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Designs Balanced for Neighbor Effects in Blocks of Size Nine

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Abstract: Nearest neighbor designs have their own importance in the experiments where the performance of a treatment is affected by the treatments applied to its adjacent plots. This is a rich field of investigation in statistics and equally in combinatorics. Experiments in agriculture, horticulture, forestry, serology and industry, neighbor designs are useful because these designs ensure that treatment comparisons will be robust to neighbor effects. In this paper, these designs are constructed in circular binary blocks of size nine. A catalogue of these designs is also presented which is beneficial for practitioners to choose. **MSC(2000): 05B05, 62K10**

Key words: Neighbor designs · Nearest neighbor-balanced designs · Binary blocks · Circular binary blocks

INTRODUCTION

Rees [1] introduced neighbor designs in serology. A neighbor design is a collection of circular blocks in which any two treatments appear as neighbors equally often. In circular block, each treatment has one left and other right neighbor. Experiments in agriculture, horticulture, forestry and industry, neighbor designs are useful because these designs ensure that treatment comparisons will be as little affected by neighbor effects as possible. Rees [1] constructed neighbor designs for all odd v (number of treatments) when k (block size) = v. He also presented neighbor designs for odd v up to 41 with k $\leq$ 10. Hwang [2] constructed some infinite classes of neighbor designs. Lawless [3], Das and Saha [4] and Dey and Chakravarty [5] also constructed neighbor designs for different configurations. Bermond and Faber [6] constructed neighbor designs (i) for v = 2 (mod 4), v=k, b (number of blocks) = v-1 = r and λ' (number of times each pairs of distinct treatment appears as neighbor) = 2 and (ii) for k = v-1, b = v, v is even and λ' = 2. Azais *et al.* [7] proposed several methods for constructing binary circular neighbor designs for k = v and v-1. Ahmed and Akhtar [8] presented computer generated neighbor balanced designs. Iqbal *et al.* [9], Ahmed and Akhtar [10], Akhtar and Ahmed [11] and Iqbal *et al.* [12] considered several cases of neighbor designs. Ai *et al.* [13] constructed complete block all order neighbor balanced designs when v is odd prime. They also constructed all order neighbor balanced designs for v prime or prime power when k $\leq$ v. Ai *et al.* [13] also suggested that the circular blocks neighbor designs are universally optimal. Recently, optimality of circular neighbor-balanced designs for total effects with autoregressive correlated observations is discussed by Ai *et al.* [14].

Neighbor designs for all v in linear blocks of size 3 are constructed by Jacroux [15]. Akhtar *et al.* [16] presented a catalogue of circular block nearest neighbor balanced designs in blocks of size five for v $\leq$ 100. In this study designs balanced for neighbor effects are constructed in circular blocks of size nine. How neighbor designs are generated through initial block (s), it is explained in section 2 with the help of two examples. Ahmed and Akhtar [17] constructed NNBD by cyclic shifts which are presented in section 3 for block size nine when v is prime. Construction methods of NNBD of block size nine are presented in section 4. A catalogue of these designs is presented in section 5. Some of the designs presented here might be somewhere in the literature but their presence in the catalogue is useful for practitioners to choose.

Developing the Designs Through Initial Block(S): The method of construction is to derive the i initial blocks such that the combined set of forward and backward differences between neighboring elements (including the difference of first and last elements of the initial block) takes on all the values 1 to v-1 equally often, λ' times, for further detail see Rees [1]. The procedure to generate the neighbor designs through initial blocks presented in this article is explained here with the help of following two examples.

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Example 2.1: Neighbor design for $v = 11$ and k = 5 is generated through initial block (0,1,10,2,6) in 11 blocks as: First block (B_1) is the initial block itself. Add 1 to each element of B_1 mod 11 to get B_2 and so on.

B_1 = (0,1,10,2,6), B_2 = (1,2,0,3,7), B_3 = (2,3,1,4,8), B_4 = (3,4,2,5,9),
B_5 = (4,5,3,6,10), B_6 = (5,6,4,7,0), B_7 = (6,7,5,8,1), B_8 = (7,8,6,9,2),
B_9 = (8,9,7,10,3), B_{10} = (9,10,8,0,4), B_{11} = (10,0,9,1,5)

Example 2.2: Neighbor design for $v = 6$, k = 5 is generated in six blocks through initial block (0,1,2,4,∞) with augmented block (0,2,4,1,3), where ∞ = 5 as: First block (B_1) is the initial block itself. Add 1 to each element of B_1 mod 5 to get B_2 and similarly B_3 … B_5 then include the augmented block (s) in the blocks obtained through initial block(s) to get required design.

B_1 = (0,1,2,4,∞), B_2 = (1,2,3,0,∞), B_3 = (2,3,4,1,∞),
B_4 = (3,4,0,2,∞), B_5 = (4,0,1,3,∞), B_6= (0,2,4,1,3)

Construction of Nnbd for Odd Prime v and K = 9: If $v = i$k+1 (prime) and k = 9 is relatively prime to (v-1) then circular block NNBD can be generated with $\lambda' = 9$ by developing following i initial blocks cyclically mod v (see Ahmed and Akhtar [17]).

$$I_j = \{(0, j, 2j, 3j, 4j, 5j, 6j, 7j, 8j) \bmod v\ ; j = 1, 2, \ldots, i\}$$

Example 3.10. If $v = 17$ and k = 9 then NNBD can be generated by developing the following eight initial blocks cyclically mod 17.

I_1 = (0,1,2,3,4,5,6,7,8), I_2 = (0,2,4,6,8,10,12,14,16),
I_3 = (0,3,6,9,12,15,1,4,7), I_4 = (0,4,8,12,16,3,7,11,15),
I_5 = (0,5,10,15,3,8,13,1,6) I_6 = (0,6,12,1,7,13,2,8,14),
I_7 = (0,7,14,4,11,1,8,15,5), I_8 = (0,8,16,7,15,6,14,5,13)

Construction of Nearest Neighbor Designs for Block of Size Nine: In this section, nearest neighbor designs are constructed in circular binary blocks of size 9 for several cases.

Case (i). NNBD for $v = 2i$k+1

If $v = 2i\,k + 1$; i is an integer and k = 9 then block NNBD with $\lambda' = 1$ can be generated in $i\,v$ blocks by developing i initial blocks cyclically mod v.

Example 4.1: If $v = 37$ and k = 9 then NNBD can be generated by developing the following two initial blocks cyclically mod 37.

I_1 = (0,1,3,6,10,15,21,28,8), I_2 = (0,28,1,12,24,11,25,3,19)

Case (ii). NNBD for $v = i$k+1; i is odd

If $v = i\,k + 1$; i is an odd and k = 9 then circular block NNBD $\lambda' = 2$ can be generated in $i\,v$ blocks by developing either i initial blocks cyclically mod v or i initial blocks (one of these blocks contains∞) cyclically mod (v-1) along with (v-1)/9 augmented blocks.

Example 4.2: If $v = 28$ and k = 9 then NNBD can be generated by developing the following three initial blocks cyclically mod 28.

$I_1 = I_2$ = (0,1,3,6,10,15,23,2,11), I_3 = (0,10,16,1,13,27,12,24,6)

Example 4.3: If v = 46 and k = 9 then NNBD can be constructed by developing the following five initial blocks cyclically mod 45 with five augmented blocks.

$I_1 = I_2$ = (0,1,3,6,10,16,23,43,19), $I_3 = I_4$ = (0,8,17,27,38,5,18,32,17),
I_5 = (0,5,21,37,10,32,9,27,∞), where ∞ = 45

Augmented Blocks

(0,5,10,15,20,25,30,35,40), (1,6,11,16,21,26,31,36,41), (2,7,12,17,22,27,32,37,42), (3,8,13,18,23,28,33,38,43), (4,9,14,19,24,29,34,39,44)

Case (iii). NNBD for $v = i$k

If $v = i$ k; i is a positive integer and k = 9 then circular binary block NNBD with λ' = 2 can be generated in $i(v$-1) blocks by developing v/3 initial blocks (one of these blocks contains ∞) cyclically mod (v-1).

Example 4.4: If v = 27 and k = 9 then NNBD can be generated by developing the following three initial blocks cyclically mod 26.

$I_1 = I_2$ = (0,1,3,6,11,15,21,2,10), I_3 = (0,9,20,6,19,5,16,25,∞), where ∞ = 26

Case (iv). NNBD when HCF of v and k is 3

If highest common factor (HCF) of v and k = 9 is 3 then circular block NNBD with λ' = 6 can be generated by developing v/3 initial blocks (three of these blocks contain ∞) cyclically mod (v-1).

Example 4.5: If v = 12 and k = 9 then NNBD can be generated by developing the following four initial blocks cyclically mod 11.

I_1 = (0,4,6,9,3,7,10,1,2), I_2 = (0,1,4,8,2,6,9,7,∞),
I_3 = (0,1,4,8,2,6,9,7,∞), I_4 = (0,1,2,3,5,10,4,9,∞), where ∞ = 11

Example 4.6: If v = 15 and k = 9 then NNBD can be generated by developing the following five initial blocks cyclically mod 14.

$I_1 = I_2$ = (0,1,3,7,10,2,8,13,6), I_3 = (0,5,10,12,2,3,6,13,∞),
I_4 = (0,1,3,6,10,11,2,7,∞), I_5 = (0,2,4,7,10,11,1,5,∞), where ∞ = 14

Case (v). NNBD for v odd when HCF of v-1 and k is 3

If v is odd, HCF of v-1 and k = 9 is 3 then circular block NNBD with λ' = 3 can be generated by developing (v-1)/6 initial blocks cyclically mod v.

Example 4.7: If v = 13 and k = 9 then NNBD can be generated by developing the following two initial blocks cyclically mod 13.

I_1 = (0,1,3,7,10,11,4,9,2), I_2 = (0,5,10,12,8,11,1,2,6)

Example 4.8: If v = 25 and k = 9 then NNBD can be generated by developing the following four initial blocks cyclically mod 25.

$I_1 = I_2 = I_3$ = (0,1,4,6,10,15,21,3,11), I_4 = (0,15,24,12,21,5,18,3,13)

Case (vi). NNBD for *v* even when HCF of *v*-1 and k is 3

If *v* is even, HCF of *v*-1 and k = 9 is 3 then circular block NNBD with λ′ = 6 can be generated by developing (*v*-1)/3 initial blocks cyclically mod *v*.

Example 4.9: If *v* = 16 and k = 9 then NNBD can be generated by developing the following five initial blocks cyclically mod 16.

I_1 = (0,1,3,6,10,15,5,12,4), I_2 = (0,1,3,6,10,15,5,12,4),
I_3 = (0,1,3,6,10,15,5,12,4), I_4 = (0,2,3,6,11,1,8,15,5),
I_5 = (0,1,3,4,6,9,12,2,11)

Catalogue of Neighbor-balanced Designs for K = 9

v	λ′	Initial Blocks
18	2	(0,1,3,6,10,15,4,11,2),(0,3,4,8,13,2,9,1,∞), where ∞= 17
19	1	(0,14,15,17,1,5,11,18,10)
21	6	(0,2,6,1,7,14,15,18,10), (0,2,6,1,7,14,15,18,10), (0,2,6,1,7,14,15,18,10), (0,1,3,6,10,15,8,14,2), (0,1,4,8,13,19,6,14,∞), (0,1,4,8,13,19,6,14,∞), (0,9,18,7,16,5,14,12,∞), where ∞ = 20
24	6	(0,1,3,6,10,15,21,5,13), (0,1,3,6,10,15,21,5,13), (0,1,3,6,10,15,21,5,13), (0,1,3,6,10,15,21,5,13), (0,1,3,6,10,15,21,5,13), (0,9,18,4,15,3,14,22,∞), (0,1,3,6,10,15,21,5,∞), (0,9,18,4,15,3,14,1,∞), where ∞ = 23
30	6	(0,1,3,6,10,15,21,28,8), (0,1,3,6,10,15,21,28,8), (0,1,3,6,10,15,21,28,8), (0,1,3,6,10,15,21,28,8), (0,1,3,6,10,15,21,28,8), (0,1,3,6,10,15,21,28,8), (0,10,21,4,17,2,15,27,12), (0,10,20,1,11,21,3,14,∞), (0,11,22,4,16,28,12,24,∞), (0,13,26,10,24,9,23,8,∞), where ∞ = 29
31	2	(0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,11,21,2,15,29,13,27,10), (0,21,1,12,24,5,18,2,15)
33	6	(0,1,3,6,10,5,11,4,12), (0,1,3,6,10,5,11,4,12), (0,1,3,6,10,5,11,4,12), (0,1,3,6,10,5,11,4,12), (0,1,3,6,10,5,11,4,12), (0,1,3,6,10,5,11,4,12), (0,9,19,6,17,31,14,30,13), (0,9,19,6,17,31,14,30,13), (0,10,19,29,6,17,30,9,∞), (0,10,19,29,6,17,30,9,∞), (0,14,28,10,24,7,22,6,∞), where ∞ = 32
36	2	(0,1,3,6,10,24,31,12,17), (0,1,3,6,10,24,31,12,17), (0,6,14,23,33,9,21,34,12), (0,6,14,23,33,9,24,4,∞), where ∞ = 35
39	6	(0,1,3,6,10,15,9,16,24), (0,1,3,6,10,15,9,16,24), (0,1,3,6,10,15,9,16,24), (0,1,3,6,10,15,9,16,24), (0,1,3,6,10,15,9,16,24), (0,1,3,6,10,15,9,16,24), (0,9,19,30,4,17,32,10,27), (0,9,19,30,4,17,32,10,27), (0,9,19,30,4,17,32,10,27), (0,9,19,30,4,17,32,10,27), (0,9,19,30,4,17,32,10,∞), (0,9,19,30,4,17,32,10,∞), (0,19,36,16,34,15,32,13,∞), where ∞ = 38
40	6	(0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,9,19,30,2,15,29,4,21), (0,9,19,30,2,15,29,4,21), (0,9,19,30,2,15,29,4,21), (0,9,19,30,2,15,29,4,21), (0,9,19,30,2,15,29,4,21), (0,16,36,12,32,8,39,9,20), (0,16,32,8,20,33,19,4,21)
42	6	(0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,9,19,30,1,14,28,2,18), (0,9,19,30,1,14,28,2,18), (0,9,19,30,1,14,28,2,18), (0,9,19,30,1,14,28,2,18), (0,9,19,30,1,14,28,2,18), (0,9,19,30,1,14,28,2,∞), (0,16,33,9,26,5,25,4,∞), (0,17,34,10,30,9,29,6,∞), where ∞ = 41
43	3	(0,1,3,6,10,15,21,28,20), (0,1,3,6,10,15,21,28,20), (0,1,3,6,10,15,21,28,20), (0,9,19,30,42,12,26,41,16), (0,9,19,30,42,12,26,41,16), (0,9,19,30,42,12,26,41,16), (0,17,38,16,33,7,26,2,21)
45	2	(0,1,3,6,10,15,21,28,20), (0,1,3,6,10,15,21,28,20), (0,9,19,30,42,11,25,10,26), (0,9,19,30,42,11,25,10,26), (0,17,34,9,28,5,26,4,∞), where ∞ = 44
48	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,31,44,11,26,42,25), (0,9,19,31,44,11,26,42,25), (0,9,19,31,44,11,26,42,25), (0,9,19,31,44,11,26,42,25), (0,9,19,31,44,11,26,42,25), (0,9,19,31,44,11,26,42,25), (0,18,37,10,31,7,28,1,20), (0,18,37,10,31,7,28,9,∞), (0,18,37,10,31,7,28,9,∞), (0,18,36,7,27,3,26,2,∞), where ∞ = 47

v	*λ′*	Initial Blocks
49	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,7,22,6,23), (0,9,19,30,42,7,22,6,23), (0,9,19,30,42,7,22,6,23), (0,18,37,8,29,2,26,48,21), (0,18,36,6,26,7,27,48,24)
51	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,5,20,4,21), (0,9,19,30,42,5,20,4,21), (0,9,19,30,42,5,20,4,21), (0,9,19,30,42,5,20,4,21), (0,9,19,30,42,5,20,4,21), (0,9,19,30,42,5,20,4,21), (0,18,37,7,29,2,26,1,24), (0,18,37,7,29,2,26,1,24), (0,18,37,5,24,44,14,36,∞), (0,18,37,5,24,44,14,36,∞), (0,22,44,17,40,14,38,13,∞), where ∞ = 50
52	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,20,30,42,3,17,2,19), (0,9,20,30,42,3,17,2,19), (0,9,20,30,42,3,17,2,19), (0,9,20,30,42,3,17,2,19), (0,9,20,30,42,3,17,2,19), (0,9,20,30,42,3,17,2,19), (0,26,44,12,37,6,28,51,23), (0,26,44,12,37,6,28,51,23), (0,26,44,12,37,6,28,51,23), (0,20,38,4,24,45,15,46,18), (0,20,41,11,33,5,29,2,27)
54	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,20,30,42,2,16,1,19), (0,9,20,30,42,2,16,1,19), (0,16,36,4,26,49,20,48,21), (0,16,36,5,28,52,24,50, ∞), where ∞ = 53
55	1	(0,1,3,6,10,15,21,28,36), (0,9,19,30,42,11,24,49,20), (0,41,1,17,34,16,37,4,27)
57	6	(0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,1,3,6,10,15,21,14,22), (0,9,19,30,42,55,13,54,16), (0,9,19,30,42,55,13,54,16), (0,9,19,30,42,55,13,54,16), (0,9,19,30,42,55,13,54,16), (0,9,19,30,42,55,13,54,16), (0,9,19,30,42,55,13,54,16), (0,39,2,22,43,10,34,3,29), (0,39,2,22,43,10,34,3,29), (0,39,2,22,43,10,34,3,29), (0,39,2,22,43,10,34,3,29), (0,17,36,1,21,44,12,37,∞), (0,17,36,1,21,44,12,37,∞), (0,26,54,24,52,23,51,22,∞), where ∞ = 56
60	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,58,10,22,35,49,5,21), (0,9,58,10,22,35,49,5,21), (0,9,58,10,22,35,49,5,21), (0,9,58,10,22,35,49,5,21), (0,9,58,10,22,35,49,5,21), (0,9,58,10,22,35,49,5,21), (0,17,57,16,36,14,38,4,30), (0,17,57,16,36,14,38,4,30), (0,17,57,16,36,14,38,4,30), (0,17,57,16,36,14,38,4,30), (0,17,57,16,36,14,38,4,30), (0,17,35,54,15,37,2,27,∞), (0,26,53,21,48,17,45,14,∞), (0,29,56,24,51,20,48,17,∞), where ∞ = 59
61	3	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,8,23,39), (0,9,19,30,42,55,8,23,39), (0,18,1,20,41,21,44,7,33), (0,18,1,20,41,21,44,7,33), (0,18,1,20,41,21,44,7,33), (0,9,20,32,45,59,44,60,30), (0,29,2,12,44,10,37,8,39),
63	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,53,1,12,24,37,51,4,20), (0,53,1,12,24,37,51,4,20), (0,45,1,44,3,25,48,10,35), (0,45,1,44,3,25,48,10,35), (0,28,56,23,52,20,50,19,∞), where ∞ = 62
64		(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,5,20,37), (0,9,19,30,42,55,5,20,37), (0,16,34,53,9,30,52,11,35), (0,16,34,53,9,30,52, 11,35), (0,25,51,17,48,16,47,13,39)
66	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,4,20,35), (0,9,19,30,42,55,4,20,35), (0,9,19,30,42,55,4,20,35), (0,9,19,30,42,55,4,20,35), (0,9,19,30,42,55,4,20,35), (0,9,19,30,42,55,4,20,35), (0,17,35,54,9,30,52,10,34), (0,17,35,54,9,30,52,10,34), (0,17,35,54,9,30,52,10,34), (0,17,35,54,9,30,52,10,34), (0,17,35,54,9,30,52,10,34), (0,17,35,54,9,30,52,10,34), (26,51,13,41,8,36,63,25), (0,25,51,13,41,8,40,3,∞), (0,25,51,13,41,8,40,3,∞), (0,25,50,11,37,63,30,57,∞) where ∞ = 65
67	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,2,17,35), (0,9,19,30,42,55,2,17,35), (0,9,19,30,42,55,2,17,35), (0,16,33,52,5,26,48,4,28), (0,16,33,52,5,26,48,4,28), (0,16,33,52,5,26,48,4,28), (0,25,51,11,40,3,36,65,27), (0,25,50,9,35,8,38,1,34)
69	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,2,16,33), (0,9,19,30,42,55,2,16,33), (0,9,19,30,42,55,2,16,33), (0,9,19,30,42,55,2,16,33), (0,9,19,30,42,55,2,16,33), (0,9,19,30,42,55,2,16,33), (0,16,34,53,5,26,48,3,27), (0,16,34,53,5,26,48,3,27), (0,16,34,53,5,26,48,3,27), (0,16,34,53,5,26,48,3,27), (0,16,34,53,5,26,48,3,27),(0,16,34,53,5,26,48,3,27), (0,25,51,11,40,2,33,67,30), (0,25,51,11,40,2,33,67,30), (0,25,50,8,34,62,22,51,∞), (0,25,50,8,34,62,22,51,∞), (0,29,58,21,52,14,44,10,∞), where ∞ = 68
73	1	(0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,11,27), (0,17,35,54,1,22,55,16,51), (0,23,72,47,21,49,20,63,32)

v	λ'	Initial Blocks
75	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,41,56,40), (0,9,19,30,42,55,41,56,40), (0,9,19,30,42,55,41,56,40), (0,9,19,30,42,55,41,56,40), (0,9,19,30,42,55,41,56,40), (0,9,19,30,42,55,41,56,40), (0,17,35,54,5,25,47,68,44), (0,17,35,54,5,25,47,68,44), (0,17,35,54,5,25,47,68,44), (0,17,35,54,5,25,47,68,44), (0,17,35,54,5,25,47,68,44), (0,17,35,54,5,25,47,68,44), (0,51,4,32,61,19,50,9,35), (0,51,4,32,61,19,50,9,35), (0,51,4,32,61,19,50,9,35), (0,51,4,32,61,19,50,9,35), (0,13,40,68,23,55,12,45,∞), (0,13,40,68,23,55,12,45,∞), (0,26,63,15,52,13,50,11,∞), where ∞ = 74
76	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,8,24), (0,9,19,30,42,55,69,8,24), (0,9,19,30,42,55,69,8,24), (0,9,19,30,42,55,69,8,24), (0,9,19,30,42,55,69,8,24),(0,9,19,30,42,55,69,8,24), (0,17,35,54,74,19,41,64,39), (0,17,35,54,74,19,41,64,39), (0,17,35,54,74,19,41,64,39), (0,17,35,54,74,19,41,64,39), (0,17,35,54,74,19,41,64,39), (0,17,35,54,74,19,41,64,39), (0,26,53,5,51,4,35,67,34), (0,26,53,5,51,4,35,67,34), (0,26,53,5,51,4,35,67,34), (0,26,53,5,51,4,35,67,34), (0,26,53,5,51,4,35,67,34), (0,38,73,32,67,29,55,6,35), (0,38,73,32,60,14,45,1,34)
78	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,7,23), (0,9,19,30,42,55,69,7,23), (0,9,19,30,42,55,69,7,23), (0,9,19,30,42,55,69,7,23), (0,9,19,30,42,55,69,7,23), (0,9,19,30,42,55,69,7,23), (0,17,35,54,74,18,40,64,39), (0,17,35,54,74,18,40,64,39), (0,17,35,54,74,18,40,64,39), (0,17,35,54,74,18,40,64,39), (0,17,35,54,74,18,40,64,39), (0,17,35,54,74,18,40,64,39), (0,26,53,25,54,24,55,10,43), (0,35,70,28,65,25,62,18,∞), (0,26,53,4,33,63,17,49,∞), (0,35,70,28,65,25,62,18,∞), where ∞ = 77
79	3	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,10,21,33,46,60,75,12,29), (0,10,21,33,46,60,75,12,29), (0,18,37,57,78,21,44,68,27), (0,18,37,57,78,21,44,68,27), (0,18,37,57,78,21,44,68,27), (0,54,1,29,59,11,43,76,34), (0,54,1,29,59,11,43,76,34), (0,54,1,29,59,11,43,76,34), (0,10,78,11,24,38,53,69,29), (0,44,61,21,30,65,74,4,39)
81	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,4,20), (0,9,19,30,42,55,69,4,20), (0,17,35,54,75,18,40,16,41), (0,17,35,54,75,18,40,16,41), (0,26,53,1,30,79,49,2,34), (0,26,53,1,30,79,49,2,34), (0,35,70,27,64,22,60,20,∞) where ∞ = 80
82		(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,10,21,33,46,60,75,11,27), (0,10,21,33,46,60,75,11,27), (0,18,37,57,78,19,42,66,40), (0,18,37,57,78,19,42,66,40) (0,17,36,56,77,18,40,64,39), (0,17,36,56,77,18,40,64,39), (0,56,3,32,62,12,44,77,43), (0,56,3,32,62,12,44,77,43), (0,35,70,26,63,21,60,69,∞), where ∞ = 81
		With Augmented Blocks
		(0,9,18,27,36,45,54,63,72), (1,10,19,28,37,46,55,64,73), (2,11,20,29,38,47,56,65,74), (3,12,21,30,39,48,57,66,75), (4,13,22,31,40,49,58,67,76), (5,14,23,32,41,50,59,68,77), (6,15,24,33,42,51,60,69,78), (7,16,25,34,43,52,61,70,79), (8,17,26,35,44,53,62,71,80),
84	6	(0,1,2,3,4,5,6,32,58), (0,2,5,9,14,20,27,35,44), (0,2,5,9,14,20,27,35,44), (0,2,5,9,14,20,27,35,44), (0,2,5,9,14,20,27,35,44), (0,2,5,9,14,20,27,35,44), (0,2,5,9,14,20,27,35,44), (0,10,21,33,46,60,75,8,27), (0,10,21,33,46,60,75,8,27), (0,10,21,33,46,60,75,8,27), (0,10,21,33,46,60,75,8,27),(0,10,21,33,46,60,75,8,27),(0,10,21,33,46,60,75,8,27), (0,17,35,55,76,15,38,62,34), (0,17,35,55,76,15,38,62,34), (0,17,35,55,76,15,38,62,34), (0,17,35,55,76,15,38,62,34), (0,17,35,55,76,15,38,62,34), (0,17,35,55,76,15,38,62,34), (0,29,59,7,39,6,41,5,43), (0,29,59,7,39,6,41,5,43), (0,29,59,7,39,6,41,5,43), (0,29,59,7,39,6,41,5,43), (0,29,59,7,39,6,41,5,43), (0,29,59,7,39,6,41,5,43), (0,25,50,75,17,42,68,11,∞), (0,26,52,6,43,80,34,71,∞), (0,37,78,36,77,35,76,34,∞), where ∞ = 83
87	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,84,14), (0,9,19,30,42,55,69,84,14), (0,9,19,30,42,55,69,84,14), (0,9,19,30,42,55,69,84,14), (0,9,19,30,42,55,69,84,14),(0,9,19,30,42,55,69,84,14), (0,69,3,22,43,65,2,26,51), (0,69,3,22,43,65,2,26,51), (0,69,3,22,43,65,2,26,51), (0,69,3,22,43,65,2,26,51), (0,69,3,22,43,65,2,26,51), (0,69,3,22,43,65,2,26,51), (0,60,1,59,2,32,63,9,42), (0,60,1,59,2,32,63,9,42),(0,60,1,59,2,32,63,9,42), (0,60,1,59,2,32,63,9,42), (0,60,1,59,2,32,63,9,42), (0,60,1,59,2,32,63,9,42), (0,34,71,23,62,22,63,14,52), (0,34,71,23,62,22,63,14,52), (0,34,68,22,62,13,50,2,∞), (0,38,77,30,69,22,62,16,∞), (0,43,84,39,82,37,80,35,∞), where ∞ = 86

v	λ'	Initial Blocks
90	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,17,35,55,76,9,32,8,33), (0,17,35,55,76,9,32,8,33), (0,63,1,29,59,30,61,4,38), (0,63,1,29,59,30,61,4,38), (0,35,72,22,62,14,56,10,54), (0,37,76,27,68,21,64,19,∞), where ∞ = 89
91	1	(0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,84,16), (0,17,35,54,74,4,26,70,24), (0,25,51,78,15,44,74,14,48), (0,33,65,9,46,84,32,83,42)
93	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,17,35,54,74,3,26,2,27), (0,17,35,54,74,3,26,2,27), (0,17,35,54,74,3,26,2,27), (0,17,35,54,74,3,26,2,27), (0,17,35,54,74,3,26,2,27), (0,17,35,54,74,3,26,2,27), (0,66,2,31,61,30,62,3,37), (0,66,2,31,61,30,62,3,37), (0,66,2,31,61,30,62,3,37), (0,66,2,31,61,30,62,3,37), (0,66,2,31,61,30,62,3,37), (0,66,2,31,61,30,62,3,37), (0,35,73,20,60,9,51,2,46), (0,44,89,41,86,40,85,39,∞), (0,35,72,19,59,8,50,1,∞), (0,35,72,19,59,8,50,1,∞),where ∞ = 92
96	6	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36),(0,1,3,6,10,15,21,28,36),(86,1,85,2,15,29,44,60),(86,1,85,2,15,29,44,60),(86,1,85,2,15,29,44,60), (86,1,85,2,15,29,44,60), (86,1,85,2,15,29,44,60), (86,1,85,2,15,29,44,60), (0,17,35,54,74,1,22,45,69), (0,17,35,54,74,1,22,45,69), (0,17,35,54,74,1,22,45,69), (0,17,35,54,74,1,22,45,69), (0,17,35,54,74,1,22,45,69), (0,17,35,54,74,1,22,45,69), (0,25,52,80,14,44,75,12,45), (0,25,52,80,14,44,75,12,45), (0,25,52,80,14,44,75,12,45), (0,25,52,80,14,44,75,12,45), (0,25,52,80,14,44,75,12,45), (0,25,52,80,14,44,75,12,45), (0,34,71,14,53,13,54,1,44), (0,34,71,14,53,13,54,1,44), (0,34,71,14,53,13,54,1,44), (0,34,71,14,53,13,54,1,44), (0,34,71,14,53,13,54,1,44), (0,34,71,14,53,93,39,81,∞), (0,46,92,43,90,42,89,37,∞), (0,46,92,43,90,42,89,38,∞), where ∞ = 95
97	3	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,84,68), (0,9,19,30,42,55,69,84,68), (0,17,35,54,74,95,20,43,67), (0,17,35,54,74,95,20,43,67), (0,17,35,54,74,95,20,43,67), (0,25,51,78,9,40,72,8,42), (0,25,51,78,9,40,72,8,42), (0,25,51,78,9,40,72,8,42), (0,35,72,34,73,16,57,3,47), (0,35,72,34,73,16,57,3,47), (0,35,72,34,73,16,57,3,47), (0,87,78,89,77,90,7,22,68), (0,16,64,15,60,8,53,2,48)
99	2	(0,1,3,6,10,15,21,28,36), (0,1,3,6,10,15,21,28,36), (0,9,19,30,42,55,69,54,70), (0,9,19,30,42,55,69,54,70), (0,17,35,54,74,95,19,42,66), (0,17,35,54,74,95,19,42,66), (0,25,51,78,9,40,70,5,39), (0,25,51,78,9,40,70,5,39), (0,35,72,12,52,11,53,96,44), (0,35,72,12,52,11,53,96,44), (0,45,90,39,86,36,84,35,∞), where ∞ = 98
100		(0,1,3,6,10,15,21,28,36),(0,1,3,6,10,15,21,28,36),(0,9,19,30,42,55,69,84,52), (0,9,19,30,42,55,69,84,52),(0,17,35,54,74,95,18,40,65),(0,17,35,54,74,95,18,40,65), (0,24,50,77,5,34,64,95,33),(0,24,50,77,5,34,64,95,33),(0,34,71,10,50,91,33,90,46), (0,34,71,10,50,91, 33,90,46),(0,55,8,24,75,25,76,92,45)

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REFERENCES

1. Rees, D.H., 1967. Some designs of use in serology, Biometrics, 23: 779-791.
2. Hwang, F.K., 1973. Constructions for some classes of neighbor designs. Annals of Statistics, 1(4): 786-790.
3. Lawless, J.F., 1971. A note on certain types of BIBDs balanced for residual effects. Annals of Mathematical Statistics, 42: 1439-1441.
4. Das, A.D. and G.M. Saha, 1976. On construction of neighbor designs. Calcutta Statist. Assoc. Bull., 25: 151-164.
5. Dey, A. and R. Chakravarty, 1977. On the construction of some classes of neighbor designs. J. Indian Soc. Agri. Statist., 29: 97-104.
6. Bermond, J.C. and V. Faber, 1976. Decomposition of the complete directed graph into k-circuits. J. Comb. Theory B, 21: 146-155.
7. Azais, J.M., R.A. Bailey and H. Monod, 1993. A catalogue of efficient neighbour designs with border plots. Biometrics, 49(4): 1252-61.
8. Ahmed, R. and M. Akhtar, 2009. Computer generated neighbor balanced designs. World Applied Science General. In press.
9. Iqbal, I., M. Amanullah and J.A. Nasir, 2006. The construction of second order neighbor designs. Journal of Research (Science), BZU, Multan, Pakistan, 17(3): 191- 199.
10. Ahmed, R. and M. Akhtar, 2008. Construction of neighbor balanced block designs. Journal of Statistical Theory and Practice, 2(4): 551- 558.

11. Akhtar, M. and R. Ahmed, 2009. Circular binary block second and higher order neighbor designs. Communication in Statistics-Simulation and Computation, 38: 821- 828.
12. Iqbal, I., M.H. Tahir and S.S.A. Ghazali, 2009. Circular neighbor-balanced designs using cyclic shifts. Journal of Science in China Series A: Mathematics. Sci. China Ser, 52(10): 2243-2256.
13. Ai, M., G. Ge and L.Y. Chan, 2007. Circular neighbor-balanced designs universally optimal for total effects. Journal of Science in China Series A: Mathematics, 50: 821-828.
14. Ai, M., Y.L. Yu and S.Y. He, 2009. Optimality of circular neighbor-balanced designs for total effects with autoregressive correlated observations. J. Statist. Plann. Inference, 139: 2293-2304.
15. Jacroux, M., 1998. On the construction of efficient equineighbored incomplete block designs having block size 3. Sankhya 60(B3): 488–495.
16. Akhtar, M., R. Ahmed and F. Yasmin, 2010. A catalogue of nearest neighbor balanced- designs in circular blocks of size five. Pakistan Journal of Statistics. In press.
17. Ahmed, R. and M. Akhtar, 2009. On construction of one dimensional all order neighbor balanced designs by cyclic shifts. Pakistan Journal of Statistics, 25(2): 121-126.