World Applied Sciences Journal 6 (12): 1665-1672, 2009
ISSN 1818-4952

A New Approach Based on Benders Decomposition for Unit Commitment Problem

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Abstract: This study presents a new formulation based on generalized benders decomposition approach to solve thermal unit commitment problem. This new approach decomposes the problem into a master problem and a sub-problem. The master problem is an integer program and sub-problem is a nonlinear program. The master problem applies the integer programming method to solve (Unit Commitment) UC and finds proper on/off states of the units. In the proposed approach, the sub-problem is modeled as a nonlinear problem and it uses the outputs of master problem to form appropriate cuts and adds them to the master problem for solving the next iteration of UC. Appropriate formulations of constraints for both parts of problem are presented. The results obtained, considering a commonly used system, show the efficiency of this approach in comparison with other methods.

Key words: Benders decomposition . Mixed integer programming . Unit commitment

INTRODUCTION

Unit Commitment (UC) is a very significant optimization task, which plays a major role in the daily operation planning of power systems, especially in the framework of the deregulated power markets. The UC objective is to minimize the total operating cost of the generating units during the scheduling horizon, subject to a number of system and unit constraints. The overall problem can be divided into two sub-problems: the mixed-integer nonlinear programming problem of determining the on/off state of the generating units for every hour of the dispatch period and the quadratic programming problem of dispatching the load among them. The simultaneous solution of both problems is a very complicated procedure, the difficulty of which grows proportionally to the number of units and constraints taken into consideration. For several decades, UC has been an active research topic because of potential savings in operation costs. As a consequence, several solution techniques have been proposed such as heuristics [1-3], dynamic programming [4-6], mixed-integer linear programming (MILP) [7, 8], Lagrangian relaxation [9-15], simulated annealing [16-18] and evolution-inspired approaches [19-23]. A recent extensive literature survey on unit commitment can be found in [24]. Among the aforementioned methodologies, Lagrangian relaxation is the most widely used approach because of its capability of solving large-scale problems. The main disadvantage of this method is that, due to the non-convexities of the unit commitment problem, heuristic procedures are needed to find feasible solutions, which may be suboptimal. In contrast, MILP guarantees convergence to the optimal solution in a finite number of steps [25] while providing a flexible and accurate modeling framework. In addition, during the search of the problem tree, information on the proximity to the optimal solution is available. Efficient mixed-integer linear software such as the branch-and-cut algorithm has been developed and optimized commercial solvers with large-scale capabilities are currently available [26-28]. As a consequence, a great deal of attention has been paid to MILP-based approaches. In [7], MILP was first applied to solve the unit commitment problem. The formulation in [7] was based on the definition of three sets of binary variables to, respectively, model the startup, shutdown and on/off states for every unit and every time period. This mixed-integer linear formulation was extended in [7] to model the self-scheduling problem faced by a single generating unit in an electricity market. Non-convex production costs, time-dependent startup costs and intertemporal constraints such as ramping limits and minimum up and down times were accounted for at the expense of increasing the number of binary variables. For realistic power systems comprising several tens of generators, the models of [7] require a large number of binary variables. Thus, the resulting MILP problems might be computationally intensive for state-of-the-art

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implementations of branch-and-cut algorithms [26, 27] and current computing capabilities. Startup costs and minimum up and down times were formulated using linear expressions that required a single type of binary variables [29]. However, the unit commitment model did not consider ramping limits and their influence on the spinning reserve constraints. In addition, shutdown costs were not modeled either.

In this study, a new formulation based on generalized benders decomposition approach for solving UC is presented. This approach decomposes UC into a master problem and a sub-problem. The master problem is an integer program and sub-problem is a nonlinear program. The master problem applies the Mixed Integer Programming (MIP) method to solve UC and find proper on/off states of the units and the sub-problem uses this solution to form appropriate cuts and adds them to the master problem for solving the next iteration of UC. The iterative process will continue until predefined gap is obtained and a converged optimal solution is found. In the proposed decomposition approach both sub-problem and master problem would have constraints. The objective function and constraints of the master problem are modified in a way that there exists no continuous variable in this problem, which helps to obtain the solution faster.

PROBLEM FORMULATION

In electric power systems, the unit commitment scheduling mainly determines the startup, shutdown and generation output of all units from an initial status while satisfying the load demand, spinning reserve requirement and other operational constraints.
The UC is formu lated as follow:

$$\operatorname{Min} C(U,P)=\sum_{i=1}^{N}\sum_{t=1}^{T}\left[F_i\left(P_i(t)\right)u_i(t)+SU_i(t)+SD_i(t)\right] \quad (1)$$

The production cost of unit i, Fi (Pi (t)), is conventionally taken in a quadratic form:

$$F_i\left(P_i(t)\right)=a_i+b_iP_i(t)+c_iP_i^2(t) \quad (2)$$

The constraints of the optimization problem are:

- System lo ad balance

$$\sum_{i=1}^{N}u_i(t)P_i(t)=D(t) \quad (3)$$

- System spinning reserve requirements

$$\sum_{i=1}^{N}u_i(t)P_i^{max}\geq D(t)+R(t) \quad (4)$$

- Unit generation output limits

$$P_i^{min}\leq P_i(t)\leq P_i^{max} \quad (5)$$

- Ramp rate limits

$$P_i(t)\leq P_i(t-1)+RU_i \quad (6)$$

$$P_i(t)\geq P_i(t-1)-RD_i \quad (7)$$

- Minimum up time and down time constraints

$$UT_i\leq T_i^{on}(t) \quad (8)$$

$$DT_i\leq T_i^{off}(t) \quad (9)$$

- Other constraints

There exist some additional constraints such as Transmission flow and bus voltage limits, load shedding, bilateral contracts, limitations on state and control variables and scheduled outages.

MATERIALS AND METHODS

As we will mention later, there are some non-linearities in the objective function and on/off variables are binary variables. So we have a mixed integer nonlinear programming model and need an efficient algorithm to solve these kinds of problems. We use Generalized Benders Decomposition (GBD) method to solve UC. The GBD method solves in a cycle of iterations a mixed integer linear programming problem (MILP) and a relaxed NLP, with fixed integer variables. The local minimum of the NLP gives a solution (xk,yk) for the continues variables. The corresponding cost function and the nonlinear constraints are then linearized around this solution. In our problem only objective function is nonlinear and all the constraints are linear.

Assume we have an MINLP model in the form of [30-36]:

$$\begin{aligned}&\operatorname{Min}\ Cy+f(x)\\&\text{s.t:}\quad h(x)=0\end{aligned}$$

$$\begin{array}{lll} g(x)\leq 0, & Ax+By\leq b_1, & Dx+Ey=b_2\\ Gy\leq a, & x\in R^n, & y\in \text{Integer}\end{array} \quad (10)$$

Here x are the nonlinear variables and y are the binary variables appearing linearly in the model. The GBD algorithm can be stated as:

Generalized Benders Decomposition
{Initialization}
$K = 1$
$UB = \infty$
Select an initial value for the integer variables $\overline{y}^{(k)}$
{Step 1 : NLP sub problem}

$$\begin{aligned} \text{solve} \quad & \text{Min} \quad Z_{NLP} = C\overline{y}^{(k)} + f(x) \\ \text{s.t:} \quad & h(x) = 0 \\ & g(x) \le 0 \\ & Ax \le b_1 - B\overline{y}^{(k)} \\ & Dx = b_2 - E\overline{y}^{(k)} \\ & x \in X \end{aligned}$$

Where λ re the duals for inequalities $Ax \le b_1 - B\overline{y}^{(k)}$ and $Dx = b_2 - E\overline{y}^{(k)}$

Let the solution be $\overline{x}^{(k)}$ and $\lambda_p^{(k)}$

If NLP is infeasible (for this $\overline{y}^{(k)}$) then
Solve removing infeasibility

$$\begin{aligned} \text{Min} \quad & S = s_1 + s_2 + s_3 \\ \text{s.t:} \quad & h(x) = 0 \\ & g(x) \le 0 \\ & Ax - s_1 \le b_1 - B\overline{y}^{(k)} \\ & Dx + s_2 - s_3 = b_2 - E\overline{y}^{(k)} \\ & x \in X \end{aligned}$$

Let the solution be $\overline{x}^{(k)}, s_i^{(k)}, \lambda_r^{(k)}$
Else if $Z_{NLP} \prec UB$ then
{Record better solution}

$$\begin{aligned} UB &= Z_{NLp} \\ x^* &= \overline{x}^{(k)} \\ y^* &= \overline{y}^{(k)} \end{aligned}$$

End if
End if

Step 2: MIP Master problem}
If NLP (for $\overline{y}^{(k)}$) was feasible
Add this cut

$$\begin{aligned} Z_{MIP} \ge cy + f(\overline{x}^{(k)}) + \lambda_{p_1}^{(k)}[b_1 - By - A\overline{x}^{(k)}] \\ + \lambda_{p_2}^{(k)}[b_2 - Ey - D\overline{x}^{(k)}] \qquad K = 1,...,k \end{aligned}$$

To the last master problem

$$\begin{aligned} \text{Min} \quad & Z_{MIP} \\ \text{s.t:} \quad & \text{cuts from before iterations} \\ & Gy \le a \\ & y \in \text{integer} \end{aligned}$$

Else add this infeasibility cut to the last master problem

$$\lambda_{r_1}^{(k)}[b_1 - By] + \lambda_{r_2}^{(k)}[b_2 - Ey] \le 0$$

If $Z_{MIP} \ge UB$ then
Stop
Else
K=K+1
Go to step 1
End if
{End of algorithm}

By embedding GBD to the UC problem we see that the master problem deals with integer variables, which represent the commitment states of thermal units. Hence, it is actually the unit commitment problem. The NLP sub problem considers the generating output of individual unit so it is the economic dispatch problem.

Once the master problem is solved and commitment states of generating units are determined, the designated states will be imposed on the NLP sub problem for an economic dispatch. After the sub problem is solved, a set of dual values will be returned to the master problem and benders cut will be generated from these dual values, which will govern determination of the solution of the master problem.

Each problem has its own constraints. The constraints of the master problem include system spinning reserve requirement, minimum up time and down time constraints and ramping limits. All of the constraints in master are formulated using linear expressions that required a single type of binary variables. The objective function of the NLP sub-problem is quadratic but all the constraints are linear and its variables are continuous.

The objective function of the NLP only considers items from the original objective with continuous variables and master only takes items with integer variables. Because of the special structure of the UC problem if we take this process, the objective function of the first master doesn't have any background from the production function and it is possible this solution won't be a good starting point. For this reason we introduce new variables Q_i that are:

$$p_i(t) = Q_i(t) + p_i^{min}.V_i(t) \forall i \in N, \forall t \in T \quad (11)$$

After substitution in (1) and some changes, the objective function is as follows:

$$\text{MIN } C(u,p) = \sum_{t=1}^{T}\sum_{i=1}^{N}[(b_i + 2c_i p_{min}).Q_i(t) + c_i.Q_i^2(t) + (a_i + b_i p_i^{min} + c_i p_i^{min2}).V_i(t) + SU_i(t) + SD_i(t)] \quad (12)$$

With this change of variables the global solution is obtained in less iteration with more confidence.

The first master problem: the objective function of the first master problem consists of total production cost of committed units while generates their minimum output powers, startup cost of committed units and shut down cost of uncommitted units that were online in the previous period. The constraints of this problem are system spinning reserve requirements, Minimum up time constraints and minimum down time constraints.

The formulation is as follows:

$$\text{Min } Z_{MIP} = \left(a_i + b_i p_i^{min} + C_i P_i^{min2}\right)V_i(t) + SU_i(t) + SD_i(t) \quad (13)$$

here the start up and shut down costs are formulated as follows [32]:

$$SU_i(t) \geq SU_i^n\left[v_i(t) - \sum_{k=1}^{n} v_i(t-k)\right] \qquad SU_i(t) \geq 0 \quad (14)$$

SU_i^n models the start up cost as a stair wise function and is more accurate as the number of intervals increase. This function is usually defined as fallows:

$$SU^{(n)}(t) = \begin{cases} hC_i & \text{if } n \leq t_i^{cold} + DT_i \\ CC_i & \text{if } n > t_i^{cold} + DT_i \end{cases} \quad (15)$$

The minimization of the master problem is subject to the following constraints:
System spinning reserve requirements

$$\sum_{i=1}^{N} P_i^{max}.V_i(t) \geq D(t) + R(t) \quad (16)$$

This constraint holds adequate units online in order to meet the required spinning reserve. Also this constraint ensures that there are sufficient online units to serve the load demand. We use this fact and eliminate the system load balance equation from the master problem.

Minimum up time constraint

$$\sum_{t=1}^{G_i} V_i(t) = G_i \quad \forall i \in N$$
$$\sum_{n=t}^{t+\min(UT_i-1,\,T-t)} V_i(n) \geq \min(UT_i, T-t+1)\left[V_i(t) - V_i(t-1)\right] \quad 17)$$
$$\forall t = G_j + 1, \ldots, T$$

where Gi is the number of initial periods during which unit i must be online.

$$G_i = \text{Min}\left\{T, [UT_i - T_i^{on}(0)]u_i(0)\right\} \quad (18)$$

Constraint (17) holds the units on in relation to the last states of the units at the previous time span. Constraint (17) satisfies the minimum up time constraint during all the possible sets of consecutive periods of size UTi and the final periods in which if unit i is started up, it remains on until the end of the time span.
Minimum down time constraint

$$\sum_{t=1}^{L_j} V_i(t) = 0 \forall i \in N \quad (19)$$

$$\sum_{n=t}^{t+\min(UT_i-1,\,T-t)} [1 - V_i(n)] \geq \min(DT_i, T-t+1).[V_i(t-1) - V_i(t)] \quad (20)$$
$$t = L_j + 1, \ldots, T$$

Where Li is the number of initial periods during which unit i must be offline.

$$L_i = \text{Min}\left\{T, [DT_i - T_i^{off}(0)][1 - u_i(0)]\right\} \quad (21)$$

Constraint (19) holds the units off in relation to the last states of the units at the previous time span. Constraint (20) satisfies the minimum down time constraint during all the possible sets of consecutive periods of size DTi and the final periods in which if unit i is shutdown, it remains off until the end of the time span.

NLP sub problem: In NLP sub problem for every combination of commitment states, a quadratic economic dispatch algorithm does the allocation of the power generation among committed units. In this problem, integer variables are fixed by the solution of the master problem and show them by $\overline{V}_i$, so only continuous variables are handled. The continuous variable in this step is Qi(t).The objective function and related constraints are as follows:

$$\text{Min}\ \ Z_{NLp} = \sum_{i=1}^{N}\sum_{t=1}^{T}(b_i + 2c_i P_i^{min})Q_i(t) + c_i Q_i^2(t) \qquad (22)$$

The minimization of the NLP sub problem is subject to the following constraints:
Unit maximum and minimum generation output limit.

$$\begin{aligned} &Q_i(t) \le \left(P_i^{max} - P_i^{min}\right).\overline{V}_i(t) \qquad \forall t \in T, \forall i \in N \\ &Q_i(t) \ge 0 \end{aligned} \qquad (23)$$

System load balance

$$\sum_{i=1}^{N}\left[Q_i(t) + P_i^{min}.\overline{V}_i(t)\right] = D(t) \qquad \forall t \in T \qquad (24)$$

Ramp up and start up ramp rate

In order to satisfy ramp up rate limit we introduce system ramp up rate. The expression for the system ramp up rate is as follow:

$$\begin{aligned} &Q_i(t) - Q_i(t-1) \le RU_i.\overline{V}_i(t-1) + \\ &\left(RSU_i - P_i^{min}\right)[\overline{V}_i(t) - \overline{V}_i(t-1)] + P_i^{max}\left(1 - \overline{V}_i(t)\right) \end{aligned} \qquad (25)$$

The system ramp up rate shows the maximum ability of the system to respond to the increase of the load demand in two consecutive time periods. Variables Q_i are constrained by ramp up and startup ramp rate.
Shut down ramp rates

$$\begin{aligned} &Q_i(t) \le (p_i^{max} - p_i^{min}).\overline{V}_i(t+1) + \\ &(RSD_i - p_i^{min})[\overline{V}_i(t) - \overline{V}_i(t+1) \end{aligned} \qquad (26)$$

Ramp down and Shut down ramp rates

Similarly, in order to satisfy ramp down rate limit we introduce system ramp down rate. The expression for the system ramp down rate is as follows:

$$\begin{aligned} &Q_i(t-1) - Q_i(t) \le RD_i.\overline{V}_i(t) + (RSD_i - p_i^{min}) \\ &[\overline{V}_i(t-1) - \overline{V}_i(t)] + (p_i^{max} - p_i^{min})[1 - \overline{V}_i(t-1)] \end{aligned} \qquad (27)$$

Master problem: Master problems are like the first master problem but in each interaction a new cut is added to the previous master problem. There are two kinds of cuts that depend on the result of NLP sub problem. When we obtain $\overline{V}_i$ from the master problem and then fix it in NLP sub problem, one of the two bellow cases may happen:

NLP is feasible with respect to $\overline{V}_i$: In this case dual variables of the NLP constraints play essential roles and with a penalty multiplier we add active constraints to the objective function of the master problem. The cut is as follows:

$$\begin{aligned} &Z_{MIP} \ge (a_i + b_i p_i^{min} + c_i p_i^{min^2})\ V_i(t) + SU_i(t) + SD_i(t) + \\ &(b_i + 2c_i p_i^{min})\overline{Q}_i(t) + c_i \overline{Q}_i^{\,2}(t) + \lambda_{i1}^p\left[(p_i^{max} - p_i^{min})\ V_i(t)\right] \\ &+\lambda_{i2}^p\left[D(t) - \sum_{i=1}^{N} p_i^{min}.V_i(t)\right] + \\ &\lambda_{i3}^p\begin{bmatrix} RU_i.V_i(t-1) + (RSU_i - P_i^{min})[V_i(t) - V_i(t-1)] \\ +P_i^{max}(1 - V_i(t)) \end{bmatrix} + \\ &\lambda_{i4}^p\left[(P_i^{max} - P_i^{min}).V_i(t+1) + (RSD_i - P_i^{min})(V_i(t) - V_i(t+1))\right] + \\ &\lambda_{i5}^p\begin{bmatrix} RD_i.V_i(t) + (RSD_i - P_i^{min})(V_i(t-1) - V_i(t)) + \\ (P_i^{max} - P_i^{min})(1 - V_i(t-1)) \end{bmatrix} \end{aligned} \qquad (28)$$

$\overline{Q_i}(t)$ are the values of $Q_i(t)$ that obtain from solving NLP sub problem and $\lambda_{ij}^{\ p}, j = 1,...,5, \forall i \in N$ are the values of dual variables of NLP sub problem.

NLP is infeasible with respect to $\overline{V}_i$: In this case we solve a new feasibility check sub problem to calculate λ^r. This process is done to transform an infeasible NLP problem to a feasible one.

Feasibility check sub problem:

$$\begin{aligned} &\text{Min } S = \sum_{i=1}^{N}\sum_{j=1}^{6} S_{ij} \\ &s.t:\ \ Q_i(t) - S_{i1} \le (P_i^{max} - P_i^{min}).\overline{V}_i(t) \\ &\qquad \sum_{i=1}^{N} Q_i(t) + S_{i2} - S_{i3} = D(t) - \sum_{i=1}^{N} p_i^{min}.\overline{V_i}(t) \\ &\qquad Q_i(t) - Q_i(t-1) - S_{i4} \le RU_i.\overline{V_i}(t-1) + (RSU_i - p_i^{min}). \\ &\qquad [\overline{V}_i(t) - \overline{V}_i(t-1)] + p_i^{max}(1 - \overline{V}_i(t)) \\ &\qquad Q_i(t) - S_{i5} \le (P_i^{max} - P_i^{min}).\overline{V}_i(t+1) + \\ &\qquad (RSD_i - P_i^{min})\left[\overline{V}_i(t) - \overline{V}_i(t+1)\right] \\ &\qquad Q_i(t-1) - Q_i(t) - S_{i6} \le RD_i.\overline{V}_i(t) + (RSD_i - P_i^{min}) \\ &\qquad \left[\overline{V}_i(t-1) - \overline{V}_i(t)\right] + (P_i^{max} - P_i^{min})\left[1 - \overline{V}_i(t-1)\right] \end{aligned} \qquad (29)$$

We obtain $\lambda_{ij}^{\ r}, \forall i \in N, j = 1,...,5$. Then we add below cut to the previous master problem:

$$\begin{aligned}
&\lambda_{i1}^{r}(P_i^{max} - P_i^{min})V_i(t) + \lambda_{i2}^{r}[D(t) - \sum_{i=1}^{N} P_i^{min}.V_i(t)] \\
&+\lambda_{i3}^{r}[RU_i.V_i(t-1) + (RSU_i - P_i^{min}).(V_i(t) - V_i(t-1)) + p_i^{max}(1 - V_i(t))] \\
&+\lambda_{i4}^{r}[(P_i^{max} - P_i^{min})V_i(t+1) + (RSD_i - P_i^{min})(V_i(t) - V_i(t+1))] \\
&+\lambda_{i5}^{r}\begin{bmatrix} RD_i.V_i(t) + (RSD_i - P_i^{min})(V_i(t-1) - V_i(t)) + \\ (P_i^{max} - P_i^{min})(1 - V_i(t+1)) \end{bmatrix} \le 0
\end{aligned} \quad (30)$$

Stopping criteria:
The algorithm stops when

$$Z_{MIP} \ge Z_{NLP} + \sum_t \sum_i [(a_i + b_i p_i^{min} + c_i p_i^{min\,2}).\overline{V_i}(t) + \overline{SU_i}(t) + \overline{SD_i}(t) \quad (31)$$

SIMULATION RESULTS

We apply two case studies to illustrate the performance of the proposed technique.

Case study 1: The objective of the first study is to prove the effectiveness of the proposed algorithm on a system of 10, 20, 40, 60, 80 and 100 units considering a dispatch period of 24 hours.

The model has been implemented on a dell inspiron 6000 with a processor at 1.86 GHZ and 512 MB of RAM memory using CPLEX 9 to solve master problem and MINOS 5.51 to solve the quadratic economic dispatch in the sub problem. Both solvers have been called from GAMS. We run the model with three different gaps for a system of 100 units in case 1 and the results were shown in Table 1.

Also we run the model with 0.4% Gap-for every master problem-for 10, 20, 40, 80, 100 units and compare the results with other methods (Table 2).

The results shown in Table 2 prove that the proposed algorithm provides high quality solutions in terms of the total cost especially for systems with a lot of units.

Table 1: The model results for a system of 100 units without ramp rate limits with different gaps

Gap (%)	Cost ($)	Time (Sec)	# iterations
0.5	5605417	79	4
0.4	5603496	64	3
0.3	5602638	653	3

Table 2: Comparison of the model results with the best answer already obtained

# units	Cost ($)	Time (sec)	# iterations	Best cost ($)
100	5603496	64	3	5605189 [34]
80	4485443	110	4	4485633 [35]
60	3363876	69	4	3363491 [35]
40	2243646	71	4	2242178 [36]
20	1123619	31	5	1122622 [36]
10	565537	10	5	563977 [37]

Table 3: Comparison of results this method with [6] for 40-unit system with ramp rates (case study 2)

Method	Cost	Time
GBD	2246776	186.0
Ref. [6]	2256971	199.5

Case study 2: In this case we compare obtained results with [33] for a system of 40 units with ramp rate limitations. Considering the ramp rates of the units, it has been assumed that the value of rate up of a unit is equal to the value of rate down. The ramp rate of each unit was taken to be 20% of its maximum power output per hour. Table 3 compares the results of this model with [6] for the 40-unit system with ramp rates.

It can be seen that, as expected, the inclusion of ramp rate constraints led to a slight increase of the total cost of the system. The execution time of the algorithm also increased, but it remained in acceptable levels. The results prove that the proposed GBD method treats efficiently with the ramp rate constraints and the result and time of the algorithm is better than the result of [33].

CONCLUSION

This paper has embedded generalized benders decomposition on mixed integer nonlinear programming to solve thermal unit commitment problem. Also we have formulated ramp rate constraints in linear form. With generalized benders decomposition method we can break a large and difficult problem into two sub problems, IP and NLP. This procedure has implemented in lower time with better solutions in comparison with other methods, especially for systems with a large number of units.

The problem is then solved by using a commercially available optimization package. The proposed model has been successfully tested on a realistic case study. Numerical results have revealed the efficient performance of this method.

REFERENCES

1. Lee, F.N., 1988. Short-term thermal unit commitment-A new method. IEEE Trans. Power Syst., 3 (2): 421-428.
2. Li, C., R.B. Johnson and A.J. Svoboda, 1997. A new unit commitment method. IEEE Trans. Power Syst., 12 (1): 113-119.

3. Senjyu, T., K. Shimabukuro, K. Uezato and T. Funabashi, 2003. A fast technique for unit commitment problem by extended priority list, IEEE Trans. Power Syst., 18 (2): 882-888.
4. Snyder, W.L., H.D. Powell and J.C. Rayburn, 1987. Dynamic-programming approach to unit commitment. IEEE Trans. Power Syst., 2 (2): 339-350.
5. Hobbs, W.J., G. Hermon, S. Warner and G.B. Sheblé, 1988. An enhanced dynamic programming approach for unit commitment. IEEE Trans. Power Syst., 3 (3): 1201-1205.
6. Ouyang, Z. and S.M. Shahidehpour, 1991. An intelligent dynamic-programming for unit commitment application. IEEE Trans. Power Syst., 6 (3): 1203-1209.
7. Dillon, T.S., K.W. Edwin, H.D. Kochs and R.J. Tand, 1978. Integer programming approach to the problem of optimal unit commitment with probabilistic reserve determination. IEEE Trans. Power App. Syst., PAS-97 (6): 2154-2166.
8. Medina, J., V.H. Quintana and A.J. Conejo, 1999. A clipping-off interior point technique for medium-term hydro-thermal coordination. IEEE Trans. Power Syst., 14 (1): 266-273.
9. Merlin, A. and P. Sandrin, 1983. A new method for unit commitment at Electricité de France. IEEE Trans. Power App. Syst., PAS-102 (5): 1218-1225.
10. Zhuang, F. and F.D. Galiana, 1988. Towards a more rigorous and practical unit commitment by Lagrange relaxation. IEEE Trans. Power Syst., 3 (2): 763-773.
11. Wang, C. and S.M. Shahidehpour, 1994. Ramp-rate limits in unit commitment and economic dispatch incorporating rotor fatigue effect. IEEE Trans. Power Syst., 9 (3): 1539-1545.
12. Wang, C. and S.M. Shahidehpour, 1995. Optimal generation scheduling with ramping costs. IEEE Trans. Power Syst., 10 (1): 60-67.
13. Svoboda, A.J., C.-L. Tseng, C.-A. Li and R.B. Johnson, 1997. Short-term resource scheduling with ramp constraints. IEEE Trans. Power Syst., 12 (1): 77-83.
14. Lai, S.-Y. and R. Baldick, 1999. Unit commitment with ramp multipliers. IEEE Trans. Power Syst., 14 (1): 58-64.
15. Ongsakul, W. and N. Petcharaks, 2004. Unit commitment by enhanced adaptive Lagrangian relaxation. IEEE Trans. Power Syst., 19 (1): 620-628.
16. Zhuang, F. and F.D. Galiana, 1990. Unit commitment by simulated annealing. IEEE Trans. Power Syst., 5 (1): 311-318.
17. Mantawy, A.H., Y.L. Abdel-Magid and S.Z. Selim, 1998. A simulated annealing algorithm for unit commitment. IEEE Trans. Power Syst., 13 (1): 197-204.
18. Purushothama, G.K. and L. Jenkins, 2003. Simulated annealing with local search-A hybrid algorithm for unit commitment. IEEE Trans. Power Syst., 18 (1): 273-278.
19. Kazarlis, S.A., A.G. Bakirtzis and V. Petridis, 1996. A genetic algorithm solution to the unit commitment problem. IEEE Trans. Power Syst., 11 (1): 83-92.
20. Juste, K.A., H. Kita, E. Tanaka and J. Hasegawa, 1999. An evolutionary programming solution to the unit commitment problem, IEEE Trans. Power Syst., 14 (4): 1452-1459.
21. Arroyo, J.M. and A.J. Conejo, 2002. A parallel repair genetic algorithm to solve the unit commitment problem. IEEE Trans. Power Syst., 17 (4): 1216-1224.
22. Rajan, C.C.A. and M.R. Mohan, 2004. An evolutionary programming based tabu search method for solving the unit commitment problem. IEEE Trans. Power Syst., 19 (1): 577-585.
23. Damousis, I.G., A.G. Bakirtzis and P.S. Dokopoulos, 2004. A solution to the unit commitment problem using integer-coded genetic algorithm. IEEE Trans. Power Syst., 19 (2): 1165-1172.
24. Padhy, N.P., 2004. Unit commitment-A bibliographical saurvey. IEEE Trans. Power Syst., 19 (2): 1196-1205.
25. Nemhauser, G.L. and L.A. Wolsey, 1999. Integer and Combinatorial Optimization. New York: Wiley-Interscience.
26. Bixby, R.E., M. Fenelon, Z. Gu, E. Rothberg and R. Wunderling, 2000. MIP: Theory and practice closing the gap, in System Modeling and Optimization: Methods, Theory and Applications. Powell, M.J.D. and S. Scholtes (Eds.). Norwell, MA: Kluwer, pp: 19-50.
27. The *ILOG* CPLEX Website, 2006. [Online]. Available: http://www.ilog.com/products/cplex/.
28. Ervin Kalvelagen, some MINLP solution Algorithms, http://www.gams.com/~erwin/minlp.pdf
29. Nowak, M.P. and W. Römisch, 2000. Stochastic Lagrangian relaxation applied to power scheduling in a hydro-thermal system under uncertainty. Ann. Oper. Res., 100: 251-272.
30. Geoffrion, A.M., 1972. Generalized Benders decomposition. Journal of optimization Theory and Applications, 10 (4): 237-260.

31. Shahidehpour, M. and Y. Fu, 2005. Benders decomposition: applying Benders decomposition to power systems, IEEE Power and Energy Magazine, 3(2):20-21.
32. Carrion, M. and M. Arroyo, 2006. A computationally efficient mixed integer linear formulation for the thermal unit commitment problem. IEEE Trans actions on power system, 21 (3): 1371-1378.
33. Simopoulos, D.N., S.D. Kavatza and C.D. Vournas, 2006. Unit commitment by an enhanced simulated annealing algorithm. Power Systems Conference and Exposition, pp: 193-201.
34. Ongsakul, W. and N. Petcharaks, 2004. Unit Commitment by enhanced adaptive Lagrangian relaxation. IEEE Trans. Power Syst., 19: 620-628.
35. Cheng, C.P., C.W. Liu and G.C. Liu, 2000. Unit commitment by Lagrangian relaxation and genetic algorithms. IEEE Trans. Power Syst., 15: 707-714.
36. Valenzuela, J. and A.E. Smith, 2002. A seeded memetic algorithm for large unit commitment problems . J. Heuristics, 8: 173-195.