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Numerical Simulation of the Jaulent-miodek Equation by He's Homotopy Perturbation Method

Ahmet Yildirim and Alev Kelleci

Department of Mathematics, Ege University, 35100 Bornova, Izmir, Turkey

Abstract: The Jaulent-Miodek (JM) equation, with some initial conditions, is solved numerically by homotopy perturbation method. This method is useful for obtaining numerical solutions with high degree of accuracy. The homotopy perturbation solution for the JM equation converges to its exact solution. In order to show the efficiency of this method, exhaustive numerical examples are presented.

Key words: Homotopy perturbation method . jaulent-miodek equation . numerical solution of differential equations . exact traveling wave solutions

INTRODUCTION

The Jaulent-Miodek equation is considered in [1] as the following form:

$$L_t u = -L_x u - \frac{3}{2} v v_{xxx} - \frac{9}{2} v_x v_{xx} + 6uu_x + 6uvv_x + \frac{3}{2} u_x v^2$$
$$L_t v = -L_x v + 6u_x v + 6uv_x + \frac{15}{2} v_x v^2 \qquad (1)$$

with the initial conditions u(x, 0) = f(x), v (x, 0) = g (x),

Most of scientific problems occur nonlinearly. Except a limited number of these problems, most of them do not have precise analytical solutions so that they have to be solved using other methods. Many different new methods have recently presented some techniques to eliminate the small parameter; for example, the variational iteration method (VIM) [2, 3], the Adomian's decomposition method (ADM) [4, 5] and the homotopy perturbation method [5, 26]. In this study, HPM is used to solve nonlinear equations of Jaulent-Miodek.

Homotopy perturbation method is a straightforward and convenient method for both linear and nonlinear equations. This method does not depend on a small parameter. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p \in [0,1]$, which is considered as a "small parameter" [6, 9]. When $p = 0$, the system of equations usually reduces to a sufficiently simplied form, which normally admits a rather simple solution. As p gradually increases to 1, the system goes through a sequence of deformations, the solution for each of which is close to that at the previous stage of deformation. Eventually at $p = 1$, the system takes the original form of the equation and the final stage of deformation gives the desired solution. One of the most remarkable features of the HPM is that usually just few perturbation terms are sufficient for obtaining a reasonably accurate solution. Considerable research works have been conducted recently in applying this method to a class of linear and non-linear equations [10, 22]. The interested reader can see the Refs. [23, 26] for last development of HPM.

In this paper, we will use HPM for solving JM equation. Recently, Kaya and El-Sayed [1] used ADM for solving JM equation. Also Ganji *et al*. [27] used ADM and VIM for solving JM equation.

APPLICATION OF HOMOTOPY PERTURBATION METHOD

We first consider the application of the method to the JM equation (1) with the initial conditions

$$\begin{cases} u(x,0) = f(x) = \frac{c_2}{2} + 2c_2 \sec h^2(kx) \\ v(x,0) = g(x) = 2k \sec h(kx) \end{cases} \qquad (2)$$

where c_2 and k are arbitrary constants.

According to homotopy perturbation method, we construct the following homotopy

$$\frac{\partial u}{\partial t} - \frac{\partial u_0}{\partial t} = p\left(-\frac{\partial^3 u}{\partial x^3} - \frac{3}{2} v \frac{\partial^3 v}{\partial x^3} - \frac{9}{2} \frac{\partial v}{\partial x} \frac{\partial^2 v}{\partial x^2} + 6u \frac{\partial u}{\partial x} + 6uv \frac{\partial v}{\partial x} + \frac{3}{2} \frac{\partial u}{\partial x} v^2 - \frac{\partial u_0}{\partial t} \right) \qquad (3)$$

Corresponding Author: Dr. Ahmet Yildirim, Department of Mathematics, Ege University, 35100 Bornova, Izmir, Turkey

$$\frac{\partial v}{\partial t}-\frac{\partial v_0}{\partial t}=p\left(-\frac{\partial^3 v}{\partial x^3}+6\frac{\partial u}{\partial x}v+6u\frac{\partial v}{\partial x}+\frac{15}{2}\frac{\partial v}{\partial x}v^2-\frac{\partial v_0}{\partial t}\right) \tag{4}$$

In view of the HPM, we use the homotopy parameter p to expand solution,

$$u=u_0+pu_1+p^2u_2+p^3u_3+\ldots \tag{5}$$

Substituting (5) into (3-4) and equating the coefficients of like powers of p, we get the following set of differential equations

$$p^0:\quad \frac{\partial^\alpha u_0}{\partial t^\alpha}-\frac{\partial^\alpha u_0}{\partial t^\alpha}=0 \tag{6}$$

$$p^0:\quad \frac{\partial^\beta v_0}{\partial t^\beta}-\frac{\partial^\beta v_0}{\partial t^\beta}=0$$

$$p^1:\quad \frac{\partial u_1}{\partial t}=\left(-\frac{\partial^3 u_0}{\partial x^3}-\frac{3}{2}v_0\frac{\partial^3 v_0}{\partial x^3}-\frac{9}{2}\frac{\partial v_0}{\partial x}\frac{\partial^2 v_0}{\partial x^2}+6u_0\frac{\partial u_0}{\partial x}+6u_0v_0\frac{\partial v_0}{\partial x}+\frac{3}{2}\frac{\partial u_0}{\partial x}{v_0}^2\right) \tag{7}$$

$$p^1:\quad \frac{\partial v_1}{\partial t}=\left(-\frac{\partial^3 v_0}{\partial x^3}+6\frac{\partial u_0}{\partial x}v_0+6u_0\frac{\partial v_0}{\partial x}+\frac{15}{2}\frac{\partial v_0}{\partial x}{v_0}^2\right)$$

$$p^2:\quad \frac{\partial u_2}{\partial t}=\left(\begin{array}{l}-\frac{\partial^3 u_1}{\partial x^3}-\frac{3}{2}v_0\frac{\partial^3 v_1}{\partial x^3}-\frac{3}{2}v_1\frac{\partial^3 v_0}{\partial x^3}-\frac{9}{2}\frac{\partial v_0}{\partial x}\frac{\partial^2 v_1}{\partial x^2}-\frac{9}{2}\frac{\partial v_1}{\partial x}\frac{\partial^2 v_0}{\partial x^2}+6u_0\frac{\partial u_1}{\partial x}+6u_1\frac{\partial u_0}{\partial x}+\\ 6u_0v_1\frac{\partial v_0}{\partial x}+6u_0v_0\frac{\partial v_1}{\partial x}+6u_1v_0\frac{\partial v_0}{\partial x}+\frac{3}{2}\frac{\partial u_1}{\partial x}{v_0}^2+3\frac{\partial u_0}{\partial x}v_0v_1\end{array}\right) \tag{8}$$

$$p^2:\quad \frac{\partial v_2}{\partial t}=\left(-\frac{\partial^3 v_1}{\partial x^3}+6\frac{\partial u_0}{\partial x}v_1+6\frac{\partial u_1}{\partial x}v_0+6u_0\frac{\partial v_1}{\partial x}+6u_1\frac{\partial v_0}{\partial x}+\frac{15}{2}\frac{\partial v_1}{\partial x}{v_0}^2+15\frac{\partial v_0}{\partial x}v_0v_1\right)$$

$$p^3:\quad \frac{\partial u_3}{\partial t}=\left(\begin{array}{l}-\frac{\partial^3 u_2}{\partial x^3}-\frac{3}{2}v_0\frac{\partial^3 v_2}{\partial x^3}-\frac{3}{2}v_1\frac{\partial^3 v_1}{\partial x^3}-\frac{3}{2}v_2\frac{\partial^3 v_0}{\partial x^3}-\frac{9}{2}\frac{\partial v_0}{\partial x}\frac{\partial^2 v_2}{\partial x^2}-\frac{9}{2}\frac{\partial v_1}{\partial x}\frac{\partial^2 v_1}{\partial x^2}-\frac{9}{2}\frac{\partial v_2}{\partial x}\frac{\partial^2 v_0}{\partial x^2}\\ +6u_0\frac{\partial u_2}{\partial x}+6u_1\frac{\partial u_1}{\partial x}+6u_2\frac{\partial u_0}{\partial x}+6u_0v_1\frac{\partial v_1}{\partial x}+6u_0v_2\frac{\partial v_0}{\partial x}+6u_0v_0\frac{\partial v_2}{\partial x}+6u_1v_1\frac{\partial v_0}{\partial x}\\ +6u_1v_0\frac{\partial v_1}{\partial x}+6u_2v_0\frac{\partial v_0}{\partial x}+\frac{3}{2}\frac{\partial u_2}{\partial x}{v_0}^2+\frac{3}{2}\frac{\partial u_0}{\partial x}v_1^2+3\frac{\partial u_0}{\partial x}v_0v_2+3\frac{\partial u_1}{\partial x}v_0v_1\end{array}\right) \tag{9}$$

$$p^3:\quad \frac{\partial v_3}{\partial t}=\left(\begin{array}{l}-\frac{\partial^3 v_2}{\partial x^3}+6\frac{\partial u_0}{\partial x}v_2+6\frac{\partial u_1}{\partial x}v_1+6\frac{\partial u_2}{\partial x}v_0+6u_0\frac{\partial v_2}{\partial x}+6u_1\frac{\partial v_1}{\partial x}+6u_2\frac{\partial v_0}{\partial x}\\ +\frac{15}{2}\frac{\partial v_0}{\partial x}{v_1}^2+\frac{15}{2}\frac{\partial v_2}{\partial x}v_0^2+15\frac{\partial v_0}{\partial x}v_0v_2+15\frac{\partial v_1}{\partial x}v_0v_1\end{array}\right)$$

….

Solving the above equations and using the initial conditions yield

$$\begin{aligned}u_0&=\frac{c_2}{2}+2c_2\operatorname{sech}^2(kx)\\ v_0&=2k\operatorname{sech}(kx)\end{aligned} \tag{10}$$

$$u_1 = -2c_2 kt\left(27c_2 + 20k^2 + 3c_2\cosh(2kx) - 4k^2\cosh(2kx)\right)\operatorname{sech}^4(kx)\tanh(kx) \tag{11}$$

$$v_1 = \frac{kt}{2}\left(-135c_2^2 - 21k^3\cosh(kx) - 15c_2^2\cosh(2kx) + k^3\cosh(3kx)\right)\operatorname{sech}^4(kx)\tanh(kx)$$

$$u_2 = \frac{-c_2k^2t^2}{8}\begin{pmatrix} 10440c_2^2 + 29376c_2k^2 + 19328k^4 - 6255c_2^2\cosh(2kx) \\ -25752c_2k^2\cosh(2kx) - 19056k^4\cosh(2kx) - 576c_2^2\cosh(4kx) \\ +1344c_2k^2\cosh(4kx) + 1920k^4\cosh(4kx) - 9c_2^2\cosh(6kx) \\ +24c_2k^2\cosh(6kx) - 16k^4\cosh(6kx) \end{pmatrix}\operatorname{sech}^8(kx) \tag{12}$$

$$v_2 = \frac{k^2t^2}{64}\begin{pmatrix} -10440c_2^3 + 293760c_2k^2 - 13005k^5\cosh(kx) + 62550c_2^3\cosh(2kx) \\ +257520c_2^2k^2\cosh(2kx) + 9821k^5\cosh(3kx) + 5760c_2^3\cosh(4kx) \\ -13440c_2^2k^2\cosh(4kx) - 721k^5\cosh(5kx) + 90c_2^3\cosh(6kx) \\ -240c_2^2k^2\cosh(6kx) + k^5\cosh(7kx) \end{pmatrix}\operatorname{sech}^8(kx)$$

$$u_3 = \frac{-c_2k^3t^3}{48}\begin{pmatrix} -3875607c_2^3 - 23376708c_2^2k^2 - 48393936c_2k^4 \\ -28863680k^6 + 1273644c_2^2\cosh(2kx) + 15259536c_2^2k^2\cosh(2kx) \\ +40078656c_2k^4\cosh(4kx) + 26135296k^6\cosh(2kx) \\ +177660c_2^3\cos(4kx) - 376272c_2^2k^2\cosh(4kx) \\ -35666016c_2k^4\cosh(4kx) - 2996992k^6\cosh(4kx) \\ +5076c_2^3\cosh(6kx) - 28944c_2^2k^2\cosh(6kx) + 29376c_2k^4\cosh(6kx) \\ +64768k^6\cosh(6kx) + 27c_2^3\cosh(8kx) - 108c_2^2k^2\cosh(8kx) \\ +144c_2k^4\cosh(8kx) - 64k^6\cosh(8kx) \end{pmatrix}\operatorname{sech}^{10}(kx)\tanh(kx) \tag{13}$$

$$v_3 = \frac{k^3t^3}{768}\begin{pmatrix} 77512140c_2^4 + 467534160c_2^3k^2 + 967878720c_2^2k^4 \\ +29470686k^7\cosh(kx) - 25472880c_2^4\cosh(2kx) \\ -305190720c_2^3k^2\cosh(2kx) - 801573120c_2^2k^4\cosh(2kx) \\ -18234636k^7\cosh(3kx) - 3553200c_2^4\cos(4kx) \\ +7525440c_2^3k^2\cosh(4kx) + 71320320c_2^2k^4\cosh(4kx) \\ +1716996k^7\cosh(5kx) - 101520c_2^4\cosh(6kx) + 578880c_2^3k^2\cosh(6kx) \\ -587520c_2^2k^4\cosh(6kx) - 19671k^7\cosh(7kx) - 540c_2^4\cosh(8kx) \\ +2160c_2^3k^2\cosh(8kx) - 2880c_2^2k^4\cosh(8kx) + k^7\cosh(9kx) \end{pmatrix}\operatorname{sech}^{10}(kx)\tanh(kx)$$

and so on, the other components of the homotopy perturbation series (5) can be determined in a similar way. Here, setting p = 1 and substituting (6)-(13) into (5) and using the homotopy perturbation series (5) which is a Taylor series, we obtain the closed form solutions

$$u(x,t) = \frac{c_2}{2} + 2c_2\operatorname{sech}^2\left(k(Rt+x)\right),\quad v(x,t) = 2k\operatorname{sech}\left(k(Rt+x)\right), \tag{14}$$

where $R = \frac{1}{2}\left(b_0^2 + c_2\right)$ and b_0, c_2, k are arbitrary constants. These solutions are constructed by Fan [28].

Secondly, we consider an other initial conditions of the JM equation (1) as

$$u(x,0) = s - \frac{b_0k\operatorname{sech}(kx)}{2} - \frac{3c_2\operatorname{sech}^2(kx)}{4},\quad v(x,0) = b_0 + k\operatorname{sech}(kx), \tag{15}$$

Fig. 1a-1f show exact and approximate solutions of *u(x, t)* and *v(x,t)* for Eqs.(1-2).

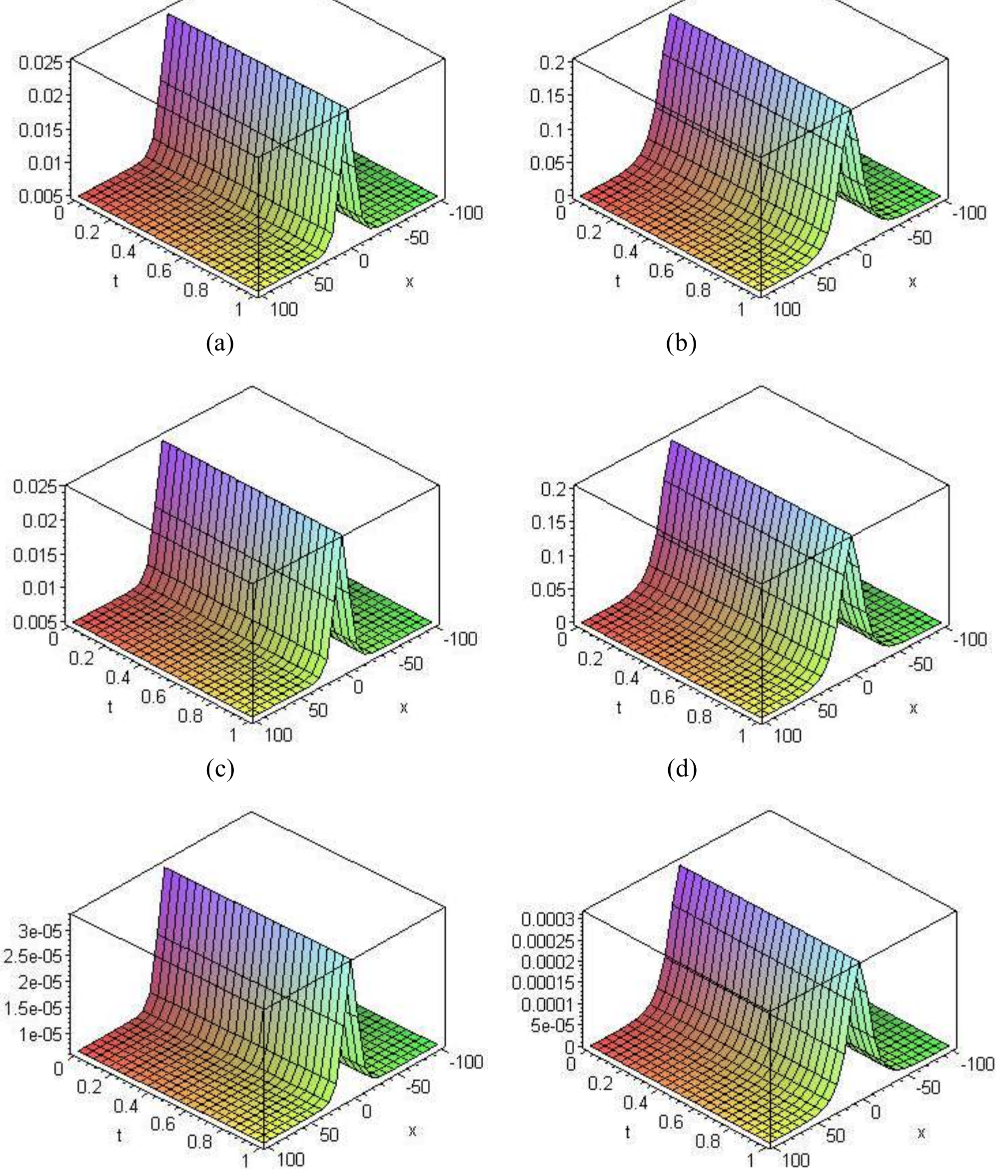

Fig. 1: The surfaces show exact and approximate solutions of $u(x, t)$ and $v(x,t)$ for Eqs.(1-2) $(b_0 = c_2 = 0.01, k = 0.1)$
$(a)\, u_{ex}$, $(b)\, v_{ex}$, $(c)\, u_{hpm}\,(4\text{thorder})$, $(d)\, v_{hpm}\,(4\text{thorder})$, $(e)\, |u_{ex} - u_{hpm}|$, $(f)\, |v_{ex} - v_{hpm}|$

where $s = \frac{1}{4}\left(c_2 - b_0^2\right)$ and b_0, c_2, k are arbitrary constants. Using the same homotopy perturbation procedure, we obtain the exact solution

$$u(x,t) = s - \frac{b_0 k \operatorname{sech}(k(Rt + x))}{2} - \frac{3c_2 \operatorname{sech}^2(k(Rt + x))}{4}, \quad v(x,t) = b_0 + k \operatorname{sech}(k(Rt + x)), \tag{16}$$

where $R = \frac{1}{2}\left(b_0^2 + c_2\right)$.

CONCLUSION

In this study, homotopy perturbation method has been successfully applied to find the solution of nonlinear Jaulent-Miodek equation. Examples show that the results of the present method are in excellent agreement with exact solution and the obtained solutions are shown graphically. In our paper, we use the Maple Package to calculate the functions obtained from the homotopy perturbation method. Some of the advantages of HPM are that the initial solution can be freely chosen with some unknown parameters and that we can easily achieve the unknown parameters in the initial solution.An interesting point about HPM is that with the fewest number of iterations or even in some cases, once, it can converge to correct results.

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