World Applied Sciences Journal 7 (1): 94-101, 2009
ISSN 1818-4952

Geometric Programming Problems with Fuzzy Parameters and its Application to Crane Load Sway

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Abstract: In this work an approach is proposed to solve geometric programming problems under uncertainty. The proposed approach derives the lower and upper bounds of the objective of geometric programming problems with fuzzy parameters. A pair of two-level mathematical programs is formulated to calculate the lower and upper bounds of the objective value. The solution is in a range. Two illustrative examples are presented to clarify the proposed approach. The problem of suppressing the crane load sway has been also considered as a practical application to illustrate the effectiveness of the proposed approach.

Key words: Optimization . geometric programming . fuzzy parameters . duality theorem . crane load sway

INTRODUCTION

Geometric programming provides a methodology for solving nonlinear optimization problems where nonlinear relations can be presented by exponential function. In most practical applications, the possible values of the model parameters are provided by fuzzy data. Several engineering applications [1-7] have investigated the effectiveness and importance of geometric programming. Effective algorithms have been developed for solving geometric programming problems [8-11]. In geometric programming, the parameters in the problem are not allowed to vary simultaneously while calculating the bounds of the objective value. Many applications of geometric programming are engineering problems in which some of the parameters are estimates of the actual values [12]. A solution procedure for solving posynomial geometric programming with parametric uncertainty has been also introduced [13]. The parameters are represented by ranges and the derived results are also represented by ranges.

This paper presents an approach for solving nonlinear optimization problems under uncertainty. The proposed approach derives the lower and upper bounds of the objective of geometric programming problems with fuzzy parameters. The parameters are represented by triangular fuzzy numbers. A pair of two-level mathematical programs is formulated to calculate the lower and upper bounds of the objective value. The solution is in a range. Two illustrative examples are presented to clarify the proposed approach.

The problem of suppressing the crane load sway is also considered as a practical application of present approach. Our approach implements fuzzy partition to the state variables of the load sway based on Lyapunov synthesis. So that, the resulting control law is stable and able to exploit the dynamic variables of the system in a linguistic manner. The proposed method enables the designer to systematically derive the rule base of the control. The numerical simulation illustrates the effectiveness of the proposed approach.

GEOMETRIC PROGRAMMING PROBLEM

The geometric programming problem with fuzzy parameters (FGPP) can be formulated as follows:
(FGPP):

$$\text{Min } (Z = C X_0) \tag{1}$$

Subject to

$$A X_1 \le b \tag{2}$$

$$X > 0 \tag{3}$$

Where

- X is a column vector of n dimensional representing the decision variables,
- X_o is a column vector of s_o dimensional representing terms of the objective function,
- X_1 is a column vector of s_m dimensional representing terms of the m constraints,
- C is a row vector of so dimensional representing objective function coefficients,
- A is a matrix of m x s_m dimensional representing constraints coefficients,

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b is a column vector of s_m dimensional of positive numbers representing right-hand sides of the constraints.

FUZZY APPROACH FOR GEOMETRIC PROGRAMMING PROBLEM

The objective function coefficients may be represented in triangular fuzzy numbers. Furthermore, the constraints coefficients and the right-hand sides of the constraints may also be represented in triangular fuzzy numbers. This means that:

$$\left.\begin{array}{l} C_{to} = \left[C_{to}^{L}, C_{to}^{m}, C_{to}^{U}\right],\ a_{it} = \left[a_{it}^{L}, a_{it}^{m}, a_{it}^{U}\right] \\ b_t = \left[b_t^{L}, b_t^{m}, b_t^{U}\right], t_o = 1,2,\ldots,s_o \ , t=1,2,\ldots,s_m \\ i= 1,2,\ldots,m \end{array}\right\} \quad (4)$$

The major difficulty lies on how to deal with fuzzy numbers in the objective function and the constraints. Let

$$\left.\begin{array}{l} T = \{\ \tilde{c}_{to}, \tilde{a}_{it}, \tilde{b}_t \mid \\ C_{to}^{L} + \alpha\left(C_{to}^{m} - C_{to}^{L}\right) \le \tilde{c}_{to} \le C_{to}^{U} - \alpha\left(C_{to}^{U} - C_{to}^{m}\right) \\ a_{it}^{L} + \alpha\left(a_{it}^{m} - a_{it}^{L}\right) \le \tilde{a}_{it} \le a_{it}^{U} - \alpha\left(a_{it}^{U} - a_{it}^{m}\right) \\ b_t^{L} + \alpha\left(b_t^{m} - b_t^{L}\right) \le \tilde{b}_t \le b_t^{U} - \alpha\left(b_t^{U} - b_t^{m}\right) \\ \qquad\qquad\qquad 0 \le \alpha \le 1 \ \} \end{array}\right\} \quad (5)$$

For each $\left(\tilde{c}_{to}, \tilde{a}_{it}, \tilde{b}_t\right) \in T$ the value Z ($\tilde{c}_{to}$, $\tilde{a}_{it}$, $\tilde{b}_t$) is denoted by the objective value of the Model (1-3). Let Z^L and Z^U be the minimum and maximum of Z($\tilde{c}_{to}$, $\tilde{a}_{it}$, $\tilde{b}_t$) on T, respectively as follows:

$$\left.\begin{array}{l} Z^{L} = \mathrm{Min}\{Z\left(\tilde{c}_{to},\tilde{a}_{it},\tilde{b}_t\right) \mid \left(\tilde{c}_{to},\tilde{a}_{it},\tilde{b}_t\right) \in T\ \} \\ Z^{L} = \mathrm{Max}\{Z\left(\tilde{c}_{to},\tilde{a}_{it},\tilde{b}_t\right) \mid \left(\tilde{c}_{to},\tilde{a}_{it},\tilde{b}_t\right) \in T\ \} \end{array}\right\} \quad (6)$$

which can be reformulated as the following pair of two-level mathematical problems:

$$Z^{L} = \underset{(\tilde{c}_{to},\ \tilde{a}_{it},\ \tilde{b}_t) \in T}{\mathrm{Min}} \ \underset{x}{\mathrm{Min}}\ C X_o$$

Subject to

$$A X_1 \le b \qquad (7a)$$
$$X > 0$$

$$Z^{U} = \underset{(\tilde{c}_{to},\ \tilde{a}_{it},\ \tilde{b}_t) \in T}{\mathrm{Max}} \ \underset{x}{\mathrm{Min}}\ C X_o$$

Subject to

$$A X_1 \le b \qquad (7b)$$
$$X > 0$$

In Model (7) the right–hand side value b may not be equal to the constant value 1. In this case, the constraint coefficients A can be divided by the right – hand side value b for every constraint i, thus, the following standard geometric programming problems will be as follows:

$$Z^{L} = \underset{(\tilde{c}_{to},\ \tilde{a}_{it},\ \tilde{b}_t) \in T}{\mathrm{Min}} \ \underset{x}{\mathrm{Min}}\ C X_o$$

Subject to

$$A_1 X_1 \le I \qquad (8a)$$
$$X > 0$$

$$Z^{U} = \underset{(\tilde{c}_{to},\ \tilde{a}_{it},\ \tilde{b}_t) \in T}{\mathrm{Max}} \ \underset{x}{\mathrm{Min}}\ C X_o$$

Subject to

$$A_1 X_1 \le I \qquad (8b)$$
$$X > 0$$

where, I is a column vector of s_m dimensional of identity numbers representing right-hand sides of the constraints.

Model (8-a) is to derive the lower bound of the objective value of the Model. The minimum value can be obtained by setting all C to their lower bounds in the objective function. Moreover, since the values of the right-hand sides are the identity values, the lower the ratio of $\tilde{a}_{it}/\tilde{b}_t$ in constraints, the larger the feasible region is. Thus, the value of $\tilde{a}_{it}$ must be set to its lower bound and $\tilde{b}_t$ to its upper bound for every i and t. The Model (8-a) can be transformed to the following fuzzy primal geometric programming problem (FPGPP) as follows: (FPGPP):

$$Z^{L} = \underset{x}{\mathrm{Min}} \sum_{t=1}^{s_o}\left(C_{to}^{L} + \alpha\left(C_{to}^{m} - C_{to}^{L}\right)\right)\prod_{j=1}^{n} x_j^{\beta_{tj}}$$

Subject to

$$\prod_{t=s_o+1}^{s_o+s_m}\left(a_{it}^{L} + \alpha\left(a_{it}^{m} - a_{it}^{L}\right)\right)\left(b_i^{U} - \alpha\left(b_i^{U} - b_i^{m}\right)\right)^{-1}$$
$$\times\prod_{j=1}^{n} x_j^{\beta_{tj}} \le 1,2,\ldots,m \qquad (9)$$
$$0 \le \alpha \le 1,\ x_j > 0,\ j=1,2,\ldots,n$$

The solution techniques for geometric programming problem may be treated as either primal-based algorithms that solve the nonlinear primal problems at different cuts α, or dual-based algorithms that solve the equivalent linear dual problems at also at different cuts α.

Beightler and Phillips [12] and Duffin *et al*. [14], the Model (9) can be transformed to the corresponding fuzzy dual geometric programming problem (FDGPP) as follows:

(FDGPP):

$$Z^L = \underset{y}{\mathrm{Max}} \prod_{t=1}^{s_o} \left(\left(C_{to}^L + \alpha \left(C_{to}^m - C_{to}^L \right) \right) \Big/ y_t \right)^{y_t} \times$$

$$\prod_{i=1}^{m} \prod_{t=s_o+1}^{s_o+s_m} \left[\frac{\left(a_{it}^L + \alpha \left(a_{it}^m - a_{it}^L \right) \right) \left(b_i^U - \alpha \left(b_i^U - b_i^m \right) \right)^{-1} y_t}{y_{it}} \right]^{y_{it}}$$

Subject to

$$\prod_{j=1}^{m} \prod_{t=s_o+1}^{s_o+s_m} \beta_{tj}\, y_t = 0$$

$$\sum_{t=1}^{s_o} y_t = 1$$

$$0 = \alpha = 1,\ y_t = 0,\ t = 1,2,\ldots,s_o + s_m \qquad (10)$$

Model (10) is to find a stationary point of Lagrangian function for a concave objective function subject to a set of convex constraints. Model (10) has a unique stationary point of Lagrangian function – a global maximum [12, 14]. Thus, the lower bound of the objective value of the Model can be obtained.

Model (8-b) is to derive the upper bound of the objective value of the Model. The maximum value can be obtained by setting all C to their upper bounds in the objective function. Moreover, since the values of the right-hand sides are the identity values, the upper the ratio of $\tilde{a}_{it}/\tilde{b}_t$ in constraints, the larger the feasible region is. Thus, the value of $\tilde{a}_{it}$ must be set to its upper bound and $\tilde{b}_t$ to its lower bound for every i and t. The Model (8-b) can be transformed to the following fuzzy primal geometric programming problem (FPGPP) as follows:

(FPGPP):

$$Z^U = \underset{x}{\mathrm{Min}} \sum_{t=1}^{s_o} \left(C_{to}^U - \alpha \left(C_{to}^U - C_{to}^m \right) \right) \prod_{j=1}^{n} x_j^{\beta_{tj}}$$

Subject to

$$\sum_{t=s_o+1}^{s_o+s_m} \left(a_{it}^U - \alpha \left(a_{it}^U - a_{it}^m \right) \right) \left(b_i^L + \alpha \left(b_i^m - b_i^L \right) \right)^{-1} \times \prod_{j=1}^{n} x_j^{\beta_{tj}} \le 1$$

$$i = 1,2,\ldots,m,\ 0 = \alpha = 1,\ x_j > 0,\ j = 1,2,\ldots,n \qquad (11)$$

The solution techniques for geometric programming problem may be treated as either primal-based algorithms that solve the nonlinear primal problems at different cuts α, or dual-based algorithms that solve the equivalent linear dual problems at also at different cuts α.

Model (11) can be transformed to the corresponding fuzzy dual geometric programming problem (FDGPP) as follows:

(FDGPP):

$$Z^U = \underset{y}{\mathrm{Max}} \prod_{t=1}^{s_o} \left(\left(C_{to}^U + \alpha \left(C_{to}^U - C_{to}^m \right) \right) \Big/ y_t \right)^{y_t} \times$$

$$\prod_{i=1}^{m} \prod_{t=s_o+1}^{s_o+s_m} \left[\frac{\left(a_{it}^U + \alpha \left(a_{it}^U - a_{it}^m \right) \right) \left(b_i^L - \alpha \left(b_i^m - b_i^L \right) \right)^{-1} y_t}{y_{it}} \right]^{y_{it}}$$

Subject to

$$\prod_{j=1}^{m} \prod_{t=s_o+1}^{s_o+s_m} \beta_{tj}\, y_t = 0$$

$$\sum_{t=1}^{s_o} y_t = 1$$

$$0 = \alpha = 1,\ y_t = 0,\ t = 1,2,\ldots,s_0 + s_m \qquad (12)$$

Model (12) is to find a stationary point of Lagrangian function for a concave objective function subject to a set of convex constraints. Model (12) has a unique stationary point of Lagrangian function – a global maximum [12, 14]. Thus, the upper bound of the objective value of the Model can be obtained.

The lower bound Z^L and upper bound Z^U of the objective value are solved from Models (10) and (12), respectively. In this paper, the fuzzy parameters in the problem are allowed to vary simultaneously.

ILLUSTRATIVE EXAMPLES

Two geometric programming problems are presented in this paper to illustrate the proposed fuzzy approach.

Illustrative Example 1: Consider the fuzzy geometric programming problem (FGPP1) as follows:

Table 1: Results of fuzzy optimization for the lower bound objective function of Example 1

α	Z^L	y_1	y_2	y_3	y_4	y_5	y_6
0	0.2050	0.9951	0.0049	16.3121	32.3677	4.8388	0.0316
0.1	0.2346	0.9939	0.0061	13.9775	27.6141	4.2137	0.0348
0.2	0.2685	0.9924	0.0077	11.9750	23.9633	3.7014	0.0381
0.3	0.3072	0.9905	0.0095	10.2503	20.4937	3.2245	0.0418
0.4	0.3517	0.9880	0.0120	8.7730	17.2864	2.7895	0.0457
0.5	0.4029	0.9852	0.0148	7.5069	14.9123	2.4393	0.0497
0.6	0.4620	0.9815	0.0185	6.4113	12.6588	2.1126	0.0539
0.7	0.5309	0.9771	0.0230	5.4632	10.7332	1.8250	0.0584
0.8	0.6115	0.9716	0.0285	4.6434	9.1085	1.5741	0.0627
0.9	0.7067	0.9648	0.0352	3.9325	7.6651	1.3475	0.0673
1.0	0.8202	0.9566	0.0434	3.3174	6.4325	1.1473	0.0717

(FGPP1):
$$\mathrm{Min}\left(Z = C^{(1)} X_o^{(1)}\right) \tag{13}$$

Subject to
$$A_1 X_1^{(1)} \le b^{(1)} \tag{14}$$

$$X^{(1)} > 0 \tag{15}$$

where
$$C^{(1)} = \left[(6,8,10) \quad (8,10,12)\right],$$

$$X_o^{(1)} = \left[x_1 x_2^{-1} x_3^{2} x_4^{-2} \quad x_1^{3} x_2^{-2} x_3^{2}\right]^t$$

$$A^{(1)} = \begin{bmatrix} (6,8,10) & 1 & 0 & 0 \\ 0 & 0 & (3,5,7) & 1 \end{bmatrix},$$

$$X_1^{(1)} = \left[x_1 x_3^{3} \quad x_1^{-1} x_3^{-2} \quad x_2 x_3^{-1} x_4^{2} \quad x_1 x_2^{2} x_4\right]^t$$

$$b^{(1)} = \begin{bmatrix} (5,8,10) \\ 1 \end{bmatrix}, \quad X^{(1)} = \left[x_1 \; x_2 \; x_3 \; x_4\right]^t$$

where (6,8,10) and (8,10,12) represent the triangular fuzzy numbers of the coefficient in the objective function, (6,8,10) represent the triangular fuzzy numbers of constraints and (5,8,10) represents the triangular fuzzy number of the right-hand side constraints. In this example, s_0=2 and s_m=4.

According to Models (10) and (12), the problem can be transformed to the following pair of geometric programming problems:

(FDGPP1):

$$Z^L = \underset{y}{\mathrm{Max}}\left(\frac{6+2\alpha}{y_1}\right)^{y_1}\left(\frac{8+2\alpha}{y_2}\right)^{y_2}\left(\frac{(5+2\alpha)(y_3+y_4)}{(10-2\alpha)\,y_3}\right)^{y_3}$$
$$\left(\frac{y_3+y_4}{(10-2\alpha)\,y_4}\right)^{y_4}\left(\frac{y_5+y_6}{y_5}\right)^{y_5}\left(\frac{(3+2\alpha)(y_5+y_6)}{y_6}\right)^{y_6}$$

Subject to:
$$C^{(1)} Y^{(1)} = B^{(1)}$$

$$yt=0,\ t=1,2,\ldots,6 \tag{16}$$

where

$$C^{(1)} = \begin{bmatrix} 1 & 3 & 1 & -1 & 0 & 1 \\ -1 & -2 & 0 & 0 & 1 & 2 \\ 2 & 2 & 3 & -2 & -1 & 0 \\ -2 & 0 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \; Y^{(1)} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}, \; B^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Results of fuzzy optimization for the lower bound of the geometric programming problem at different values of α are given in Table 1.

(FDGPP1):

$$Z^U = \underset{y}{\mathrm{Max}}\left(\frac{10-2\alpha}{y_1}\right)^{y_1}\left(\frac{12-2\alpha}{y_2}\right)^{y_2} \times$$
$$\left(\frac{(9-2\alpha)(y_3+y_4)}{(5+3\alpha)\,y_3}\right)^{y_3}\left(\frac{y_3+y_4}{(5+3\alpha)\,y_4}\right)^{y_4}$$
$$\times\left(\frac{y_5+y_6}{y_5}\right)^{y_5}\left(\frac{(7-2\alpha)(y_5+y_6)}{y_6}\right)^{y_6}$$

Subject to:
$$C^{(1)} Y^{(1)} = B^{(1)}$$
$$y_t=0,\ t=1,2,\ldots,6 \tag{17}$$

where

$$C^{(1)} = \begin{bmatrix} 1 & 3 & 1 & -1 & 0 & 1 \\ -1 & -2 & 0 & 0 & 1 & 2 \\ 2 & 2 & 3 & -2 & -1 & 0 \\ -2 & 0 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \; Y^{(1)} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}, \; B^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Table 2: Results of fuzzy optimization for the upper bound objective function of Example 1

α	Z^U	y_1	y_2	y_3	y_4	Y_5	y_6
0	27.098	0.5634	0.4366	0.0666	0.1180	0.0118	0.0251
0.1	14.953	0.6216	0.3784	0.1274	0.2272	0.0283	0.0386
0.2	8.9117	0.6814	0.3186	0.2249	0.4035	0.0589	0.0534
0.3	5.6958	0.7389	0.2611	0.3684	0.6677	0.1086	0.0669
0.4	3.8683	0.7908	0.2092	0.5665	1.0363	0.1811	0.0775
0.5	2.7638	0.8355	0.1645	0.8254	1.5263	0.2779	0.0840
0.6	2.0575	0.8725	0.1276	1.1512	2.1542	0.3996	0.0864
0.7	1.5824	0.9018	0.0982	1.5528	2.9342	0.5466	0.0855
0.8	1.2483	0.9250	0.0750	2.0380	3.8905	0.7191	0.0822
0.9	1.0042	0.9429	0.0572	2.6204	5.0456	0.9183	0.0773
1.0	0.8202	0.9566	0.0434	3.3174	6.4325	1.1473	0.0717

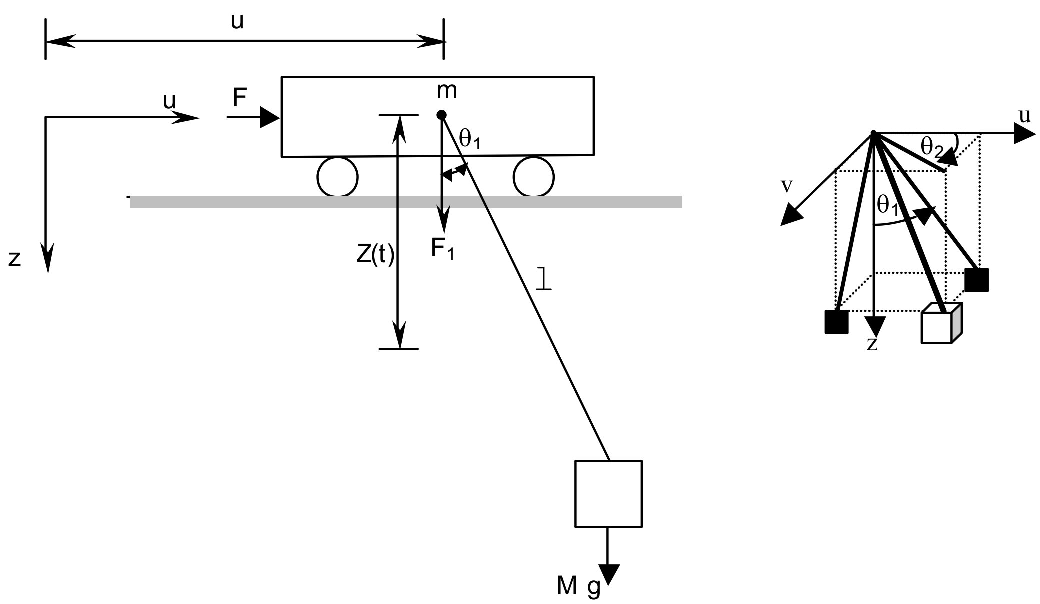

Fig. 2: Simplified Crane Model, Container Mass: M = 100 Kg., Trolley Mass m = 300 Kg

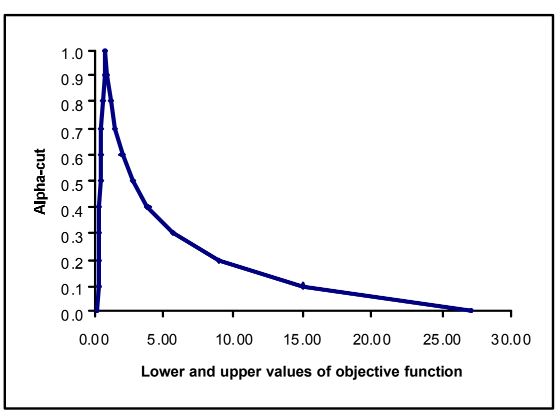

Fig. 1: Lower and upper objective values at different values of α for Example 1

Results of fuzzy optimization for the upper bound of the geometric programming problem at different values of α are given in Table 2.

Figure 1 shows the lower and upper bounds of the objective values for the geometric programming problem at different values of α of Example 1.

Illustrative example 2: Fuzzy approach to suppress crane load sway: The crane model is illustrated in the Fig. 2. A fuzzy synthesis is applied to the design of a controller using the Lyapunov direct method. Since extracting knowledge, in many cases, is a tedious task, the basic physical information about the system is assumed according to previous work [15]. The details of model equations can be found in Appendix A.

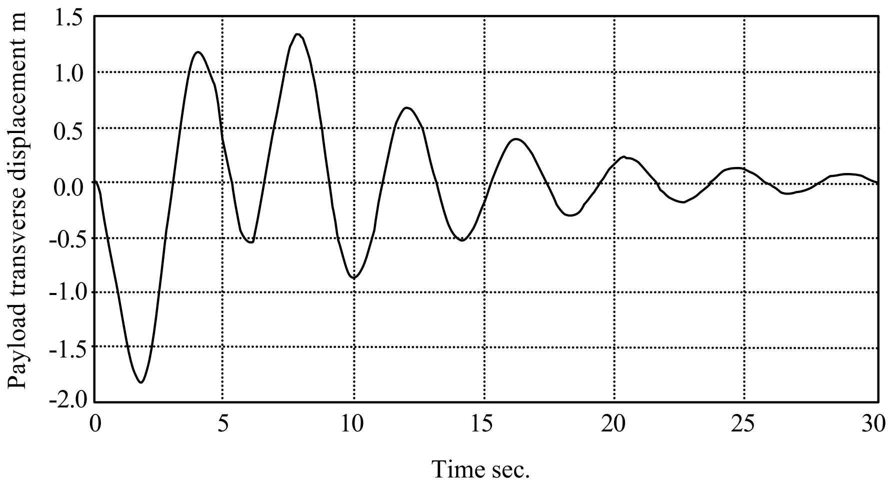

Fig. 3: The time response of the payload transverse displacement

Assuming the wire rope length is constant "l=6m" and the girder is stationary "$\nu=0$", the equations of motion, according to Appendix, can be reduced to;

$$(M+m)\ddot{u}+Ml\cos\theta_1\ddot{\theta}_1 = F \tag{18}$$

$$(Ml\cos\theta_1)\ddot{u}+Ml^2\ddot{\theta}_1+Mgl\sin\theta_1 = 0 \tag{19}$$

As shown in Fig. 2, the crane sway can be represented by two angles; the angle between the wire rope and the vertical θ_1 and the angle rope twist in the horizontal plane θ_2. According to Appendix A, the analysis will be restricted to the angle θ_1 because θ_2 is not controllable [15, 16]. However, θ_2 will be decreased if the first state "$x_1=\theta_1$" is suppressed [15, 16].

Hence, the analysis will consider only $x_1=\theta_1$ and $x_2=\dot{\theta}_1$. Regarding the controlling force, "F" it has two components; the nominal force component "F_0" which has to overcome the inertia forces and "δF" [15] which is determined using the fuzzy controller, i.e. $F = F_0+\delta F$.

During each phase of the crane motion, the objective of δF is to minimize the load swing around the corresponding nominal position. Therefore, the control force component δF is proportional to $x_2=\dot{\theta}_1$ only.

The second method of Lyapunov is the most general for determining the stability of a nonlinear and/or time-varying system of any order. This method will be implemented to stabilize the control scheme and to extract the fuzzy rules. The following Lyapunov geometric function candidate;

$$L(x_1,x_2)=\frac{1}{2}(x_1^2+x_2^2) \tag{20}$$

Table 3: Fuzzy partition of x_1, x_2 and δF

x_1	x_2	δF
+	+	+
-	-	-
+	-	0
-	+	0

Differentiating with respect to time gives;

$$\dot{L}=x_1\dot{x}_1+x_2\dot{x}_2 \tag{21}$$

In order to achieve the asymptotic stability, the required necessary condition is to find $\delta F(x_1, x_2)$ so that

$$\dot{L}=x_1 x_2+x_2\dot{x}_2<0 \tag{22}$$

in some neighborhood of the equilibrium of the Lyapunov function. Sufficient conditions for the above stability condition can be stated as follows:

If x_1 and x_2 have opposite signs and $\dot{x}_2$ is zero, the stability condition holds;

If x_1 and x_2 are both positive, the stability condition will hold if $\dot{x}_2<-x_1$; and

If x_1 and x_2 are both negative, the stability condition will hold if $\dot{x}_2>-x_1$.

Using the observations and our knoweledge that δF is proportional to $\dot{x}_2$, the following logic rules can be obtained Table 3;

These rules are simply the fuzzy partition of x_1, x_2 and δF which follow directly from the stabilizing condition of the Lyapunov function. Each rule represents the model characteristic in an operting region. The models are linked together by the fuzzy membership functions to form a global model.

Using the data of the considered crane, with a transportation plane of 0.1 m/sec2 acceleration for 3 seconds, constant velocity of 0.3 m/sec and finally constant deceleration of 0.1 m/sec2 for the last 3 seconds, the load sway angle can be obtained. Fig. 3 illustrates a simulation example that the load sway has been suppressed by appling the proposed scheme.

CONCLUSION

In geometric programming, the possible values of the parameters required in the modeling of the problem are provided by either fuzzy or crisp data. This paper presents an approach for solving geometric programming problems under uncertainty. The proposed approach derives the lower and upper bounds of the objective of geometric programming problems with fuzzy parameters. The parameters are represented by triangular fuzzy numbers. A pair of two-level mathematical programs is formulated to calculate the lower and upper bounds of the objective value. The solution is in a range. Two illustrative examples are presented to clarify the proposed approach. The problem of suppressing the load sway of cranes is also considered as a practical application to illustrate the effectiveness of the present approach.

Appendix A

The equations of motion for the different coordinates "u,v,l, θ_1,θ_2" can be written as follows [15];

$$\begin{aligned}&(M+m)\ddot{u}+M\theta_1\sin\theta_2\ddot{l}+Ml\cos\theta_1\ddot{\theta}_1\\&+C_1(\dot{l},\dot{\theta}_1,\dot{\theta}_2)+(Ml\theta_1\cos\theta_2)\ddot{\theta}_2=F\end{aligned} \tag{A1}$$

$$\begin{aligned}&(M_{total})\ddot{v}+Ml\cos\theta_2\ddot{\theta}_1+C_2(\dot{l},\dot{\theta}_1,\dot{\theta}_2)\\&-Ml\theta_1\sin\theta_2\ddot{\theta}_2+(M\theta_1\cos\theta_2)\ddot{l}=F_{girder}\end{aligned} \tag{A2}$$

$$\begin{aligned}&(M\theta_1\sin\theta_2)\ddot{u}+(M\theta_1\cos\theta_2)\ddot{v}+M\ddot{l}\\&+C_3(\dot{l},\dot{\theta}_1,\dot{\theta}_2)=Mg-F_{hoisting}\end{aligned} \tag{A3}$$

$$\begin{aligned}&(Ml\cos\theta_1)\ddot{u}+(Ml\cos\theta_2)\ddot{v}+Ml^2\ddot{\theta}_1\\&+C_4(\dot{l},\dot{\theta}_1,\dot{\theta}_2)+Mgl\sin\theta_1=0\end{aligned} \tag{A4}$$

$$\begin{aligned}&(Ml\theta_1\cos\theta_2)\ddot{u}+(-Ml\theta_1\sin\theta_2)\ddot{v}\\&+Ml^2\ddot{\theta}_2+C_5(\dot{l},\dot{\theta}_1,\dot{\theta}_2)+D_5(\theta_1,\theta_2)=0\end{aligned} \tag{A5}$$

These equations of motion for the case of no-hoisting; i.e. "l=0" and for stationary girder, i.e. "v=0", can be reduced to the equations A1, A4 and A5. The elements $C_j(\dot{l},\dot{\theta}_1,\dot{\theta}_2)$, j=1,2,.., 5 and D5 ($\theta_1$, θ_2) can be neglected for small perturbation about the nominal value [15]. Regarding the coordinate θ_2, it can be shown that it is not controllable or observable. However, it will be decreased if the first coordinate θ_1 is suppressed [15, 16]. Therefore, the equations of motion can be reduced to:

$$(M+m)\ddot{u}+Ml\cos\theta_1\ddot{\theta}_1=F \tag{A6}$$

$$(Ml\cos\theta_1)\ddot{u}+Ml^2\ddot{\theta}_1+Mgl\sin\theta_1=0 \tag{A7}$$

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