World Applied Sciences Journal 6 (12): 1685-1690, 2009
ISSN 1818-4952

Hybrid Single Sampling Plan

S. Sampath

University of Madras, Chennai 600005, India

Abstract: In the manufacturing processes, quantities like the proportion of defective units in a production lot may not be precisely known and most of the times the practitioners have to compromise with some imprecise (approximate) values. Prior knowledge of such quantities is required to evaluate the quality of a produced lot. In this paper, the properties of single sampling plan under situations involving both impreciseness and randomness are considered. Using the Theory of Chance due to Liu [6], the process of drawing an Operating Characteristic curve and the issue of identifying optimal sampling plans are also addressed for fuzzy random environment.

Key words: Sampling plan . fuzzy variable . OC curve . optimal sampling plan . risk

INTRODUCTION

Statistical Quality Control is one of the oldest branches of Statistics which finds wide usage by practitioners in manufacturing industry. Acceptance sampling is a vital branch of study in SQC. While applying the theoretical developments related to Acceptance Sampling in real life situations quite often problems of different kinds are faced by practitioners. For example, while drawing Operating Characteristic (OC) curves associated with a given sampling plan one need to have prior knowledge about the proportion of defective units in the lot so that probabilities of acceptance can be evaluated. However, in real life situations rarely those values are known and in such cases practitioners rely either on their past experience or they take the opinion of experts. Hence such assumed values are likely to be only approximate and they are bound to be of imprecise nature. Hence it is meaningful to treat such parameters as fuzzy quantities. While designing sampling plans, specifically single sampling plans, it is well known that probability distribution plays a crucial role. Since in the conventional probability distributions the parameters are assumed to be precise values, difficulties arise when the parameters become imprecise. Hence one needs a totally new approach in designing of sampling plans. In this paper, the Theory of Chance due to Liu [1] is used to explore the possibility of introducing a sampling plan suitable for a situation having both randomness and impreciseness. It is pertinent to note that the problem of designing sampling plans under fuzzy environment has been initially studied by Kanagawa and Ohta [2]. The application of fuzzy set theory in various contexts under SQC has been considered by many including Ohta and Ichihashi [3], Kanagawa *et al*. [4], Raz and Wang [5], Wang and Raz [6] and Hryniewicz [7]. For a recent review on articles related to applications of fuzzy set theory in SQC one can refer to Noori *et al*. [8] and Hryniewicz [9]. This review work clearly highlights majority of the works are related to the theory of control charts for dealing with imprecise data. This paper is devoted to the Theory of Acceptance Sampling. The organization of the paper is as follows. The second section briefly reviews Chance theory. The third section describes binomial distribution involving fuzzy parameter which is used in making an in depth study on the sampling plan developed in this paper. The fourth section addresses the problem of determining optimum single determining sampling plans under random situation involving impreciseness. The theory considered in this paper is illustrated numerically.

CHANCE THEORY

The introduction of Chance theory requires an understanding of the Credibility theory which provides the foundation for the introduction of fuzzy variables and Probability theory.

Credibility theory: Let Θ be a nonempty set and P be the power set of Θ. Each element of P is called an *event*. For every event A, we associate a number denoted by Cr {A}, which indicates the credibility that A will occur. In credibility theory the following four axioms are accepted.

Corresponding Author: Dr. S. Sampath, University of Madras, Chennai 600005, India

Axiom 1: (Normality) Cr (Θ) = 1

Axiom 2: (Monotonicity) Cr (A)≤Cr (B) whenever A⊂B

Axiom 3: (Self duality) Cr (A)+Cr (A^c) = 1 for any event A

Axiom 4: (Maximality) Cr (U_iA_i) = Sup_iCr (A_i) for any events {A_i} with Sup_iCr (A_i)<0.5

Credibility measure: The set function Cr is called a credibility measure if it satisfies the normality, monotonicity, self-duality and maximality axioms

Credibility space: Let Θ be a nonempty set, P be the power set of Θ and Cr a credibility measure. Then the triplet (Θ, P, Cr) is called a credibility space.

Fuzzy variable: A fuzzy variable is a measurable function from a credibility space (Θ, P, Cr) to the set of real number

Membership function: Let ξ be a fuzzy variable on the credibility space (Θ, P, Cr). Then its membership function is derived from the credibility measure by

$$\mu(x) = \{2Cr\{\xi = x\}\} \wedge 1, x \in \Re$$

Credibility distribution: The credibility distribution Φ: ℜ →[0,1] of a fuzzy variable ξ is defined by Φ (x) = Cr {θ ∈ Θ|ξ(θ)≤x}

Probability theory: Let Ω be a nonempty set and A be the power set of Ω. Each element of A is called an *event*. For every event A, we associate a number denoted by Pr{A}, which indicates the probability that A will occur. In probability theory the following three axioms are accepted.

Axiom 1: (Normality) Pr (Ω) = 1

Axiom 2: (Nonnegativity) Pr (A)≥0 for any event A

Axiom 3: (Countable additivity)

$$\Pr(\bigcup_i A_i) = \sum_i \Pr(A_i)$$

for every countable sequence of disjoint events {A_i}

Probability measure: The set function Pr is called a probability measure if it satisfies the normality, nonnegativity and countable additivity axioms.

Probability space: Let Ω be a nonempty set, A be the power set of Ω and Pr a credibility measure. Then the triplet (Ω, A, Pr) is called a probability space.

Random variable: A random variable is a measurable function from a probability space (Ω, A, Pr) to the set of real numbers.

Probability distribution: The probability distribution Φ: ℜ → [0,1] of a random variable ξ is defined by Φ (x) = Pr {ϖ ∈ Ω|ξ(ω)≤x}.

Chance theory: Using the above definitions related to Credibility and Probability spaces, Liu [1] developed ideas relevant to handle situations where both impreciseness and randomness play simultaneous roles in the given system. The hybrid development based on credibility and probability space has been named as Chance theory. The following definitions are due to Liu [1].

Chance space: Suppose that (Θ, P, Cr) is a credibility space and (Ω, A, Pr) is a probability space. The product (Θ, P, Cr)× (Ω, A, Pr) is called a chance space.

Let (Θ, P, Cr)× (Ω, A, Pr) be a chance space. A subset Λ⊂Θ×Ω is called an event if Λ(θ)∈A for each θ∈Θ

Chance measure: Let (Θ, P, Cr)× (Ω, A, Pr) be a chance space. Then a chance measure of an event Λ is defined as:

$$Ch(\Lambda) = \begin{cases} \sup_{\theta\in\Theta}\{Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\} & \text{if} \sup_{\theta\in\Theta}\{Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\} < 0.5 \\ 1 - \sup_{\theta\in\Theta}\{Cr\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\} & \text{if} \sup_{\theta\in\Theta}\{Cr\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\} \geq 0.5 \end{cases}$$

To describe a quantity with both fuzziness and randomness, the concept of hybrid variable is used which is defined as follows.

Hybrid variable: A hybrid variable is a measurable function from a chance space (Θ, P, Cr)× (Ω, A, Pr) to the set of real numbers. That is, for any Borel set B of real numbers, {ξ∈B} = {(θ,ϖ)∈Θ×Ω|ξ(θ,ω)∈ B} is an event.

Liu [1] has identified five different approaches for defining Hybrid variable. The following is the model (Model IV of Liu [1]) which will be used in our further discussion. This model is suitable for dealing with situations where the parameters involved in a given probability distribution are fuzzy by nature. The model proposed by Liu is explained below

Let ξ be a random variable with probability density function $\phi(x;\theta)$ where θ is a fuzzy variable. Clearly ξ is a hybrid variable and if μ is a membership function associated with θ then it has been shown by Qin and Liu [10], for any Borel set B of real numbers, the chance Ch ($\xi \in B$) is given by:

$$Ch(\xi \in B) = \begin{cases} \sup_{\theta\in\Theta}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \phi(x,\theta)dx\right\} \\ \text{if} \sup_{\theta\in\Theta}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \phi(x,\theta)dx\right\} < 0.5 \\ 1-\sup_{\theta\in\Theta}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \int_{B^c} \phi(x,\theta)dx\right\} \\ \text{if} \sup_{\theta\in\Theta}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \int_B \phi(x,\theta)dx\right\} \geq 0.5 \end{cases}$$

Chance distribution: The chance distribution $\Phi: \Re \to [0,1]$ of a hybrid variable ξ is defined by $\Phi(x) = Ch\{(\theta,\omega) \in \Theta \times \Omega \mid \xi(\theta,\omega) \leq x\}$

Chance density function: The chance density function $\phi: \Re \to [0,\infty]$ of a hybrid variable ξ is a function such that $\Phi(x) = \int_{-\infty}^{x}\phi(y)dy,\ \forall x \in \Re$ and $\int_{-\infty}^{\infty}\phi(y)dy = 1$ where Φ is the chance distribution of ξ.

The definitions presented are relevant for further discussion made in this paper. For more detailed and exhaustive discussion one can refer to Liu [1].

HYBRID BINOMIAL DISTRIBUTION

Let ξ be a discrete random variable having Binomial distribution with probability of success θ be a fuzzy variable with triangular membership function. That is, the pdf of ξ and the membership function of θ are respectively given by

$$\phi(x) = \begin{cases} \binom{n}{x}\theta^x(1-\theta)^{n-x}, \text{ if } x = 0,1,2,....n \\ 0 \quad \text{otherwise} \end{cases}$$

and

$$\mu(\theta) = \begin{cases} \frac{\theta-a}{b-a}, \text{ if } a \leq \theta \leq b \\ \frac{b-\theta}{c-b}, \text{ if } b \leq \theta \leq c \\ 0 \quad \text{otherwise} \end{cases}$$

Clearly ξ is a hybrid version of Binomial distribution.

The chance values associated with hybrid binomial distribution are computed using the expression.

Table 1: Chance distribution values

r	Ch ($\xi = r$)	Ch ($\xi \geq r$)	Ch ($\xi \leq r$)
0	0.07886	1	0.007886
1	0.66812	0.992114	0.072109
2	0.217659	0.927891	0.258348
3	0.416353	0.741652	0.565266
4	0.434734	0.434734	1

$$Ch(\xi = r) = \begin{cases} \sup_{a\leq\theta\leq b}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \binom{n}{r}\theta^r(1-\theta)^{n-r}\right\} \\ \text{if } \sup_{a\leq\theta\leq b}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \binom{n}{r}\theta^r(1-\theta)^{n-r}\right\} < 0.5 \\ 1-\sup_{\theta\in\Theta}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \left(1-\binom{n}{r}\theta^r(1-\theta)^{n-r}\right)\right\} \\ \text{if } \sup_{a\leq\theta\leq b}\left\{\min_{\theta}\left(\frac{\mu(\theta)}{2}\right) \wedge \binom{n}{r}\theta^r(1-\theta)^{n-r}\right\} \geq 0.5 \end{cases}$$

It is to be mentioned here that unlike binomial probabilities theses quantities can not be calculated in a straight forward manner. Hence one has to use some high end tools like evolutionary algorithms. In this work, Genetic algorithm is used to compute the chance values for various choices of the parameters in the binomial distribution and the membership function. In the genetic algorithm, initial population of 50 chromosomes each representing a point in the solution space is considered. The process of selection has been implemented using Roulette wheel selection and single point cross over as suggested by Liu [1] is adopted. Further randomized bidirectional mutation has been performed. In order to reach the solution 1000 generations have been considered.

Illustration 1: Assume that $\xi \sim B(4,\theta)$ where $\theta \sim$Triangular (0.7, 0.8, 0.9)
In this case

$$Ch(\xi = r) = \sup_{0.7\leq\theta\leq0.9}\left\{\frac{\mu(\theta)}{2} \wedge P_\theta(r)\right\}, r = 0,1,2,3,4$$

Where

$$\mu(\theta) = \begin{cases} \frac{\theta-0.7}{0.1} & \text{if } 0.7 \leq \theta \leq 0.8 \\ \frac{0.9-\theta}{0.1} & \text{if } 0.8 \leq \theta \leq 0.9 \end{cases}$$

and

$$P_\theta(r) = \begin{cases} \binom{4}{r}\theta^r(1-\theta)^{4-r} \text{ if } r = 0,1,2,3,4 \\ 0 \text{ otherwise} \end{cases}$$

Table 1 gives chance values of different kinds for various choices of r.

Table 2: Comparisons of Chance and Probability values

a	b|p_d	c	Chance of acceptance	Probability of Acceptance	Percentage deviation
0.005	0.01	0.015	0.992442	0.998153	0.572163
0.015	0.02	0.025	0.961033	0.979765	1.911895
0.025	0.03	0.035	0.900000	0.929537	3.177643
0.035	0.04	0.045	0.814601	0.845989	3.710267
0.045	0.05	0.055	0.713862	0.738317	3.312317
0.055	0.06	0.065	0.607602	0.619594	1.935441
0.065	0.07	0.075	0.501847	0.501847	5.02E-05
0.075	0.08	0.085	0.402941	0.393763	2.330837
0.085	0.09	0.095	0.315940	0.300280	5.215159
0.095	0.10	0.105	0.240918	0.223187	7.944411
0.105	0.11	0.115	0.179974	0.162066	11.05014
0.115	0.12	0.125	0.130567	0.115198	13.34097

The hybrid binomial distribution is suitable for dealing with situations similar to those explained in the introductory section of this paper. The details related to Single Sampling and the way in which parameters related to such a sampling plan in the presence of both impreciseness and uncertainty are explained in the following Section.

HYBRID SINGLE SAMPLING PLAN

The single sampling plan for attributes is explained below. Consider a lot consisting of N units. Take a random sample of size n and count the number of defective units in the lot. If the number of defective units d in the sample is less than or equal to a predetermined value c then the lot will be accepted else it will be rejected as a bad lot. If the lot size N is very large then the number of defective units in the sample d is a binomial random variable. When the proportion of defective units in the lot is not known precisely and is an uncertain value, we can treat the distribution of defective units as hybrid binomial distribution. In our discussion, we shall assume that the proportion of defective units is a fuzzy variable with triangular membership function. Hence the expression given Section 3 can be used for computing the Chance values for various choices of n and c. The Operating Characteristic Curve for the single sampling plan described can be drawn by using the Chance distribution. The chance of the hybrid variable d taking a value less than or equal to r is given by

$$Ch(d \le r) = \sum_{i=1}^{r} Ch(d = i)$$

where Ch (d = i) can be computed using the expression given in Section 3. Using these credibility values one can draw the operating characteristic curve for the single sampling plan under the imprecise setup. The following example makes use of hybrid binomial distribution to draw the OC Curve.

Consider the Single Sampling Plan n = 52 and c = 3. Table 2 gives Chances of acceptance computed using hybrid binomial distribution for different choices of lot proportion of defective units. It is assumed that the proportion of defective units is explained through triangular membership functions. The first three columns of the table gives (a, b, c) values associated with the triangular membership function of the fuzzy variable p_d, denoting the proportion of defective units in the lot. The fourth column gives the chance of acceptance under the imprecise situation and the fifth column gives the probability of acceptance for the crisp situation where the proportion of defective units is taken as p_d. The last column of the table gives the percentage deviation from probability of acceptance one has while obtaining chance of acceptance. The percentage deviation is computed on multiplying the ratio of absolute difference between the chance of acceptance and probability of acceptance to chance of acceptance by 100. Figure 1 is a diagrammatic representation of the information contained in Table 2.

Determination of sampling plan: Effective implementation of a single sampling plan requires appropriate values for the sample size n and the threshold value c. The values of n and c are determined for specified choices of α, β, p_1 and p_2, where α and β are respectively producer's risk and consumer's risk. In the imprecise situation, we choose the values of n and c so that the following two conditions are two satisfied:

(i) $Ch_{P_1}(d \le c) \ge 1 - \alpha$

(ii) $Ch_{p_1}(d \le c) \le \beta$

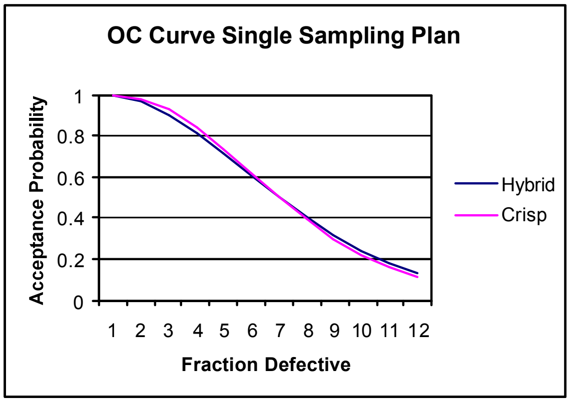

Fig. 1: Operating characteristic curves

An empirical study carried out on the behavior of the OC curve drawn using chance distribution clearly established the following properties:

For fixed p and c, Ch_p (d≤c) is a decreasing function of n
For fixed n and p, Ch_p (d≤c) is an increasing function of c
For fixed n and c, Ch_p (d≤c) is a decreasing function of p

Hence the iterative procedure suggested by Guenther [11] can be used to determine the values of n and c

Iterative procedure: The iterative procedure is as follows:

Step 1: Set c = 0.

Step 2: Find the largest n, say n_L, such that Ch_{p1} (d≤c)=1 - a (this condition is satisfied for all n=n_L).

Step 3: Find the smallest n, say n_S, such that Ch_{p2} (d≤c)=ß (this condition is satisfied for all n=n_S).

Step 4: If n_S=n_L, then the optimum plan is (n_S, c); otherwise increment c by 1 [here n_S is the minimum size that will satisfy the conditions (iii) and (iv)].

Step 5: Repeat the Steps 2, 3 and 4 until an optimum plan is obtained.

On using the above iterative procedure, optimal sampling plans are determined for a wide range of p_1 and p_2 with specified producer's and consumer's risks. Table 3 gives some single sampling plans obtained using the above iterative procedure when p_1, producer's quality level and p_2, the customers quality level are triangular fuzzy variables. For example, (125,2) is the optimal sampling plan as given by the above iterative procedure when the consumer's quality level is a triangular fuzzy variable defined on (0.04,0.05,0.06) and producer's quality level is another triangular fuzzy variable defined on (0.004,0.005,0.006).

Table 3: Hybrid Sampling Plans

	Producer's quality level (p_1)	
Consumer's quality level (p_2)	(0.004,0.005,0.006)	(0.009,0.010,0.11)
(0.04,0.05,0.06)	(125,2)	(218,5)
(0.05,0.06,0.07)	(100,2)	(151,4)
(0.06,0.07,0.08)	(84,2)	(105,3)
(0.07,0.08,0.09)	(73,2)	(91,3)
(0.08,0.09,0.10)	(64,2)	(80,3)

CONCLUSION

Thus in this paper, the way in which the Chance theory can be used to solve difficulties one faces in the form of impreciseness arises in Statistical Quality Control. The way in which Operating Characteristic Curves are drawn using the concept of Chance theory is also illustrated. Further the question of determining an optimal single sampling plan is also considered. Specific single sampling plans are also obtained to illustrate the theory discussed in this paper.

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