World Applied Sciences Journal 7 (11): 1382-1387, 2009
ISSN 1818-4952

Computer Aided First- and Second-Order Triangular Balanced Designs

[1]Muhammad Nawaz, [2]Muhammad Zafar-Yab and [3]Munir Akhtar

[1]Department of Statistics, The Islamia University of Bahawalpur, Pakistan
[2]Department of Statistics, University of Gujrat, Gujart, Pakistan
[3]COMSATS Institute of Information Technology, Wah Cantt, Pakistan

Abstract: Veevers and Boffey [1] identified 12 elementary balanced arrays. Zafar-Yab [2] confirmed the results of Veevers and Boffey [1] and investigated that among those 12 arrays, six arrays are unique and the other six are their associate partners. In this paper, there are 54 isomorphic classes containing 43-hills. Among these arrays 12 are vertically self-buildable arrays, after suitably augmentation these arrays become isomorphic to 12 elementary balanced arrays identified by Veevers and Boffey [1]. Therefore, those 12 elementary balanced arrays constitute a subset of currently identified 54 isomorphic classes. In addition, four new balanced arrays of 34-hills are constructed on triangular lattice and three of them are partially self-buildable. Another set of 93 balanced designs is identified containing 46-hills. A new family of designs balanced for first- and second-order neighbours is identified. These designs are eligible to find both the first- and second- order effect.

Key words: Elementary Balanced Arrays · First Order Balanced Arrays · Opposite Neighbours · Second Order Balanced Arrays · Variety Competition

INTRODUCTION

Intercropping has received increased attention because it offers potential advantages for resource utilization, decreased inputs and increased sustainability in crop production Andersen *et al.* [3]. It is important to differentiate between intercropping and competition because they differ on the basis of objectives behind them. In intercropping, the objective is to find the best technique to grow in mixture, however, in competition, the mechanism of competition is investigated e.g. how a genotype or a specie in a mixture tolerate the other or provide competition benefit to the other Mead and Riley [4]. If the lack of resources limits the growth of an individual then that individual has suffered from competition Stoll and Weiner [5]. Although the definition of competition has been debated from time to time (e. g. Milne [6]; Thompson [7]; Keddy [8]; and Stoll and Weiner [5]), for most plant ecologists, the core elements that define competition have never nearly departed from one of the earliest published definitions: "Competition occurs where two or more plants make demands for light, nutrients or water in excess of the supply" Weaver and Clements, [9]. According to Bulson *et al.* [10] components of a mixture use limiting resources more efficiently than pure stands. Better biological efficiency of mixtures compared with monocultures may result from differences in growing cycles and root and root architecture Wilson, [11]; Ponce, [12]; Aufhammer *et al.* [13] and Vandermeer, [14]. This phenomenon has been observed in small grains when one component of a mixture is less susceptible to lodging and provides support for the second component Sobkowicz [15]. Sobkowicz and Tendziagolska [16] assesses the productivity of mixtures of oats and wheat and compares two different approaches used in plant competition studies such as replacement designs and in additive designs.

While growing mixtures it is worth investigating how a plant of one variety will perform when surrounded by 0, 1, 2, numbers of immediate neighbouring plants of another variety. Consider a competition experiment utilizing a fifty-fifty mixture of two varieties planted on a triangular lattice. In such an arrangement a plant of one variety could be immediately surrounded by 0, 1, 2, 3, 4, 5 or 6 plants of the other variety thus providing seven levels of competition. It would be desirable to have such a design so that all levels of competition for both varieties appear equally often.

Corresponding Author: Muhammad Nawaz, Department of Statistics,
The Islamia University of Bahawalpur, Pakistan

The first real attempt was made by Martin [17] to construct balanced hexagonal designs. These designs were the key motivation in developing balanced designs on regular lattices. Thus, it laid a foundation of competition designs for two varieties on both triangular and square lattices. Martin's designs are called leveled beehives but those are not balanced. The advantage of using triangular lattice over square lattice is that planting on triangular lattice together with the honeycombing property of the implied hexagons which leads to layouts requiring less area for same plant density and having more scope than the designs for the same purpose on a square lattice (Veevers and Boffey, [18]). They extended leveled beehives and also constructed these for r=8. Veevers and Boffey [1] gave a class of symmetric designs on a triangle lattice. They introduced a method for constructing balanced designs of arbitrary size from elementary balanced arrays. They listed 12 designs possessing vertically self-building property. It is noticed that if any of the self-building elementary arrays is overlapped on its right hand four hills by the left had four hills of the reverse of a copy of itself then a double length balanced array is produced. It is concluded that these cannot be self-buildable horizontally and consequently are of limited use. Zafar-Yab [2] reproduced 12 fundamental generators developed by Veevers and Boffey [1] and confirmed their results. Since these designs occur in pairs- fundamental generator and associate partner, therefore, we need to know only six of these.

Construction of 34 Hills First Order Triangular Balanced Deigns: In investigation of self-buildable designs, let us consider an arrangement of 34 hill plots. This arrangement (balanced arrays in complementary halves) of hills contains 8 similar hills in the first and the last rows and 9 hills each in the second and the third rows. The numbers of possibilities to be investigated are 2^{34}. When one of the arbitrary varieties from 0 and 1 is fixed, say variety 0, at the left most hill of the second row it cuts down the possibilities to half. Each elementary balanced array is identified by N. There are only four isomorphic classes that are presented in Table 2. 1.

None of these arrays posses self-buildability in both directions. However, three of them with array numbers 1, 2 and 4 can said to be partially self-buildable. In these arrays the last hill of each row is the complement of the first hill in the respective row. Therefore, these are only partially self-buildable.

Buildable Arrays in Complementary Halves of Moderate Size: This arrangement of hills comprises 43 hills arranged in 5 rows. In such an arrangement the first and the last rows contain eight similar hills each. While the second, the third and the fourth rows consist of nine hills each. Following the search procedure described earlier 54 arrays are identified possessing balance in complementary halves and are presented in the Table 2. 2 in octal representations.

When two copies of any elementary balanced array constructed by Veevers and Boffey [1] are stacked by overlapping its last row and augmenting by suitable variety the overlapped row, on its right produces one of the 12 currently identified vertically self-buildable arrays. Consequently 54 arrays in Table 2. 2 must contain those 12 vertical buildable elementary balanced arrays of Veevers and Boffey [1]. Array numbers of these 12 elementary balanced arrays are presented as bold face for differentiation in Table 2. 2.

Further more, array numbers 12, 22 and 31 are vertically buildable but only for a single copy of the elementary balanced array. As an example consider array number N=31.

Rotate the elementary balanced array downward about the fifth row. By adding a suitable variety at the boxed hill (here 1), it can be verified that the array is balanced for nine rows. The array is shown in the binary form in the Fig. 2. 1. Since the array is constructed in complimentary halves, therefore basic balanced design can be obtained by placing elementary balanced array and it complement side by side. However, only elementary half is shown in Fig. 2. 1.

Construction of 46 Hills First Order Balanced Arrays on Triangular Lattice: To construct lager arrays possessing self-buildable property, elementary arrays are considered in complementary halves containing three rows. In such arrays, the first and the third rows contain 15 hills each and the second row contains 16 hills. There are 2^{31} possibilities to be investigated. Arbitrary selection of one variety on a particular hill cuts down the explicit consideration to half. Let there be a 0 fixed at the first hill of the second row then there are 62085 isomorphism classes. Unfortunately none of these arrays possesses vertically and infinitely self-buildability property. Among these 61992 arrays which leads to the same number of balanced designs when the two complementary halves are used in an experiment. Such designs can be used in an experiment where only two replications are sufficient.

Table 2.1: Octal representation of first three rows of 34-hills balanced arrays

N	Elementary array		
1	3	25	53
2	7	25	53
3	14	22	44
4	17	25	53

Table 2. 2: Octal representation of first four rows of 43-hills balanced arrays containing five rows

N	Elementary Array				N	Elementary Array				N	Elementary Array			
1	1	4	22	255	2	1	5	22	254	3	1	214	22	214
4	1	260	102	260	5	1	304	102	304	6	1	320	102	320
7	3	4	26	255	8	3	5	26	251	9	3	5	26	254
10	3	211	26	215	11	3	214	26	214	12	10	206	21	207
13	10	206	22	206	**14**	10	207	21	207	15	11	204	26	204
16	11	204	26	205	17	11	204	32	204	**18**	13	204	26	204
19	14	202	31	203	20	14	202	31	206	21	14	206	32	206
22	15	204	22	205	**23**	15	204	32	204	**24**	16	203	35	203
25	17	204	26	204	26	17	204	32	204	27	17	227	136	247
28	17	274	136	274	**29**	36	357	75	357	**30**	36	373	75	373
31	40	206	101	207	32	40	206	102	206	**33**	40	207	101	207
34	41	260	102	260	**35**	41	320	102	320	**36**	57	274	136	274
37	60	200	140	312	38	60	202	141	203	39	60	202	141	302
40	60	212	140	300	41	74	236	172	236	42	74	336	172	336
43	74	366	172	366	**44**	75	364	172	364	**45**	76	217	175	217
46	77	225	137	337	47	77	234	136	234	48	77	234	172	234
49	77	235	136	335	50	77	237	137	325	51	77	267	136	327
52	77	334	172	334	53	77	364	172	364	54	141	204	102	205

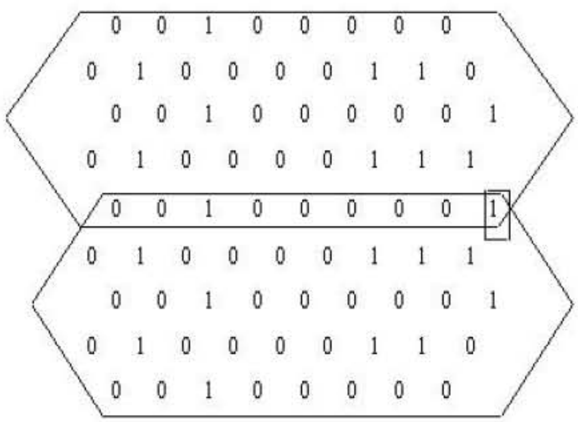

Fig. 2.1: Extension of 5-row balanced in to 9-row balanced array

The remaining 93 elementary balanced arrays are balanced designs in their own right having one replication each of the seven levels of competition for each variety. However, in any experiment it is still advisable to use both complementary halves so as to eliminate any selection bias and such use will generate two replications of the experiment. Since these 93 designs have similar hills in the first and the last rows so the octal representation of the first two rows is given in Table 2. 3.

Construction of Second Order Triangular Balanced Arrays: Veevers and Boffey [1] gave the class of symmetric balanced elementary arrays with respect first order nearest neighbours, having 50-50 mixture for two varieties on a triangular lattice. Basic balanced designs can be obtained by placing elementary array and its complement side by side. The designs consist of monoculture columns in their planting keys.

Table 2. 3: Octal representation of the first two rows containing 15 and 16 hills respectively

N	Elementary Array	N	Elementary Array	N	Elementary Array
1	00676 013217	32	03436 014257	63	07036 010557
2	00736 013057	33	03474 012563	64	07036 010573
3	00756 013643	34	03474 013523	65	07074 012563
4	00766 013613	35	03570 010753	66	07074 016523
5	01374 012723	36	03570 011273	67	07170 012473
6	01476 012617	37	03570 011353	68	07170 016453
7	01574 012353	38	03570 012273	69	07360 012713
8	01574 012743	39	03570 012353	70	07360 016513
9	01636 012157	40	03570 012473	71	07416 013643
10	01636 012173	41	03570 013453	72	07434 012273
11	01674 012563	42	03616 017243	73	07434 013513
12	01674 013523	43	03634 011273	74	07434 013523
13	01716 012743	44	03634 011653	75	07434 016253
14	01734 012473	45	03670 011273	76	07434 016513
15	01734 012713	46	03670 011653	77	07434 016523
16	01734 012723	47	03706 010753	78	07470 012273
17	01734 012743	48	03706 017053	79	07470 016253
18	01734 013453	49	03714 012723	80	07560 012353
19	01734 013513	50	03730 010753	81	07560 013453
20	01734 013523	51	03730 012713	82	07560 013513
21	01754 013523	52	03744 011353	83	07560 013613
22	01764 013513	53	03750 011353	84	07560 016253
23	01770 013507	54	03760 010567	85	07560 016453
24	02374 012723	55	04374 012723	86	07560 016513
25	02770 012273	56	04770 012273	87	07614 016523
26	03076 015217	57	05760 013513	88	07630 016513
27	03174 011353	58	06076 013217	89	07660 016513
28	03370 011273	59	06174 011353	90	07704 016453
29	03436 010557	60	06370 011273	91	07710 016453
30	03436 010573	61	06760 011653	92	07720 016453
31	03436 010657	62	06760 013613	93	07740 016427

Table 2. 4: Octal representation of the first four rows containing 43-hills

N	Elementary array	N	Elementary array	N	Elementary array
1	000 0007 305 0417	2	000 0007 705 1017	3	000 0017 305 0407
4	000 0017 705 1007	5	000 0026 257 0236	6	000 0036 257 0226
7	000 0101 270 1141	8	000 0141 270 1101	9	001 0006 305 0416

For example consider elementary array number N=1.

On a triangle lattice a plant has six nearest (first order) neighbours at a distance d and the next six nearest neighbours at a distance $d\sqrt{3}$ (second order). The plants at different distances from the central plant are likely to produce different effects. In the construction of first order elementary balanced arrays, it is assumed that the effect of second and higher order neighbours is negligible but it is possible only if plants are not planted in close proximity. However, for plantation at close proximity it seems unnatural to ignore second order neighbours, as they are not too far away to be neglected. Thus a need to develop designs, that are balanced with respect to both first as well as second order opposite neighbours separately. The aim is to find weather the second- order neighbours have significant effect on the interior-hill plot.

The term second order balanced arrays in this study stands for balanced arrays with respect to both first- as well as second-order nearest neighbours separately. There are 10 and 14 levels of competition on a square lattice and triangular lattice respectively.

The smallest possible arrangement of hills necessary for second order balanced arrays in complementary halves is shown in Fig. 2. 2. In such an arrangement there are 7 similar hills in the first and the last row and there are 10, 9 and 10 hills in the second, third and fourth row respectively. This concept is explained with the help of Fig. 2. 2.

Consider the internal hexagon in Fig. 2.2 the test-hill enclosed in the circle is surrounded by six hills lying on its perimeter. Each at a distance d are the first order

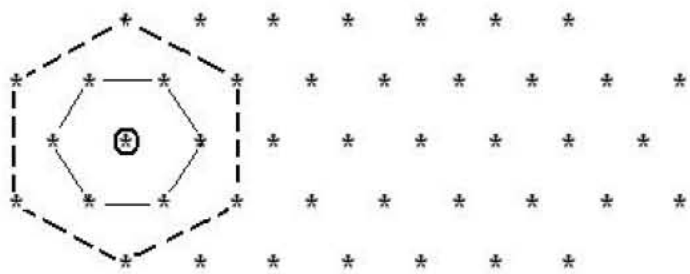

Fig. 2.2: Arrangement of 43 hill plots each hill is represented by a star (*), the internal hills in the third row are test-hills

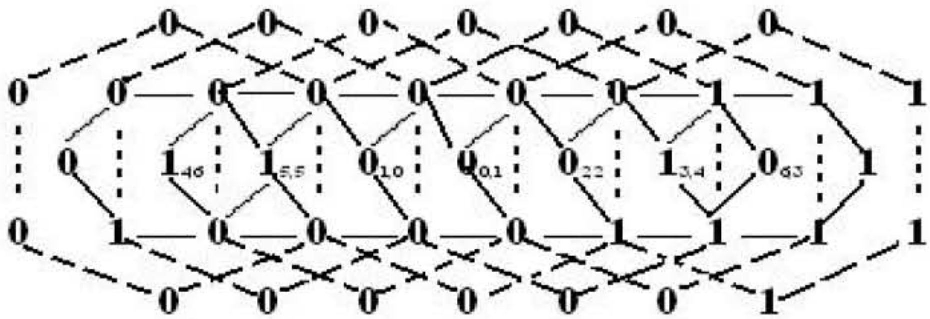

Fig. 2.3: The internal hills of the third row are test-hills, the first and second elements are the numbers of first and second order opposite neighbours

nearest neighbours. The next six hills lying on the perimeter of the external hexagon, are at a distance $d\sqrt{3}$ from the test-hill and are second order nearest neighbours. There are 2^{36} possible configurations, which are large in number to be considered. Following the procedure described earlier, the first hill in the second row is assigned variety 0. The layout of the array advocates that such arrays can not have self-buildability property. The reason for selecting this layout is the limitation of computer's ability to calculate such huge amount. There are 2152 second order balance arrays. These arrays have similar the first and the last rows, therefore four rows of binary digits converted to octal base determine the array. We do not suggest a way to classify these into groups because all are non-buildable in both horizontal and vertical directions. Therefore, only nine of these are presented in Table 2.4. However, the complete list of those 2152 elementary balanced arrays can be obtained from authors on request.

The first and the second elements in the subscript of testable hills are numbers of first and second order opposite neighbours respectively. Their respective configurations are (4, 5, 1, 0, 2, 3, 6) and (6, 5, 0, 1, 2, 4, 3). It can be seen that the array in Fig. 2. 3 is balanced for both the first and second-order opposite neighbours separately. The test ratio of these arrays is 16. 28%. Although this test ratio is very small yet it gives the opportunity to the experimenter to perform the experiment in the presence of second order neighbour effect.

Remarks: All the first order designs in the literature are constructed ignoring the effect of second and higher order effects but if the plants are at close proximity it seem unnatural to ignore the effect of second order neighbours. In the present study this problem is addressed and second order balanced designs are developed.

REFERENCES

1. Veevers, A. and T.B. Boffey, 1979. Designs for balanced observation of plant competition. J. Statisti Plan Inference, 3: 325-331.
2. Zafar-Yab, M., 1980. Construction and Statistical analysis of experimental designs for observing variety competition. Unpublished Ph. D. Thesis, University of Liverpool.
3. Andersen, M.K., H. Nielsen, J. Weiner and E.S. Jensen, 2007. Competitive dynamics in two-and three-component intercrops. J. App. Ecol., 44: 545-551.

4. Mead, R. and J. Riley, 1981. A review of statistical ideas relevant to intercropping research. J. Roy Stat So Ser A, 144: 462-509.
5. Stoll, P. and J. Weiner, 2000. A Neighborhood view of interactions among individual plants. In: The Geometry of Ecological Interactions: Simplifying Spatial complexity, eds. Dieckmann U, Law R and Metz JAJ, pp: 11-27. Cambridge Press.
6. Milne, A., 1961. Defination of competition among animals. In: Milthorpe, F. L. (ed.), Mechanisms in biological competition. Cambridge Press, pp: 61-78.
7. Thompson, K., 1987. The resource ratio hypothesis and the meaning of competition. Funct. Ecol., 1: 297-303.
8. Keddy, P.A., 1989. Competition. Champman and Hall. New York.
9. Weaver, J.E. and F.E. Clements, 1938. Plant Ecology, 2nd ed. McGraw Hill.
10. Bulson, H.A.J., R.W. Snaydon and C.E. Stopes, 1997. Effects of plant density on intercropped wheat and field beans in an organic farming system. J. Agric. Sci. Camb., 128: 59-71.
11. Wilson, J.B., 1988. Shoot competition and root competition. J. Appl. Ecol., 25: 279-296.
12. Ponce, R.G., 1998. Competition beween barley and Lolium rigidum for nitrate. Weed Res., 38: 453-460.
13. Aufhammer, W., H. Kempf, E. Ku¨bler and H. Stu¨ tzel, 1989. Effects of cultivar (wheat) and species (wheat, rye) mixtures on grain yield of disease-free stands. J. Agron. Crop Sci., 163: 319-329. (in German, abstr. in English).
14. Vandermeer, J., 1989. The Ecology of Intercropping. Cambridge University Press, Cambridge, UK.
15. Sobkowicz, P., 2003. Interspecific competition in mixtures of spring cereals. *Zesz. Nauk. AR Wroc. 458, Rozprawy, Wydawnictwo Akademii Rolniczej we Wroclawiu. 1-105 (in Polish, abstr. in English).*
16. Sobkowicz, P. and E. Tendziagolska, 2005. Competition and productivity in mixture of oats and wheat. J. Agro and Crop Sci., 191: 377-385.
17. Martin, F.B., 1973. Beehive designs for observing variety competition. Biometrics, 29: 397-402.
18. Veevers, A. and T.B. Boffey, 1975. On the existence of levelled beehive designs. Biometrics, 31: 963-967.