{2}(x-\gamma) \approx 0 \tag{7}$$

Substituting (4) into the bracket of equation (7), we obtain

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$$f(\gamma)+f'(\gamma)(x-\gamma)+[-\frac{f(\gamma)}{f'(\gamma)}]\frac{f''(\gamma)}{2}(x-\gamma)\approx 0$$

that is

$$f(\gamma)+(x-\gamma)\left[f'(\gamma)-\frac{f(\gamma)\,f''(\gamma)}{2f'(\gamma)}\right]\approx 0 \tag{8}$$

that is

$$(x-\gamma)\approx\frac{-2f(\gamma)\,f'(\gamma)}{2f'^2(\gamma)-f(\gamma)\,f''(\gamma)} \tag{9}$$

Thus we can solve equation (9) by assuming

$$2f'^2(\gamma)-f(\gamma)\,f''(\gamma)\neq 0$$

as

$$\alpha=\gamma-\frac{2f(\gamma)\,f'(\gamma)}{2f'^2(\gamma)-f(\gamma)f''(\gamma)} \tag{10}$$

which suggests the following one-step iteration method:

Algorithm 1.1: For a given x_0, compute the approximate solution x_{n+1} by the one-step iteration scheme:

$$x_{n+1}=x_n-\frac{2f(x_n)\,f'(x_n)}{2f'^2(x_n)-f(x_n)\,f''(x_n)}\quad n=0,1,2,\ldots \tag{11}$$

It will be shown that the propose method (11) has third-order convergence.

Again, based on above relations, a new one-step iteration method can be constructed with fourth-order convergence. For this reason, we rewrite equation (2) as

$$f(x)=f(\gamma)+f'(\gamma)(x-\gamma)+\frac{f''(\gamma)}{2!}(x-\gamma)^2+\frac{f'''(\gamma)}{3!}(x-\gamma)^3+HOT \tag{12}$$

Equation (12) can be approximated as

$$f(\gamma)+(x-\gamma)\left\{\begin{array}{l}f'(\gamma)+[(x-\gamma)]\frac{f''(\gamma)}{2!}\\ \quad+[(x-\gamma)^2]\frac{f'''(\gamma)}{3!}\end{array}\right\}\approx 0 \tag{13}$$

Substituting (9) into the brackets of equation (13), we find

$$f(\gamma)+(x-\gamma)\left\{\begin{array}{l}f'(\gamma)+\left[\frac{-2f(\gamma)\,f'(\gamma)}{2f'^2(\gamma)-f(\gamma)\,f''(\gamma)}\right]\frac{f''(\gamma)}{2!}\\ \quad+\left[\frac{-2f(\gamma)\,f'(\gamma)}{2f'^2(\gamma)-f(\gamma)\,f''(\gamma)}\right]^2\frac{f'''(\gamma)}{3!}\end{array}\right\}\approx 0$$

that is

$$(x-\gamma)\approx\frac{-3f(\gamma)\left[2f'^2(\gamma)-f(\gamma)\,f''(\gamma)\right]^2}{\begin{array}{c}12f'^5(\gamma)+6f^2(\gamma)\,f'(\gamma)\,f''^2(\gamma)\\-18f(\gamma)f'^3(\gamma)f''(\gamma)+2f^2(\gamma)f'^2(\gamma)f'''(\gamma)\end{array}} \tag{14}$$

Using this relation, we can suggest the following one-step iteration method as:

Algorithm 1.2: For a given x_0, compute the approximate solution x_{n+1} by the one-step iteration scheme:

$$x_{n+1}=x_n-\left.\frac{f\left(2f'^2-f\,f''\right)^2}{f'\left(\left(2f'^2-f\,f''\right)\left(2f'^2-2ff''\right)+\frac{2}{3}f^2f'f'''\right)}\right|_{x=x_n} \tag{15}$$

$$n=0,1,2,\ldots$$

It will be shown that the proposed method (15) has fourth-order convergence.

CONVERGENCE ANALYSIS

The convergence analysis of proposed one-step iteration methods (11) and (15) is studied by the following theorems.

Theorem 2.1: Let $\alpha\in I$ be a simple root of sufficiently differentiable function $f: I\subseteq R\to R$ for an open interval I. Then the one-step iteration method (15), defined by Algorithm 1.2, has fourth-order convergence.

Proof: Let α be a simple root of f. Since f is sufficiently differentiable, expanding $f(x_n)$ about α, we get

$$f(x_n)=f(\alpha)+f'(\alpha)(x_n-\alpha)+\frac{f''(\alpha)}{2}(x_n-\alpha)^2+\frac{f'''(\alpha)}{2}(x_n-\alpha)^3+\ldots \tag{16}$$

From equation (16), we have

$$f(x_n)=f'(\alpha)\left[\begin{array}{l}e_n+c_2e_n^2+c_3e_n^3+c_4e_n^4\\+c_5e_n^5+c_6e_n^6+\ldots\end{array}\right] \tag{17}$$

where $e_n-x_n-\alpha$ and

$$c_k=\frac{f^{(k)}(\alpha)}{f'(\alpha)k!}$$

for k = 2, 3,… Differentiating (17) three times with respect to x_n, we obtain

$$f'(x_n) = f'(\alpha)\left[1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + 5c_5 e_n^4 + 6c_6 e_n^5 + \dots\right] \tag{18}$$

$$f''(x_n) = f'(\alpha)\left[2c_2 + 6c_3 e_n + 12c_4 e_n^2 + 20c_5 e_n^3 + 30c_6 e_n^4 + \dots\right] \tag{19}$$

and

$$f'''(x_n) = f'(\alpha)\left[6c_3 + 24c_4 e_n + 60c_5 e_n^2 + 120c_6 e_n^3 + \dots\right] \tag{20}$$

From equations (17)-(20), we get

$$f(x_n)\left(2f'^2(x_n) - f(x_n)f''(x_n)\right)^2 = f'^5(\alpha)\left[4e_n + 28c_2 e_n^2 + (84c_2^2 + 28c_3)e_n^3 + (132c_2^3 + 184c_2c_3 + 20c_4)e_n^4 + \dots\right] \tag{21}$$

and

$$f'(x_n)\left[\left(2f'^2(x_n) - f(x_n)f''(x_n)\right)\left(2f'^2(x_n) - 2f(x_n)f''(x_n)\right) + \frac{2f^2(x_n)f'(x_n)f'''(x_n)}{3}\right] = f'^5(\alpha)\left[4 + 28c_2 e_n + (84c_2^2 + 28c_3)e_n^2 + (136c_2^3 + 180c_2c_3 + 24c_4)e_n^3 + \dots\right] \tag{22}$$

Now from equations (15), (21) and (22) we have

$$e_{n+1} = e_n - \left.\frac{f\left(2f'^2 - ff''\right)^2}{f'\left(\left(2f'^2 - ff''\right)\left(2f'^2 - 2ff''\right) + \frac{2}{3}f^2 f' f'''\right)}\right|_{x=x_n} = \frac{(4c_2^3 - 4c_2c_3 + 4c_4)e_n^4 + (12c_2^4 + 4c_2^2c_3 - 4c_3^2 + 12c_2c_4 + 16c_5)e_n^5 + \dots}{4 + 28c_2e_n + (84c_2^2 + 28c_3)e_n^2 + (136c_2^3 + 180c_2c_3 + 24c_4)e_n^3 + \dots} \tag{23}$$

that is

$$e_{n+1} = (c_2^3 - c_2c_3 + c_4)e_n^4 + (-16c_2^4 + 32c_2^2c_3 - 4c_3^2 - 16c_2c_4 + 16c_5)e_n^5 + \dots. \tag{24}$$

Thus the iterative method (15) defined by Algorithm 1.2 has fourth-order convergence.

Theorem 2.2: Let $\alpha \in I$ be a simple root of sufficiently differentiable function $f: I \subseteq R \to R$ for an open interval I. Then the one-step iteration method (11), defined by Algorithm 1.1, has third-order convergence and satisfy the error equation

$$e_{n+1} = (c_2^2 - c_3)e_n^3 + (6c_2c_3 - 3c_2^3 - 3c_4)e_n^4 + \dots \tag{25}$$

where $e_n = x_n - \alpha$ and

$$c_k = \frac{f^{(k)}(\alpha)}{f'(\alpha)k!}$$

Proof: The proof is similar to that of Algorithm 1.2's, so it is omitted.

NUMERICAL EXAMPLES

First, we present some numerical results to illustrate the efficiency of the cubic iterative method proposed in this paper. We present some numerical test results for various third-order convergence iterative schemes in Table 1. Compared were method of Homeier [13] (HM) defined by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'\left(x_n - \frac{f(x_n)}{2f'(x_n)}\right)} \tag{26}$$

Sharma's method [29] (SM) defined by

$$x_{n+1} = x_n - \frac{f^2(x_n)}{f'(x_n)\left(f(x_n) - f(y_n)\right)} \tag{27}$$

where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

the method of Noor *et al*. [27] defined by

$$x_{n+1} = x_n - \left(\frac{3\left(f'(x_n) - f'(y_n)\right)}{2f'(x_n)}\right)\frac{f(x_n)}{f'(x_n)} \tag{28}$$

where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

Chun's methods [6] (CM1) defined by

$$x_{n+1} = x_n - \left(\frac{f(x_n)+2f(y_n)}{f(x_n)+f(y_n)}\right)\frac{f(x_n)}{f'(x_n)} \quad (29)$$

and (CM2) defined by

$$x_{n+1} = x_n - \frac{3}{2}\frac{f(x_n)}{f'(x_n)} + \frac{1}{2}\frac{f(x_n)}{2f'(x_n)-f'(y_n)} \quad (30)$$

where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

and the method (11) (AL. 1.1) which introduced in the present contribution. Moreover, we present some numerical results to illustrate the efficiency of the quartic iterative method proposed in this paper. We present some numerical test results for various fourth-order convergence iterative schemes in Table 2. Compared were method of Kou *et al*. [16] (KM1) defined by

$$x_{n+1} = x_n - \left(1 - \frac{3}{4}\frac{\left(f'(y_n)-f'(x_n)\right)\left(3f'(y_n)+5f'(x_n)\right)}{\left(15f'(y_n)-7f'(x_n)\right)f'(x_n)}\right)\frac{f(x_n)}{f'(x_n)} \quad (31)$$

where

$$y_n = x_n - \frac{2}{3}\frac{f(x_n)}{f'(x_n)}$$

the method of Kou *et al*. [17] (KM2) defined by

$$x_{n+1} = y_n - \frac{f(x_n)-\frac{1}{2}f(y_n)}{f(x_n)-\frac{5}{2}f(y_n)}\frac{f(y_n)}{f'(x_n)} \quad (32)$$

where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

Kou's method [15] (KM3) defined by

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}L_f(x_n) + \frac{1}{2}L_f^2(x_n)\right)\frac{f(x_n)}{f'(x_n)} \quad (33)$$

where

$$L_f(x_n) = \frac{f''(x_n - f(x_n)/(3f'(x_n)))\,f(x_n)}{f'^2(x_n)}$$

method of Saeed and Aziz [28] (SM) defined by

$$x_{n+1} = y_n - \frac{(y_n+z_n-x_n)^2}{2f'(x_n)}f''(x_n) - \frac{(y_n+z_n-x_n)^3}{6f'(x_n)}f'''(x_n) - \frac{(y_n+z_n-x_n)^4}{24f'(x_n)}f^{(4)}(x_n) \quad (34)$$

where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

and

$$z_n = -\left[\frac{(y_n-x_n)^2}{2f'(x_n)}f''(x_n) + \frac{(y_n-x_n)^3}{6f'(x_n)}f'''(x_n) + \frac{(y_n-x_n)^4}{24f'(x_n)}f^{(4)}(x_n)\right]$$

Maheshwari's method [19] (MM) defined by

$$x_{n+1} = x_n + \frac{1}{f'(x_n)}\left\{\frac{f^2(x_n)}{f(y_n)-f(x_n)} - \frac{f^2(y_n)}{f(x_n)}\right\} \quad (35)$$

where

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

and the method (15) (AL. 1.2) which introduced in this paper. All computations are done by Maple 12 with 64 digits precision. We accept an approximate solution rather than the exact root, depending on the precision (ε) of the computer. The following stopping criteria is used for computer programs:

$$|x_{n+1} - x_n| < \varepsilon,$$

and so, when the stopping criterion is satisfied, x_{n+1} is taken as the exact root α computed. For numerical illustrations in this section we use the fixed stopping criterion $\varepsilon = 10^{-15}$. The following test problems are used and the approximate zeros x_* found up to the 30th decimal places are displayed.

$f_1(x) = \frac{1}{x} - 1$ [24], $x_* = 1$,

$f_2(x) = e^{x^2+7x-30} - 1$ [4,8,15-18,22-24,26,27,31], $x_* = 3$,

$f_3(x) = (x+2)e^x - 1$ [5, 7, 16],

$x_* = -0.442854401002388583141327999999$,

$f_4(x) = e^x - 1$ [24], $x_* = 0$,

$f_5(x) = x e^{x^2} - \sin^2(x) + 3\cos(x) + 5$ [2, 4-8, 15, 16, 18],

$x_* = -1.207647827130918927009416758356$,

$f_6(x) = \ln(x)$ [10, 19], $x_* = 1$,

Table 1: Div is the abbreviation for divergent

	IT							IT					
f (x) $x_0 < \alpha$	HM	SM	NM	CM1	CM2	AL 1.1	f (x) $x_0 > \alpha$	HM	SM	NM	CM1	CM2	AL 1.1
f_1 (x) $x_0 = 0.5$	5	5	5	6	6	2	f_1 (x) $x_0 = 2.5$	8	Div	Div	Div	Div	2
f_2 (x) $x_0 = 2.0$	Div	Div	Div	Div	Div	10	f_2 (x) $x_0 = 4.0$	13	13	15	16	16	4
f_3 (x) $x_0 = -2.5$	Div	Div	Div	46	38	6	f_3 (x) $x_0 = 1.5$	6	6	6	7	7	5
f_4 (x) $x_0 = -2.0$	8	10	Div	14	12	5	f_4 (x) $x_0 = 2.0$	6	6	6	6	6	5
f_5 (x) $x_0 = -2.5$	8	8	9	9	9	7	f_5 (x) $x_0 = 0.5$	Div	Div	Div	Div	Div	6

Comparison of various third-order convergence iterative methods

Table 2: Div" is the abbreviation for divergent

	IT							IT					
f (x) $x_0 < \alpha$	KM1	KM2	KM3	SM	MM	AL 1.2	f (x) $x_0 > \alpha$	KM1	KM2	KM3	SM	MM	AL 1.2
f_6 (x) $x_0 = 0.5$	4	4	4	4	4	4	f_6 (x) $x_0 = 2.5$	5	5	Div	Div	Div	4
f_7 (x) $x_0 = -1.5$	5	5	5	5	5	5	f_7 (x) $x_0 = 3.5$	Div	Div	Div	Div	Div	6
f_8 (x) $x_0 = 0.5$	Div	Div	Div	Div	Div	5	f_8 (x) $x_0 = 3$	4	4	5	5	5	5
f_9 (x) $x_0 = -2$	8	8	35	61	39	8	f_9 (x) $x_0 = 4$	5	5	6	6	6	5
f_{10} (x) $x_0 = -0.5$	82	82	25	39	23	6	f_{10} (x) $x_0 = 3.5$	4	4	4	4	4	4

Comparison of various Fourth-order convergence iterative methods

$f_7(x) = e^{1-x} - 1$ [24], $x_* = 1$,

$f_8(x) = e^x + 2^{-x} + 2\cos(x) - 6$ [28],

$x_* = 1.829383601933848817136212946814$,

$f_9(x) = x^3 + x^2 - 2$ [24], $x_* = 1$,

$f_{10}(x) = x^3 - 10$ [4, 8, 15, 18, 22, 23, 26-28, 31],

$x_* = 2.1544346900318837217592935665193$,

In Table 1 and 2, IT denotes the number of iterations. The test results in Table 1 and 2 show that for most of the functions we tested, the methods introduced here have at least equal performance compared to the other methods. Furthermore, for most of the functions we tested, the proposed methods converge from both left side ($x_0>\alpha$) and right side ($x_0>\alpha$) towards the root, α, whereas the other methods work usually well from one side only.

CONCLUSION

In this paper, a cubic and a quartic iterative method based on Taylor's series expansion were introduced. These methods were compared in performance to the other well known methods. The proposed iterative methods are both effective and convenient with at least equal performance. Unlike the other methods, the proposed methods work well when the starting point x_0 is chosen from both sides of the root, α. This was clearly demonstrated in the examples.

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