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A Different Perspective to Haruki's Lemma

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Abstract: In this paper we investigate the nature of the constant in Haruki's Lemma and studied a new proof of the constant and a new theorem in the beams quadrangle.

Key words: Haruki's lemma . circle . beams quadrangle

INTRODUCTION

In his papers [1, 2] Ross Honsberger mentions a remarkably beautiful lemma that he accredits to Hiroshi Haruki. The beauty and mystery of Haruki's lemma is in its apparent simplicity. Yaroslav Bezverkhnyev [3] studied Haruki's lemma and a new related locus problem. We discussed a new proof of Haruki's lemma above.
From the Fig. 1, we have the following equality

$$\frac{|AE|.|BF|}{|EF|} = \text{constant } [3]$$

Lemma 1: Given two nonintersecting chords AB and CD in a circle and a variable point P on the arc AB remote from points C and D, let E and F be the intersections of chords PC, AB and of PD, AB respectively. The following equalities hold:

$$\frac{|AF|.|BE|}{|EF|} = \frac{|AE|.|BF|}{|EF|} + |AB|$$

Proof: From the Figure 2, Following the notation and proof of Lemma 1, we have

$$\begin{aligned} |PF|.|FD| &= |EF|.|FG| \\ &= |EF|(|FB| + |BG|) \\ &= |AF|.|FB| \\ &= (|AE| + |EF|).|FB| \\ &= |AE|.|FB| + |EF|.|FB| = |EF|.|FB| + |EF|.|BG| \end{aligned} \qquad (1)$$

$$\frac{|AE|.|FB|}{|EF|} = \frac{|EF|.|BG|}{|EF|}$$

$$\frac{|AE|.|FB|}{|EF|} = |BG|$$

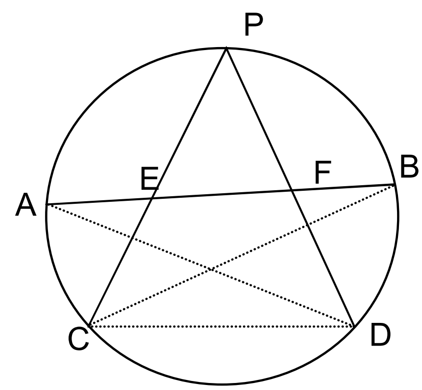

Fig. 1: Proof of the constant

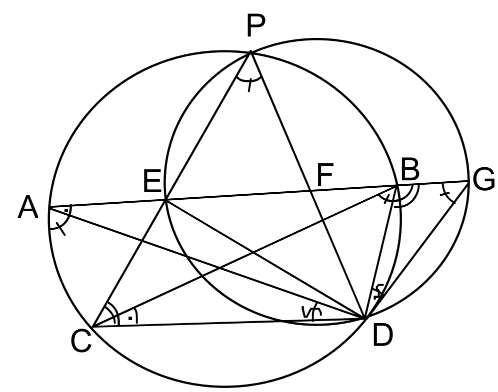

Fig. 2: Proof of the Hizarci's teorem

$$\frac{|AE|.|FB|}{|EF|} = |BG|$$

Note that in Figure 2, we have equal angles, this means that the triangles AGD and CBD are similar $\overset{\triangle}{AGD} \sim \overset{\triangle}{CBD}$

$$\overset{\triangle}{AGD} \sim \overset{\triangle}{CBD} \Rightarrow \frac{|AG|}{|BC|} = \frac{|AD|}{|CD|} \text{ and } |AG| = \frac{|BC|.|AD|}{|CD|} \qquad (2)$$

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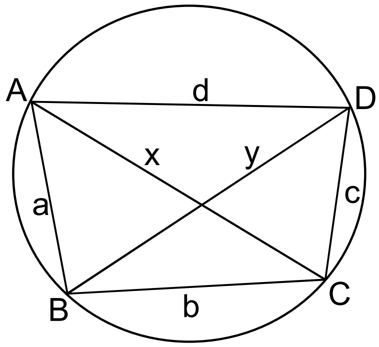

Fig. 3: Last figure

$$\begin{aligned} |AF|.|BE| &= (|AE| + |EF|).(|EF| + |BF|) \\ &= |EF|(|FB| + |BG|) \\ &= |EF|(|FB| + |BG|) \qquad (3) \\ &= |AE|.|EF| + |AE|.|BF| + |EF|.|EF| + |EF|.|BF| \end{aligned}$$

$$\frac{|AF|.|BE|}{|EF|} = \frac{|AE|.|BF|}{|EF|} + |AB|$$

From (1), (2) and (3) we have

$$\frac{|AF|.|BE|}{|EF|} = |AB| + |BG| = |AG|$$

$$\boxed{\frac{|AF|.|BE|}{|EF|} = \frac{|BC|.|AD|}{|CD|}} \qquad (4)$$

Hizarci's Theorem: When points A, B, C, D all belong to the same circle and AC, BD are diagonals

$$|AC|.|BD| + |AB|.|CD| = |BC|.|AD|$$

Proof: We have similar triangles like ACD and GBD, so from the figure we can write

From

$$ACD \;\; GBD$$

$$\frac{|AC|}{|GB|} = \frac{|CD|}{|BD|} \qquad (5)$$

$$|GB| = \frac{|AC|.|BD|}{|CD|}$$

From (3), (4) and (5) the following equalities hold:

$$|AB| + \frac{|AC|.|BD|}{|CD|} = \frac{|BC|.|AD|}{|CD|}$$

$$|AC|.|BD| + |AB|.|CD| = |BC|.|AD|$$

From the Figure 3, at ABCD beams quadrangle we have

$$a.c + b.d = x.y$$

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