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Identification of Vibrating Structures with Application to a Steel Tower

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Abstract: A stochastic identification technique is proposed to estimate both the parameters and order of multi-input multi-output vibrating structural systems. An overspecified order time-series model is considered. The extraneous modes associated with the overspecified order model are distinguished from system modes by applying a backwards approach. The poles of the extraneous modes have been shown to be canceled by the extra zeroes. These poles are found to be generally inside the unit circle. Simulation examples have been presented to illustrate the effectiveness of the proposed approach. Experimental data from the vibration of a flexible truss, representing a model of a steel tower, is used to identify the system order and natural frequencies. The results of the identified natural frequencies and system parameters will be used to study the system response and to investigate design modifications.

Key words: Multi-Input Multi-Output (MIMO) systems . flexible structures . stochastic system identification . model order . extraneous modes . steel tower

INTRODUCTION

Many mechanical and structural systems respond dynamically to random environmental loads, such as wind, sea wave or earthquake forces. Examples include flexible buildings vibrating due to turbulent wind loading and offshore structures moving as a result of combining wave and wind loading. To assess the reliability of such structures it is important to predict their dynamic response, at the design stage. However, whilst mass and stiffness parameters in the governing equations of motion can usually be computed with some accuracy, damping parameters are normally not quantifiable by theoretical means. The measured system parameters are also more accurate because they considered the actual end conditions.

Several investigators consider the problem of system identification [1-14]. To identify the parameters of these systems, their order (which roughly can be considered as the number of these parameters), must be known and the system must be also represented in a canonical form. Otherwise, biased estimates of the parameters could be obtained [1-5]. Recently, several time domain identification techniques have been proposed [1-8]. Identification accuracy is improved by increasing the model order. However, these overspecified models contain extraneous modes beside system modes. To distinguish between system modes and these extraneous modes, several techniques have been proposed, for single-input single output cases [5-7]. However, the extraneous modes cannot be distinguished from system modes, unless physical restrictions are applied to the model.

In this work, a canonical form, that presents the structural dynamics, has been proposed and its uniqueness has been proved. The backward approach has been applied to identify the system parameters, using an overspecified model. Regarding order identification of MIMO systems, a pole-zero cancellation technique has been proposed. This proposed technique is capable of identifying both the order and parameters, using the minimum prediction error method. Several simulation examples illustrate the effectiveness of the proposed technique. Thereafter, the experimental data of a flexible truss has been used to identify the system parameters which include the truss natural frequencies. The present paper is organized as follows. In the following section, the problem formulation, together with backward method and order estimation, has been introduced.

The order identification technique, for the considered canonical form, is proposed afterwards. The

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importance of overspecified model is illustrated through several simulation examples. Hence, the experimental data from a flexible truss presenting an overhead transmission tower is used to illustrate the effectiveness of the proposed approach. The conclusions are summarized in the last section.

Problem formulation: The basic Single-input Single-output (SISO) discrete time-series model of a dynamic system has the form;

$$Y(k) = - a_1 y(k-1) - a_2 y(k-2) - ... - a_n y(k-n) + b_o u(k) + b_1 u(k-1) + ... + b_n u(k-n) \quad (1)$$

where y(k) is the system response at the k^{th} discrete time, u(k) is the system input at the k^{th} discrete time and "n" is the model order. The a's and b's are the model's parameters. This model is widely used to describe the dynamics of any finite-order linear system [2]. The above representation is known as the forward form. Regarding the structural systems, the system response may be displacement, velocity or acceleration at any measuring point on the structure. Using z-transform, equation (1) can be written as;

$$T_F(z) = \frac{b_o z^n + b_1 z^{n-1} + ... + b_n}{z^n + a_1 z^{n-1} + ... + a_n} \quad (2)$$

The denominator polynomial is the model characteristic equation and its roots can be used to estimate the natural frequencies and damping factors of the structure. The problem is to estimate the model order such that the model parameters can be estimated. Unfortunately, wrong estimates of the order leads to wrong estimates of the parameters [1-7]. Several investigators have proposed efficient techniques to identify the order and parameters for SISO systems [5-7]. In general, these techniques assume that the model is of sufficient structure or over specified model, which simply means that the model order is larger than or equal to the system order. In a recent work, the authors studied the effect of tower dynamics on the vibration of transmission lines [3]. Also, a technique is proposed to identify model parameters, if the order of one of the elements in every row is known. Otherwise, it is difficult to distinguish between system and computational modes. In this work, the backward model will be implemented to overcome the above difficulty. Eq. (1) can be written in the backward time-series model as follows:

$$Y(k) = - g_1 y(k+1) - g_2 y(k+2) - ... - g_n y(k+n) + d_o u(k) + d_1 u(k-1) + ... + d_n u(k-n) \quad (3)$$

The characteristic equation of the model is the same as that of the forward one, but the total number of equations which are used for identification are effectively increased [9]. Regarding (MIMO) systems, the problem is quite difficult. Consider the following MIMO system;

$$A(z)\, Y(k) = G(z)\, U(k) + B(z)\, W(k) \quad (4)$$

where Y(k) is an m-dimensional vector which represents the system output; U(k) is a p-dimensional vector which represents the input of the system; and W(k) is an m-dimensional zero-mean stochastic white noise with positive definite covariance matrix. The elements of the matrices A(z), G(z) and B(z) are polynomials of "z". Therefore, the considered noise is colored. The parameters and order of this system are unknown. The problem is thus to identify both the order and parameters and to verify this identification.

The matrix equation (4), which represents MIMO dynamic model, must uniquely represent the system, or the system must be in Canonical form. Also, the conditions, under which the order and parameters can be identified, must be specified. In the following section, a new Canonical form will be proposed and its uniqueness will be proved. This new Canonical form will simplify the order identification problem by decoupling the identification of the whole system parameters into the identification of several smaller subsystems. Also, an identification technique is proposed to achieve the consistent estimates of the model order and parameters. Several examples are included to investigate the effectiveness of the proposed technique.

Order identification technique: This section is divided into three subsections. In the first of them, the new Canonical form is introduced and its uniqueness is proved. The identification technique for both the model order and parameters is introduced in second subsection. Several simulation examples are included in third subsection to illustrate the effectiveness of the proposed technique.

The Canonical Form: Before proceeding with the uniqueness proof, the following conditions shall be considered;

- The system matrices A(z), G(z) and B(z) are relatively left prime, or in other words the Smith form of [A(z) : G(z) : B(z)] is [I:0:0]. This consideration means that the every state of the system equation (4) is controllable either from U(k) or from W(k).

- The inputs U(k) and W(k) are independent.
- The matrix A(z) of the system, equation (4), is assumed to be diagonal.
- To ensure that the process Y(k) is stationary and invertible, all the zeroes of the determinants of A(z) and B(z) lie outside unit circle.

Theorem 1: For the system equation (4) characterized by the matrices A(z), G(z) and B(z) with A(z) being diagonal, if the Greatest Common Left Divisor (GCLD) of A(z), G(z) and B(z) is a unimodular matrix (condition 1), the representation of Y(k) by equation(4) is unique.

Proof: Appendix A.

Identification scheme: The least prediction error method will be implemented to identify the system order and parameters. To ensure consistent estimation of the parameters, the sufficient conditions, on the model structure will be obtained. Also, it will be shown that the proposed Canonical form simplifies the identification problem. The diagonality of A(z) makes it simple to obtain the greatest common left divisor (GCLD) of the estimated matrices $\hat{A}(z), \hat{G}(z)$ and $\hat{B}(z)$ by the simple method of finding the common factor between the diagonal element of $\hat{A}(z)$ and the elements of the corresponding rows of $\hat{G}(z)$ and $\hat{B}(z)$ provided that the model is of sufficient structure. The order is then identified after factoring this GCLD.

Parameter identification: The unknown parameters of A(z), G(z) and B(z) can be estimated by minimizing the following prediction error loss function;

$$P(N,D) = \frac{1}{N}\sum_{k=0}^{N-1} \xi^T(k,D)\xi(k,D) \qquad (5)$$

where "N" is the number of measured data, "P" is the loss function and the prediction error ξ (k, D) [8] is obtained from the following model;

$$\hat{A}(z)Y(k) = \hat{G}(z)U(k) + \hat{B}(z)\xi(k, \hat{\lambda}) \qquad (6)$$

where D is a vector made up of all the unknown coefficients of $\hat{A}(z), \hat{G}(z)$ and $\hat{B}(z)$, which are the model matrices. Under the ergodicity assumption, it can be shown that the loss function of equation (5) converges uniformly in D to a deterministic function P(D) with probability one [8], as the number of measured data "N" increases. This function, P(D), is the global minimum of the loss function P(N,D) [8]. The above result will be used to investigate the properties of the identified matrices $\hat{A}(z)$, $\hat{G}(z)$ and $\hat{B}(z)$ at this global minimum.

Theorem 2: Consider the system, equation (4) and the above model, equation (6). Suppose that "U(k)" is persistent excitation of order "L", where "L" is the order of the matrix "$[\hat{B}^{-1}\hat{A}(A^{-1}G - \hat{A}^{-1}\hat{G})]$" and the model is of sufficient structure. The global minimum of the asymptotic loss function "P(D)" corresponds to;

$$\hat{A}(z) = R(z)A(z)\ ,\ \hat{G}(z) = R(z)G(z)\ ,$$
$$\hat{B}(z) = R(z)B(z) \qquad (7)$$

where R(z) is a diagonal matrix representing the GCLD between the matrices $\hat{A}(D), \hat{G}(D)$ and $\hat{B}(D)$.

Proof: Appendix B.

System modes and extraneous modes: The properties of the system modes and extraneous modes will be discussed. For backward method, the system poles are outside the unit circle. If the data is noise-free, the minimum norm solution will imply a minimum phase system and the computational or extraneous poles will fall inside the unit circle [10]. Regarding the present case, stochastic systems with colored noise, sometimes the computational poles are found outside the unit circle.

Simulation examples: In the following, several simulation examples will be represented to illustrate the identification scheme. The input U(k) is considered zero, while the noise sequence W(k) is taken as a sample of a Gaussian random vectors of zero mean and unity variance. Therefore, the system equation (4) can be written as;

$$A(z)\ Y(k) = B(z)\ W(k) \qquad (8)$$

while the model equation can be written as follows;

$$\hat{A}(z)Y(k) = \hat{B}(z)\xi(k,\lambda) \quad (9)$$

The present identification scheme has been applied to the generated set of data. Three different cases have been considered. In the first case the model was an overspecified model. In the second case, the model order was taken equal to the system order. The model in the third case was not of sufficient structure and wrong estimates of the parameters had been obtained.

Case 1, Overspecified model: In this case, the system matrices are as follows;

$$A(z) = \begin{bmatrix} 1+0.8z & 0 \\ 0 & 1+0.55z \end{bmatrix}, \quad B(z) = \begin{bmatrix} 1+0.37z & 0.25z \\ 0.25z & 1+0.8z \end{bmatrix}$$

The matrices of the identified model are taken as follows;

$$\hat{A}(z) = \begin{bmatrix} 1+a_{11}z+aa_{11}z^2 & 0 \\ 0 & 1+a_{22}z+aa_{22}z^2 \end{bmatrix}$$

$$\hat{B}(z) = \begin{bmatrix} 1+b_{11}z+bb_{11}z^2 & b_{12}z+bb_{12}z^2 \\ b_{21}z+bb_{21}z^2 & 1+b_{22}z+bb_{22}z^2 \end{bmatrix}$$

The estimated values of the above matrices are as follows;

$$\hat{A}(z) = \begin{bmatrix} 1-1.196z-1.61z^2 & 0 \\ 0 & 1+4.061z-2.44z^2 \end{bmatrix}$$

$$\hat{B}(z) = \begin{bmatrix} 1-1.606z-0.82z^2 & 0.248z-0.47z^2 \\ 0.236z-1.06z^2 & 1-3.683z-4.04z^2 \end{bmatrix}$$

These matrices can be written according to equation (7) as follows;

$$\hat{A}(z) = \begin{bmatrix} 1-2z & 0 \\ 0 & 1-4.5z \end{bmatrix}\begin{bmatrix} 1+0.8z & 0 \\ 0 & 1+0.542z \end{bmatrix}$$

$$\hat{B}(z) = \begin{bmatrix} 1-2z & 0 \\ 0 & 1-4.5z \end{bmatrix}\begin{bmatrix} 1+0.394z & 0.248z \\ 0.236z & 1+0.816z \end{bmatrix}$$

Therefore, the GCLD, "R(z)", between these identified matrices can be written as follows;

$$R(z) = \begin{bmatrix} 1-2z & 0 \\ 0 & 1-4.5z \end{bmatrix}$$

and the estimate of the system matrices, $\bar{A}(z), \bar{B}(z)$, can be written as follows;

$$\bar{A}(z) = \begin{bmatrix} 1+0.8z & 0 \\ 0 & 1+0.542z \end{bmatrix}$$

$$\bar{B}(z) = \begin{bmatrix} 1+0.394z & 0.248z \\ 0.236z & 1+0.816z \end{bmatrix}$$

which are a good estimate of the system matrices, A(z) and B(z).

Case 2, Model order is almost known: Sometimes, the number of the parameters of an element of the system matrices is known beforehand. Hence, there is common factor to be sorted at the row of this element. In the present case, the system matrices will be considered as follows;

$$A(z) = \begin{bmatrix} 1+0.8z & 0 \\ 0 & 1+0.55z \end{bmatrix}, \quad B(z) = \begin{bmatrix} 1+0.35z & 0 \\ 0 & 1+0.85z \end{bmatrix}$$

The number of the system parameters of the matrix is considered to be known beforehand and hence the matrices of the identified model are taken as follows;

$$\hat{A}(z) = \begin{bmatrix} 1+a_{11}z+aa_{11}z^2 & 0 \\ 0 & 1+a_{22}z+aa_{22}z^2 \end{bmatrix}$$

$$\hat{B}(z) = \begin{bmatrix} 1+b_{11}z & b_{12}z \\ b_{21}z & 1+b_{22}z \end{bmatrix}$$

and the estimate of the system matrices, $\bar{A}(z), \bar{B}(z)$, can be written as follows;

$$\bar{A}(z) = \begin{bmatrix} 1+0.76z-0.02z^2 & 0 \\ 0 & 1+0.58z+0.03z^2 \end{bmatrix}$$

$$\bar{B}(z) = \begin{bmatrix} 1+0.33z & 0 \\ 0 & 1+0.86z \end{bmatrix}$$

and the extra elements are almost zero and the estimates are in good agreement with the system matrices.

Case 3, Model order is not sufficient: In this case the number of model parameters is less than that of the system. This will lead to wrong estimates of the parameters. The system matrices will be as follows;

$$A(z)=\begin{bmatrix}1+0.8z & 0\\ 0 & 1+0.75z\end{bmatrix}$$

$$B(z)=\begin{bmatrix}1+0.35z & 0.25z\\ 0.25z & 1+0.75z\end{bmatrix}$$

The matrices of the identified model are taken as follows;

$$\hat{A}(z)=\begin{bmatrix}1+a_{11}z & 0\\ 0 & 1\end{bmatrix},$$

$$\hat{B}(z)=\begin{bmatrix}1+b_{11}z & b_{12}z\\ b_{21}z & 1+b_{22}z\end{bmatrix}$$

The estimated values of the above matrices are as follows;

$$\overline{A}(z)=\begin{bmatrix}1+0.73z & 0\\ 0 & 1\end{bmatrix}$$

$$\overline{B}(z)=\begin{bmatrix}1+0.33z & 0.24z\\ 0.24z & 1-0.04z\end{bmatrix}$$

Wrong estimates have been obtained and this illustrates that the model must be of sufficient structure.

Identification of the structural system: The structure used in this study is illustrated in Fig. 1. It is a steel truss representing an overhead tower. It is 3 m. height and 0.55 m. wide in each side and fixed supported to the base. Two shakers have been used to disturb the truss at the points A and B, the first of them gives 10 N, while the second gives 20 N. The shakers have been driven by a random wave generator. The velocities at points C and D have been recorded using piezo-electric accelerometers and charge amplifiers. The charge amplifiers have active integrating circuits to measure velocity. The velocity at A and B have also been measured by the same manner and considered as inputs to the vibrating system. The data are collected at a rate of 130 Hz and fed to the computer to apply the proposed identification scheme. The developed program used Powell's minimization method and the minimum error is the norm of the errors at C and D.

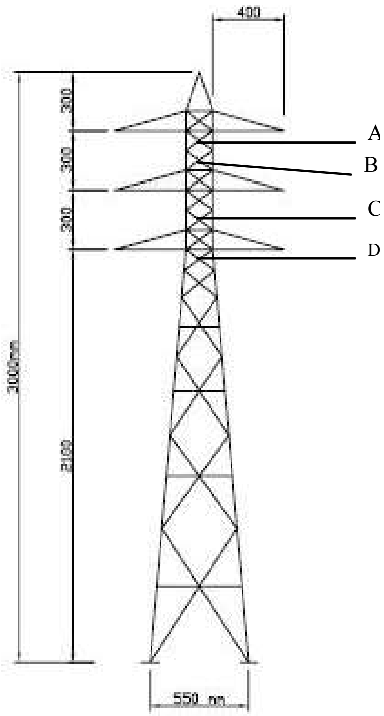

Fig. 1: Layout of the considered steel tower (all dimensions are in mm)

The program starts with a model order of four and increases the order by a step of two. The results of the identification scheme are illustrated in the following Table 1.

The results in Table1 illustrate that the system frequencies cannot be identified if the model order is less than or equal twelve. The minimum error also is a sensitive indicator to the system frequencies. For example, it has been decreased to about 25% of its value when identifying the highest natural frequencies, 88, 175 and 250 rad/sec. On the other hand, negligible changes in the minimum error have been noticed when identifying the common frequencies, which cancelled by the identified numerator part equation (7). However, some of the extraneous poles lie outside the unit circle. This can be attributed to the noise associated with the measured data. The results of Srikantha Phani and Woodhouse [10] show that the extraneous poles lie inside the unit circle if the data is noise free.

In order to verify the effectiveness of the proposed technique, the results of identifying the above flexible truss are then fed to the computer to generate simulation data. This data, after two seconds, has been compared with the measured data,

Table 1: The results of the identification scheme

Model order	Minimum error	Identified frequencies (Natural frequencies)	Common frequency GCLD (Redundant frequencies)
10	0.400	*****	*****
12	0.090	88, 175, 250 rad/sec.	*****
40	0.069	88, 175, 250 rad/sec.	65 rad/sec.
50	0.066	88, 175, 250 rad/sec.	65, 150 rad/sec.

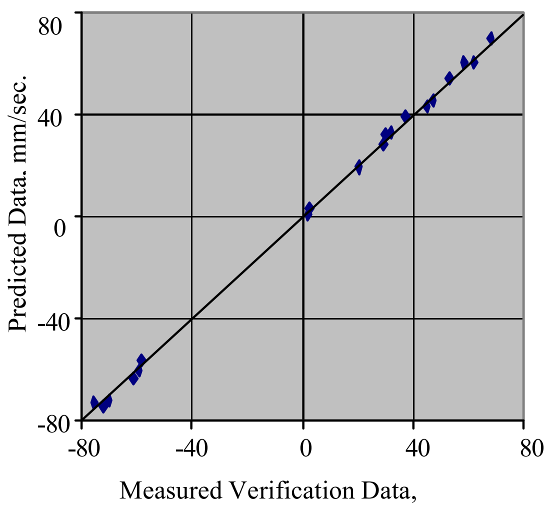

Fig. 2: Verification of the identified model, from t = 2 to 3 sec

at points C and D, for the same time period. The verification is illustrated in Fig. 2. It is clear that the dispersion from 45° is very small and consequently good agreement between the identified model and the system has been achieved.

CONCLUSION

The following conclusions may be drawn regarding the present identification technique for MIMO systems and the proposed canonical form.

- The implementation of the proposed canonical form simplifies the identification problem by decoupling the identification of the whole system into the identification of several smaller subsystems.
- The increase of the model order produces extra identified modes, often associated with a decrease of the minimum prediction error.
- The simulation examples illustrate the effectiveness of the proposed method. It is found that the extraneous modes can be distinguished from the system modes by the factorization method.
- The extraneous modes, generally, lie inside the unit circle.
- The above results have been applied into actual data. The extra modes have been identified and a small dispersion, from actual data, has been found by verification.
- The results of identifying the parameters of vibrating structures, namely the natural frequencies and order, can be used to study the response of these structures and hence to propose the necessary design requirements.

Appendix A

In the following, it will be proved that the previously proposed model uniquely represents the system. The proof will be carried out by contradiction. Assume that the matrices A(z), G(z), B(z) and Ao(z), Go(z), Bo(z) be two representations for the considered system, with A(z) being diagonal. Denote the GCLD of [A(z), G(z), B(z)] and [Ao(z), Go(z), Bo(z)] by the unimodular matrices P and Po, respectively. It can be proved that there exist two diagonal matrices H and K such that;

$$H\,A = K\,A_o, \quad H\,B = K\,B_o, \quad H\,G = K\,G_o \tag{A1}$$

since the space form is unimodular, the Smith forms of [A(z), G(z), B(z)] and [Ao(z), Go(z), Bo(z)] are [2];

$$U\,[A(z), G(z), B(z)]\,V = [I, O, O],$$

$$U_o\,[A_o(z), G_o(z), B_o(z)]\,V_o = [I, O, O] \tag{A2}$$

where U and U_o are square unimodular matrices of dimension "m" and V and Vo are unimodular matrices of dimension "2m+p". Now the above equations can be written as:

$$[HU^{-1}, \ 0, \ 0]\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} = [KU_o^{-1}, \ 0, \ 0] \quad \text{(A3)}$$

Which gives the following result;

$$HU^{-1}V_{11} = KU_o^{-1} \text{ and } V_{12} = 0 \quad \text{(A4)}$$

Since V_{11} and V_{22} are unimodular matrices, the above equation can be written as:

$$H = K\,U^0 \quad \text{(A5)}$$

where U^0 is a diagonal unimodular matrix, because H and K are diagonal matrices. Since A(0)=I, equation (A.1) gives the following result; H = K. Therefore,

$$A = A_o, \ B = B_o, \ G = G_o \quad \text{(A6)}$$

Then, the present Canonical form is a unique representation for the system.

Appendix B

The loss function of the prediction error, can be written as follows [8];

$$\begin{aligned} P(D) = \ & E\,[\hat{B}^{-1}\hat{A}(A^{-1}G - \hat{A}^{-1}\hat{G})U(k)]^T \\ & [\hat{B}^{-1}\hat{A}(A^{-1}G - \hat{A}^{-1}\hat{G})U(k)] \\ & + E[B^{-1}AA^{-1}BW(k)]^T[B^{-1}AA^{-1}BW(k)] \quad \text{(B1)} \\ & \geq E[\hat{B}^{-1}\hat{A}A^{-1}BW(k)]^T[\hat{B}^{-1}\hat{A}A^{-1}BW(k)] \\ & \geq E[W(k)]^T[W(k)] \end{aligned}$$

where "E" is the expectation and the global minimum will correspond to;

$$[\hat{B}^{-1}\hat{A}(A^{-1}G - \hat{A}^{-1}\hat{G})U(k)] = 0 \quad \text{(B2)}$$

$$\hat{A}^{-1}\hat{B} = A^{-1}B \quad \text{(B3)}$$

Since "U(k)" is persisting excitation, equation (B2) can be written as follows;

$$A^{-1}G - \hat{A}^{-1}\hat{G} = 0 \text{ or } A^{-1}G = \hat{A}^{-1}\hat{G} \quad \text{(B4)}$$

Therefore, the global minimum of "P(D)" corresponding to;

$$\hat{A}(z) = R(z)A(z), \hat{G}(z) = R(z)G(z), \hat{B}(z) = R(z)B(z) \quad \text{(B5)}$$

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