

Smarandache BCH-Algebras

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Abstract: We introduce the notion of Smarandache BCH-algebra and Smarandache (fresh, clean and fantastic) ideals, some example are given and related properties are investigated. Relationship between Q-Smarandache (fresh, clean and fantastic) ideals and other types of ideals are given. Extension properties for Q-Smarandache (fresh, clean and fantastic) ideals are established.

Key words: BCK-algebra . BCH-algebra . Smarandache BCH-algebra . Smarandache (fresh, clean and fantastic) ideals

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INTRODUCTION

A Smarandache structure on a set A means a weak structure W on A such that there exists a proper subset B of A which is embedded with a strong structure S. In [10], W.B. Vasantha Kandasamy studied the concept of Smarandache groupoids, subgroupoids, ideal of groupoids, semi-normal subgroupoids, Smarandache Bol groupoids and strong Bol groupoids and obtained many interesting results about them. Smarandache semi-groups are very important for the study of congruences and it was studied by R. Padilla [9].

As it is well known, BCK/ BCI-algebras are two classes of algebras of logic. They were introduced by Imai and Iseki [3, 4, 8]. BCI-algebras are generalizations of BCK-algebra. Most of algebras related to the t-norm based logic, such as MTL-algebras, BL-algebras, hoop, MV-algebras and Boolean algebras *et al.*, are extensions of BCK-algebras.

In 1983, Hu and Li [6, 7] introduced the notion of a BCH-algebra, which is a generalization of the notions of BCK and BCI-algebras and studied by many researchers [1, 2, 6].

It will be very interesting to study the Smarandache structure in these algebraic structures. In [5], Y. B. Jun discussed the Smarandache structure in BCI-algebras.

In this paper we introduce the notion of Smarandache BCH-algebra and we deal with Smarandache ideal structures in Smarandache BCH-algebras. We introduce the notion of Smarandache (fresh, clean and fantastic) ideal in a BCH-algebra and then we obtain some related results which have been mentioned in the abstract.

PRELIMINARIES

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCH-algebra if it satisfies the following axioms: for every $x, y, z \in X$, [6].

$$(I1) \quad x * x = 0,$$

$$(I2) \quad x * y = 0 \text{ and } y * x = 0 \text{ imply } x = y,$$

$$(I3) \quad (x * y) * z = (x * z) * y,$$

In a BCH-algebra X, the following holds for all $x, y \in X$

$$(J1) \quad x * 0 = x,$$

$$(J2) \quad (x * (x * y)) * y = 0,$$

$$(J3) \quad 0 * (x * y) = (0 * x) * (0 * y),$$

$$(J4) \quad 0 * (0 * (0 * x)) = 0 * x,$$

$$(J5) \quad x \leq y \text{ implies } 0 * x = 0 * y.$$

An algebra $(X, *, 0)$ is called a BCK-algebra if it satisfies the following conditions for every $x, y, z \in X$

$$(a1) \quad ((x * y) * (x * z)) * (z * y) = 0,$$

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- (a2) $(x * (x * y)) * y = 0$,
- (a3) $x * 0 = x$,
- (a4) $x * y = 0$ and $y * x = 0$ imply $x = y$,
- (a5) $0 * x = 0$.

In any BCH/BCK-algebra X we can define a partial order \leq by putting $x \leq y$ if and only if $x * y = 0$.

SMARANDACHE BCH-ALGEBRAS

Definition 1: A Smarandache BCH-algebra is defined to be a BCH-algebra X in which there exists a proper subset Q of X such that

- (s1) $0 \in Q$ and $|Q| \geq 2$,
- (s2) Q is a BCK-algebra under the operation of X .

Example 2: Let $X = \{0, 1, 2, 3, 4\}$. The following table shows the BCH-algebra structure on X

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	1	4
2	2	2	0	0	4
3	3	3	3	0	4
4	4	4	4	4	0

Consider $Q = \{1, 2, 3\}$, thus Q is a BCK-algebra which is properly contained in X . Then $(X, *, 0)$ is a Smarandache BCH-algebra.

In what follows, let X and Q denote a Smarandache BCH-algebra and a BCK-algebra which is properly contained in X , respectively.

Definition 3: A nonempty subset I of X is called a Smarandache ideal of X related to Q (or briefly, Q -Smarandache ideal of X) if it satisfies:

- (c1) $0 \in I$,
- (c2) $(\forall x \in Q)(\forall y \in I)(x * y \in I \Rightarrow x \in I)$.

Example 4: Let X be a Smarandache BCH-algebra of above example. It is easily checked that $I = \{0, 2\}$ is a Smarandache ideal of X .

If I is a Smarandache ideal of X related to every BCK-algebra contained in X , we simply say that I is a Smarandache ideal of X .

Proposition 5: Any ideal of X is a Q -Smarandache ideal of X .

By the following example we show that the converse of above proposition is not correct in general.

Example 6: Let $X = \{0, a, b, c, d, e, f, g, h, i, j, k, l, m, n\}$. The following table shows the BCH-algebra structure on X .

*	0	a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	0	0	0	0	0	0	0	0	h	h	h	h	l	l	n
a	a	0	a	0	a	0	a	0	h	h	h	h	m	l	n
b	b	b	0	0	f	f	f	f	i	h	k	k	l	l	n
c	c	b	a	0	g	f	g	f	i	h	k	k	m	l	n
d	d	d	0	0	0	0	d	d	j	h	h	j	l	l	n
e	e	e	a	0	a	0	e	d	j	h	h	j	m	l	n
f	f	f	0	0	0	0	0	0	k	h	h	h	l	l	n
g	g	f	a	0	a	0	a	0	k	h	h	h	a	l	n
h	h	h	h	h	h	h	h	h	0	0	0	0	n	n	l
i	i	i	h	h	k	k	k	k	b	0	f	f	n	n	l
j	j	j	h	h	h	h	j	j	d	0	0	d	n	n	l
k	k	k	h	h	h	h	h	h	f	0	0	0	n	n	l
l	l	l	l	l	l	l	l	l	n	n	n	n	0	0	h
m	m	l	m	l	m	l	m	l	n	n	n	n	a	0	h
n	n	n	n	n	n	n	n	n	l	l	h	l	h	h	0

Consider $Q = \{0, a\}$, thus Q is a BCK-algebra. Then X is a Smarandache BCH-algebra. It is clear that $I = \{0, a, b\}$ is a Q -Smarandache ideal, which is not an ideal, since $d * b = 0 \in I$ and $b \in I$, but $d \notin I$.

Proposition 7: If Q satisfies $Q * X \subset Q$, then every Q -Smarandache ideal of X satisfies in the following implication

$$(\forall x, y \in I)(\forall z \in Q)((z * y) * x = 0 \Rightarrow z \in I) \tag{1}$$

Proof: Assume that $Q * X \subset Q$ and let I be a Q -Smarandache ideal of X . Suppose that $(z * y) * x = 0$, for all $x, y \in I$ and $z \notin Q$, then $(z * y) \in Q$ by assumption and $(z * y) * x \in I$ then $z * y \in I$ by (c2) since $y \in I$, it follows that $z \in I$. This completes the proof.

Theorem 8: Let Q_1 and Q_2 are BCK-algebras which are properly contained in X and $Q_1 \subset Q_2$. Then every Q_2 -Smarandache ideal is a Q_1 -Smarandache ideal.

The following example shows that the converse of above theorem is not true in general.

Example 9: Let $X = \{0, a, b, c, d, e\}$. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e
0	0	0	0	0	d	d
a	a	0	0	a	d	d
b	b	b	0	b	d	d
c	c	c	c	0	d	d
d	d	d	d	d	0	0
e	e	d	d	e	a	0

Note that the subsets $Q_1 = \{0, b\}$ and $Q_2 = \{0, a, b, c\}$ are BCK-algebra which are properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that $I = \{0, b, c\}$ is a Q_1 -Smarandache ideal of X and it is not a Q_2 -Smarandache ideal of X, since $a * b = 0 \in I$ and $b \in I$, but $a \notin I$

Definition 10: A nonempty subset I of X is called a Smarandache fresh ideal of X related to Q (or briefly, Q-Smarandache fresh ideal of X) if it satisfies the condition (c1) and

$$(c3) \forall x, y, z \in Q((x * y) * z \in I, y * z \in I \Rightarrow x * z \in I)$$

Example 11: Let $X = \{0, 1, 2, 3\}$. The following table shows the BCH-algebra structure on X.

*	0	1	2	3
0	0	0	2	2
1	1	0	2	2
2	2	2	0	0
3	3	2	1	0

Note that $Q = \{0, 1\}$ is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that $I = \{0, 1\}$ is a Q-Smarandache fresh ideal of X.

Theorem 12: Let Q_1 and Q_2 are BCK-algebras which are properly contained in X and $Q_1 \subseteq Q_2$. Then every Q_2 -Smarandache fresh ideal of X is a Q_1 -Smarandache fresh ideal of X.

The following example shows that the converse of above theorem is not true in general.

Example 13: Let $X = \{0, 1, 2, 3, 4, 5\}$. The following table shows the BCH-algebra structure on X.

*	0	1	2	3	4	5
0	0	0	0	0	4	4
1	1	0	0	1	4	4
2	2	2	0	2	4	4
3	3	3	3	0	4	4
4	4	4	4	4	0	0
5	5	4	4	5	1	0

Note that $Q_1 = \{0, 1\}$ and $Q_2 = \{0, 1, 2, 3\}$ are BCK-algebra which are properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that a subset $I = \{0, 2, 3\}$ is a Q_1 -Smarandache fresh ideal of X and is not a Q_2 -Smarandache fresh ideal of X. Since $(1 * 2) * 3 = 0 \in I$ and $2 * 3 = 2 \in I$ but $1 * 3 = 1 \notin I$

Proposition 14: If I is a Q-Smarandache fresh ideal of X, then

$$(\forall x, y \in Q)((x * y) * y \in I \Rightarrow x * y \in I)$$

Proof: Assume that $(x * y) * y \in I$ for all $x, y \in Q$. Since $y * y = 0 \in I$. By (I1) and (c1), it follows from (c3) that $x * y \in I$. This is the desired result.

Theorem 15: Every Q-Smarandache fresh ideal which is contained in Q is a Q-Smarandache ideal.

Proof: Let I be a Q-Smarandache fresh ideal of X which is contained in Q. Let $x \in Q$ and $y \in I$ be such that $x * y \in I$. Then $(x * y) * 0 = x * y \in I$ and $y * 0 = y \in I$. Since $x \in Q$ and $y \in I \subseteq Q$ it follow from (c3) and (I3) that $x = x * 0 \in I$ so that I is a Q-Smarandache ideal of X.

The following example shows that the converse of above theorem is not true in general.

Example 16: Let $X = \{0, a, b, c, d, e\}$. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e
0	0	0	0	0	0	e
a	a	0	0	0	a	e
b	b	a	0	a	b	e
c	c	c	c	0	d	d
d	d	d	d	d	0	e
e	e	e	e	e	e	0

Note that $Q = \{0, a, b, c, d\}$ is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that a subset $I = \{0, d\}$ is a Q-Smarandache ideal of X which is not a Q-Smarandache fresh ideal. Since $(b * a) * c = 0 \in I$ and

$$a * c = 0 \in I, \text{ but } b * c = a \notin I$$

We provide conditions for a Q-Smarandache ideal to be a Q-Smarandache fresh ideal.

Theorem 17: If I is a Q-Smarandache ideal of X such that

$$(\forall x, y, z \in Q)((x * y) * z \in I \Rightarrow (x * z) * (y * z) \in I) \quad (2)$$

Then I is a Q-Smarandache fresh ideal of X.

Proof: Assume that $(x * y) * z \in I$ and $y * z \in I$, for all $x, y, z \in Q$. Then $(x * z) * (y * z) \in I$ by (2) and so $x * z \in I$ by (c2). Therefore I is a Q-Smarandache fresh ideal of X.

Proposition 18: If I is a Q-Smarandache fresh ideal of X which is contained in Q, then

$$(\forall x, y \in Q)(\forall z \in I)((x * y) * y * z \in I \Rightarrow x * y \in I) \quad (3)$$

Proof: Assume that $((x * y) * z) \in I$ for all $x, y \in Q$ and $z \in I$. If I is a Q-Smarandache fresh ideal of X, which is contained in Q, then I is a Q-Smarandache ideal of X. Using (c2), we know that $(x * y) * y \in I$ by Proposition 14, we get that $x * y \in I$.

Theorem 19: Let I and J are Q-Smarandache ideal of X and $I \subset J$. If I is a Q-Smarandache fresh ideal of X and I satisfies in following condition

$$(\forall x, y, z \in Q)((x * y) * z \in I \Rightarrow (x * z) * (y * z) \in I)$$

Then J is a Q-Smarandache fresh ideal of X.

Proof: Let $(x * y) * z \in J$ for all $x, y, z \in Q$. Using (I1) and (I3), we have $((x * ((x * y) * z)) * y) * z = ((x * y) * z) * ((x * y) * z) = 0 \in I$. Since I is a Q-Smarandache fresh ideal of X and by hypothesis we get that

$$((x * z) * (y * z)) * ((x * y) * z) = ((x * ((x * y) * z)) * z) * (y * z) \in I \subset J$$

So we get that $(x * y) * (y * z) \in J$. This proves that J is a Q-Smarandache fresh ideal of X.

SMARANDACHE CLEAN IDEALS

Definition 1: A nonempty subset I of X is called a Smarandache clean ideal of X related to Q (or briefly, Q-Smarandache clean ideal of X) if it satisfies the condition (c1) and

$$(c4) (\forall x, y \in Q)(\forall z \in I)((x * (y * x)) * z \in I \Rightarrow x \in I)$$

Example 2: Let $X = \{0, a, b, c, d, e\}$. The following table shows the BCH-algebra structure on X.

*	0	a	b	c	d	e
0	0	0	0	0	0	e
a	a	0	0	0	0	e
b	b	a	0	a	0	e
c	c	c	c	0	0	e
d	d	d	d	d	0	e
e	e	e	e	e	e	0

Note that $Q = \{0, a, b, c, d\}$ is a BCK-algebra which is properly contained in X. Then X is a Smarandache BCH-algebra. It is easily checked that $I = \{0, a, b, c\}$ is a Q-Smarandache clean ideal of X.

Theorem 3: Every Q-Smarandache clean ideal of X is a Q-Smarandache ideal of X.

Proof: Let I be a Q-Smarandache clean ideal of X. Let $x \in Q$ and $z \in I$ be such that $x * z \in I$. Taking $y = x$ in (c4), we have

$$(x * (x * x)) * z = (x * 0) * z = x * z \in I$$

and so $x \in I$. Hence I is a Q-Smarandache ideal of X.

The following example shows that converse of above theorem is not correct in general.

Example 4: Let $X = \{0, 1, 2, 3, 4\}$ with the following table shows the BCH-algebra structure on X.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	4	4	0

It is easily checked that a subset $I = \{0, 2\}$ is a Q-Smarandache ideal of X, but it is not a Q-Smarandache clean ideal of X, since

$$(3 * (3 * 4)) * 2 = (3 * 3) * 2 = 0 * 2 = 0 \in I, \text{ but } 3 \notin I$$

Remark 5: If Q is an implicative BCK-algebra, that is, Q satisfies the condition:

$$(\forall x, y \in Q)(x = x * (y * x))$$

Then every Q-Smarandache ideal is a Q-Smarandache clean ideal.

The following example shows that every Q-Smarandache fresh ideal is not a Q-Smarandache clean ideal and also every Q-Smarandache clean ideal is not a Q-Smarandache fresh ideal.

Example 6: Let $X = \{0, 1, 2, 3\}$. The following table shows the BCH-algebra structure on X .

*	0	1	2	3
0	0	0	0	0
1	1	0	3	3
2	2	0	0	2
3	3	0	0	0

Note that $Q = \{0, 1, 2, 3\}$ is BCK-algebra which is properly contained in X . Then X is a Smarandache BCH-algebra. It is easily checked that a subset $I = \{0, 3\}$ is a Q-Smarandache clean ideal of X , but it is not a Q-Smarandache fresh ideal of X . Since $(2*3)*2=0 \in I$ and $3*2=0 \in I$, but $2*3=2 \notin I$.

Example 7: Let $X = \{0, 1, 2, 3, 4\}$ with the following table shows the BCH-algebra structure on X .

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	2	4	0

it is easily checked that a subset $I = \{0, 2, 4\}$ is a Q-Smarandache fresh ideal, but is not a Q-Smarandache clean ideal of X , since $(1*(3*1))*1=0$, but $1 \notin I$.

Proposition 8: Every Q-Smarandache clean ideal I of X satisfies the following implication:

$$\forall x, y \in Q, y*(y*x) \in I \Rightarrow x*(x*y) \in I \tag{4}$$

Proof: Let I be a Q-Smarandache clean ideal of X . Then I is a Q-Smarandache ideal of X (Theorem 3). Assume that $y*(y*x) \in I$, for all $x, y \in Q$. Since $((x*(x*y))*(y*(x*(x*y))))*(y*(y*x)) = 0 \in I$ it follows from (J1) and (c2) that $((x*(x*y))*(y*(x*(x*y))))*0 = (x*(x*y))*(y*(x*(x*y))) \in I$. So we get that $(x*(x*y)) \in I$.

Theorem 9: Let I be a Q-Smarandache fresh ideal of X which is contained in Q . If I satisfy the following condition:

$$(\forall x, y \in Q)(x*(y*x)) \in I \Rightarrow x \in I \tag{5}$$

and if $x \leq y$ implies that $a*z \leq y*z$, then I is a Q-Smarandache clean ideal of X .

Proof: Let $x, y \in Q$ and $z \in I$ be such that $(x*(y*x))*z \in I$. If I is a Q-Smarandache fresh ideal which is contained in Q , then I is a Q-Smarandache ideal. Thus $x*(y*x) \in I$ by (c2). Since

$$((y*(y*x))*(y*x))*(x*(y*x)) = 0 \in I$$

it follows from (c2) that $(y*(y*x))*(y*x) \in I$ by Proposition 14, we get that $y*(y*x) \in I$. Hence by (5), $x*(x*y) \in I$ on the other hand, note that

$$((x*y)*z)*(x*(y*x)) = 0 \in I$$

Then $(x*y)*z \in I$ and so $x*y \in I$. Therefore $x \in I$.

Theorem 10: Let I be a Q-Smarandache ideal of X . Then I is a Q-Smarandache clean ideal of X if and only if I satisfies the following condition:

$$(\forall x, y \in Q)(x*(y*x) \in I \Rightarrow x \in I) \tag{6}$$

Proof: Suppose that $(x*(y*x))*z \in I$, for all $x, y \in Q$ and $z \in I$. Then $x*(y*x) \in I$ by (c2) and so by (6) $x \in I$.

Conversely, assume that I is a Q-Smarandache clean ideal of X and $x, y \in Q$ be such that $x*(y*x) \in I$. Since $0 \in I$, it follows from (J1) that $(x*(y*x))*0 = x*(y*x) \in I$, so from (c4) we get that $x \in I$.

SMARANDACHE FANTASTIC IDEALS

Definition 1: A nonempty subset I of X is called a Smarandache fantastic ideal of X related to Q (or briefly, Q-Smarandache fantastic ideal of X) if it satisfies the condition (c1) and

$$(c5) (\forall x, y \in Q)(\forall z \in I)(x*y)*z \in I \Rightarrow x*(y*(y*x)) \in I$$

In the following example we show the relationship between the Q-Smarandache fantastic ideal of X and other types of Q-Smarandache ideals which defined before.

Example 2: Let $X = \{0, 1, 2, 3, 4, 5\}$. The following table shows the BCH-algebra structure on X .

*	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0	1	0	1	5
2	2	2	0	2	0	5
3	3	1	3	0	3	5
4	4	4	4	4	0	5
5	5	5	5	5	5	0

Note that $Q = \{0, 1, 2, 3, 4\}$ is a BCK-algebra which is properly contained in X . Then X is a Smarandache BCH-algebra. It is easily checked that a subset $I_1 = \{0, 2\}$ and $I_2 = \{0, 2, 4\}$ are Q-Smarandache fantastic ideal of X , but it is not a Q-Smarandache fresh ideal of X . $I_3 = \{0, 1, 3\}$ is a Q-Smarandache fresh ideal, but is not a Q-Smarandache fantastic ideal, since $(2*4)*3 = 0 \in I_3$ and $2*(4*(4*2)) = 2 \notin I_3$.

Also $I_1 = \{0, 2\}$ is a Q-Smarandach fatastic ideal of X , but it is not a Q-Smarandache clean ideal, since $(1*(3*1))*0 = 0 = (1*1)*0 = 0 \in I_1$, but $1 \notin I_1$.

Theorem 3: Let Q_1 and Q_2 are BCK-algebras which are properly contained in X and $Q_1 \subseteq Q_2$. Then every Q_2 -Smarandache fantastic ideal is a Q_1 -Smarandache fantastic ideal.

The following example shows that converse of above theorem is not correct in general.

Example 4: Let $X = \{0, 1, 2, 3, 4, 5\}$. The following table shows the BCH-algebra structure on X .

*	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0	1	0	1	5
2	2	2	0	2	0	5
3	3	1	3	0	3	5
4	4	4	4	4	0	5
5	5	5	5	5	5	0

Note that $Q_1 = \{0, 2, 4\}$ and $Q_2 = \{0, 1, 2, 3, 4\}$ are BCK-algebra which are properly contained in X and $Q_1 \subseteq Q_2$. Then $I = \{0, 1, 3\}$ is a Q_1 -Smarandache fantastic ideal of X , but is not a Q_2 -Smarandache fantastic ideal of X .

Theorem 5: Every Q-Smarandache fantastic ideal of X is a Q-Smarandache ideal of X .

Proof: Let I be a Q-Smarandache fantastic ideal of X . Assume that $x*z \in Q$, for all $x \in Q$ and $z \in I$. Using (J1), we get that $(x*0)*z = x*z \in I$. Since $x \in Q$ and Q is a BCK-algebra, it follows from (a5), (J1) and (c5) that $x = x*(0*(0*x)) \in I$, so that I is a Q-Smarandache ideal of X .

Theorem 6: Let I be a Q-Smarandache ideal of X . Then I is a Q-Smarandache fantastic ideal of X if and only if I satisfies the following condition:

$$(\forall x, y \in Q)(x * y \in I \Rightarrow x * (y * (y * x)) \in I)$$

Proof: Suppose that I satisfies in (7), $(x*y)*z \in I$, for all $x, y \in Q$ and $z \in I$. Then $x*y \in I$ by (c2) and so $x*(y*(y*x)) \in I$.

Conversely, assume that I is a Q-Smarandache fantastic ideal of X and let $x, y \in Q$ be such that $x*y \in I$. Using (J1), we have $(x*y)*0 = x*y \in I$ and $0 \in I$. It follows from (c5) that $x*(y*(y*x)) \in I$.

Theorem 7: Let I and J are Q-Smarandache ideals of X and $I \subseteq J \subseteq Q$. If I is a Q-Smarandache fantastic ideal of X and I satisfied in following condition

$$(\forall x, y, z \in Q)((x * y) * z) \in I \Rightarrow (x * y) * (y * z) \in I$$

Then J is a Q-Smarandache fantastic ideal of X .

Proof: Assume that $x*y \in J$, for all $x, y \in Q$. Since

$$(x * (x * y)) * y = (x * y) * (x * y) = 0 \in I$$

It follows from (I3) and (7) that

$$(x * (y * (y * (x * (x * y)))) * (x * y) = (x * (x * y)) * (y * (y * (x * (x * y)))) \in I \subseteq J$$

So from (c2), $x*(y*(y*(x*(x*y))) \in J$. Since $x, y \in Q$ and Q is a BCK-algebra, we conclude that $(x*(y*(y*x)))*(x*(y*(y*(x*(x*y)))) = 0 \in J$ by using (a1). It follows that $x*(y*(y*x)) \in J$. Hence J is a Q-Smarandache fantastic ideal of X .

Theorem 8: Let I be a Q-Smarandache fresh ideal and Q-Smarandache fantastic ideal of X . Also

$$\forall x, y, z \in Q, (x \leq y) \Rightarrow x * z \leq y * z$$

Then I is a Q-Smarandache clean ideal of X .

Proof: Suppose that I is both a Q-Smarandache fresh ideal and Q-Smarandache fantastic ideal. Let $x, y \in Q$ be such that $x*(y*x) \in I$. Since

$$((y * (y * x)) * (y * x)) * (x * (y * x)) = 0 \in I$$

We get that $(y*(y*x))*(y*x) \in I$ by (c2). Since I is a Q-Smarandache fresh ideal, it follows from Proposition 14 that $y*(y*x) \in I$, then $x*y \in I$, on the other hand we have $(x*y)*(y*(y*x)) = 0 \in I$. Since I is a Q-Smarandache fantastic ideal, we obtain $x*(y*(y*x)) \in I$ by (7) and so $x \in I$. Therefore I is a Q-Smarandache clean ideal of X .

CONCLUSION

Smarandache structure occurs as a weak structure in any structure. In the present paper, we have

introduced the concept of Smarandache BCH-algebras and investigated some of their useful properties. In our opinion, these definition and main result can be similarly extended to some other algebraic systems such as BL-algebra, lattices and Lie algebras.

It is our hope that this work would other foundations for further study of the theory of BCH-algebras. Our obtained results can be perhaps applied in engineering, soft computing or even in medical diagnosis.

In our future study of Smarandache structure of BCH-algebras, may be the following topics should be considered:

- To consider the structure of quotient of Smarandache BCH-algebras;
- To get more results in Smarandache BCH-algebras and application;
- To define fuzzy structure on Smarandache BCH-algebras.

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