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Direction of Arrival (DOA) Enhancement by Array Processing

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Abstract: With multiple antennas, received and transmitted signals can be separated not only with temporal processing but also with spatial processing. We call the combination of spatial and temporal processing space-time processing [1]. Space-time processing is a tool for improving the overall economy and efficiency of a digital cellular radio system by exploiting the use of multiple antennas. Most current cellular radio modems do not, however, efficiently exploit the spatial dimension offered by multiple antennas. The spatial domain can be used to reduce co-channel interference, increase diversity gain, improve array gain and reduce inter-symbol interference. In our communication we will expose some techniques based on the application of an antenna arrays to enhance the direction of arrival (DOA) of communication signals.

Key words: Array processing . Beam-forming . DOA . Wireless communications

INTRODUCTION

In mobile radio communications, antenna arrays can be used to improve the quality and/or the capacity of the communication system. Because, a transmitted signal is subjected to reflections from buildings, mountains, or other reflectors [2], Fig. 1. This phenomenon is called multipath propagation and leads to distortion or even a temporary suppression of the received signal.

The first attempt to automatically localize signal sources using antenna arrays was through beamforming techniques [3, 4]. The idea is to steer the array in one direction at a time and measure the output power. The steering locations which result in maximum power yield the DOA estimates. The array response, Fig. 2, is steered by forming a linear combination of the sensor outputs

$$y(t) = \sum_{l=1}^{L} w_l^* x_l(t) = W^H X(t) \tag{1}$$

Given samples y(1),y(2),…,y(N), the output power is measured by

$$P(w) = \frac{1}{N}\sum_{t=1}^{N}\left|y(t)\right|^2 = \frac{1}{N}\sum_{t=1}^{N} w^H x(t) x^H(t) w = w^H \hat{R}\, w \tag{2}$$

where $\hat{R}$ is defined as

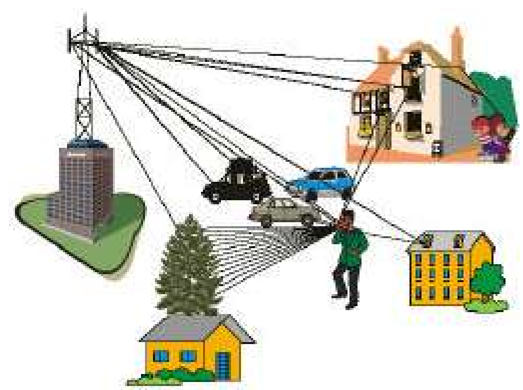

Fig. 1: Mulipath scenario

$$\hat{R} = \frac{1}{N}\sum_{t=1}^{N} x(t) x^H(t) \tag{3}$$

For which a spectral representation similar to that of R is defined as

$$\hat{R} = \hat{U}_s \hat{\Lambda}_s U_s^H + \hat{U}_n \hat{\Lambda}_n U_n^H \tag{4}$$

In the case of an ULA, the array steering vector is given by, $a(\theta) = [1, e^{jkd\sin(\theta)}, e^{j2kd\sin(\theta)}, \ldots, e^{j(L-1)kd\sin(\theta)}]$

d: Element spacing;
k: The wave number
L: Number of elements
θ: Angle of arrival

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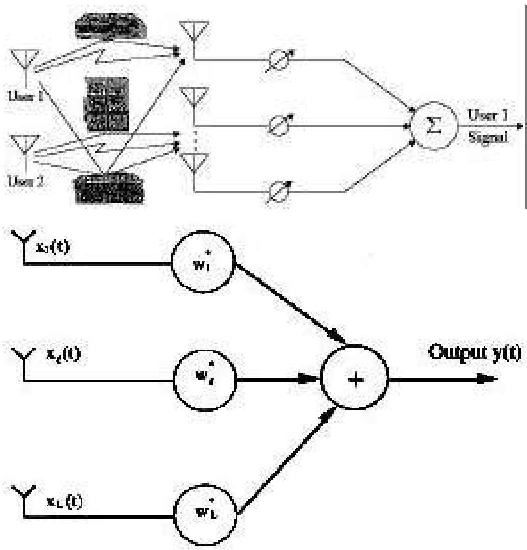

Fig. 2: Beam-former structure

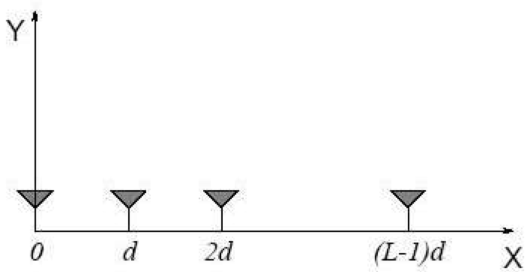

Fig. 3: Uniform Linear Array (ULA)

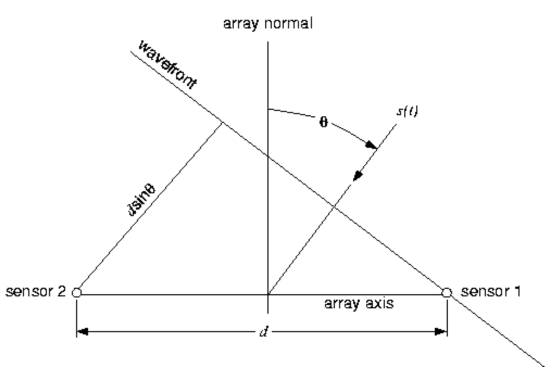

Fig. 4: ULA with 2 sensor elements

Consider an antenna array, e.g. ULA that receive various signals from several directions in space. The input signals are given by

$$x(t) = a(\theta)s(t)+n(t) \tag{5}$$

Where $a(\theta)$ is the steering vector, s(t) the transmitted signals and n(t) is the noise vector.

Different beamforming approaches correspond to different choices of the weighting vector w. For an excellent review of beamforming methods, we refer to [5].

CONVENTIONAL BEAM-FORMER

The conventional beamformer is a natural extension of classical Fourier-based spectral analysis [6] to sensor array data. For an array of arbitrary geometry, this algorithm maximizes the power of the beamforming output for a given input signal. Suppose we wish to maximize the output power from a certain direction θ. Given a signal emanating from direction θ, a measurement of the array output is corrupted by additive noise and written as (5). The problem of maximizing the output power is then formulated as:

$$\max_{w} E\left\{w^H x(t) x^H(t) w\right\} = \max_{w} w^H E\left\{x(t)x^H(t)\right\} w$$

$$\begin{aligned}\max_{w} E\left\{w^H x(t) x^H(t) w\right\} &= \max_{w} w^H E\left\{x(t)x^H(t)\right\} w \\ &= \max_{w}\left\{E\left|s(t)\right|^2 \left|w^H a(\theta)\right|^2 + \sigma^2 \left|w\right|^2\right\}\end{aligned} \tag{6}$$

The resulting solution is then :

$$w_{BF} = \frac{a(\theta)}{\sqrt{a^H(\theta)a(\theta)}} \tag{7}$$

Inserting the weighting vector (7) into (2) the classical spatial spectrum is obtained

$$P_{BF}(\theta) = \frac{a^H(\theta)\hat{R}a(\theta)}{a^H(\theta)a(\theta)} \tag{8}$$

CAPON'S BEAM-FORMER

In an attempt to alleviate the limitations of the above beam-former, such as its resolving power of two sources spaced closer than a beam-width, researchers have proposed numerous modifications. A well-known method was proposed by Capon [3]. The optimisation problem was posed as

$$\min_{w} P(w)$$

Subject to

$$w^H a(\theta) = 1 \tag{9}$$

Where P(w) is as defined in (2). Hence, Capon's beam-former attempts to minimize the power contributed by noise and any signals coming from other directions than

θ, while maintaining a fixed gain in the look direction θ. The optimal weight w can be found

$$w_{CAP} = \frac{\hat{R}^{-1}a(\theta)}{a^H(\theta)\hat{R}^{-1}a(\theta)} \tag{10}$$

Inserting the above weight into (2) leads to the following spatial spectrum

$$P_{CAP}(\theta) = \frac{1}{a^H(\theta)\hat{R}^{-1}a(\theta)} \tag{11}$$

LINEAR PREDICTION METHOD

This method estimates the output of one sensor using linear combinations of the remaining sensor outputs and minimizes the mean square prediction error, that is, the error between the estimate and the actual output [7, 8]. Thus, it obtains the array weights by minimizing the mean output power of the array subject to the constraint that the weight on the selected sensor is unity. An expression for the array weights and the power spectrum is given, respectively, by

$$w_{LP} = \frac{R^{-1}u_1}{u_1^H R^{-1} u_1} \tag{12}$$

u_1: Column vector of all zeros except one element, which is equal to one and

$$P_{LP}(\theta) = \frac{u_1^H R^{-1} u_1}{\left| u_1^H R^{-1} s_\theta \right|} \tag{13}$$

s_θ: Steering vector associated with the direction θ.

THE MUSIC ALGORITHM

MUSIC (Multiple SIgnal Classification) [9] is a Subspace-Based Method. The structure of the exact covariance matrix with the spatial white noise assumption implies that its spectral decomposition can be expressed as

$$R = APA^H + \sigma^2 = U_s\Lambda_s U_s^H + s^2 U_n U_n^H \tag{14}$$

where, assuming APA^H to be of full rank, the diagonal matrix Λ_s contains the M largest eigenvalues. Since the eigenvectors in U_n the noise eigenvectors are orthogonal to A. we have

$$U_n^H a(\theta) = 0, \quad \theta \in \{\theta_1, \ldots, \theta_M\} \tag{15}$$

To allow for unique DOA estimates, the array is usually assumed to be unambiguous, that is, any collection of L steering vectors corresponding to distinct DOAs θ_k forms a linearly independent set $\{a(\theta_1), \ldots, a(\theta_L)\}$. If a(.) satisfies these conditions and P has full rank, then APA^H is also of full rank. It then follows that $\theta 1, \ldots, \theta_M$ are the only possible solutions to the relation (15) which could therefore be used to exactly locate the DOAs.

In practice, an estimate $\hat{R}$ of the covariance matrix is obtained and its eigenvectors are separated into the signal and noise eigenvectors as in [4]. The orthogonal projector onto the noise subspace is estimated as

$$\hat{\Pi}^{\perp} = \hat{U}_n \hat{U}_n^H \tag{16}$$

The MUSIC spatial spectrum is then defined as

$$P_M(\theta) = \frac{a^H(\theta)a(\theta)}{a^H(\theta)\hat{\Pi}^{\perp}a(\theta)} \tag{17}$$

SIMULATION RESULTS

An ULA of M = 4 and 15 sensors of half-wavelength inter-element spacing is used to separate three uncorrelated emitters. The signal-to-noise ratio (SNR) for all sources is 20dB and 100dB. We have examined two cases. Table 1, illustrate the different experiments.

Table 1

DOA	SNR	Number of elements
T = [0° 5° 10°]	20 dB, 100 dB	M = 15
		M = 4
T = [0° 30° 60°]	20 dB, 100 dB	M = 15
		M = 4

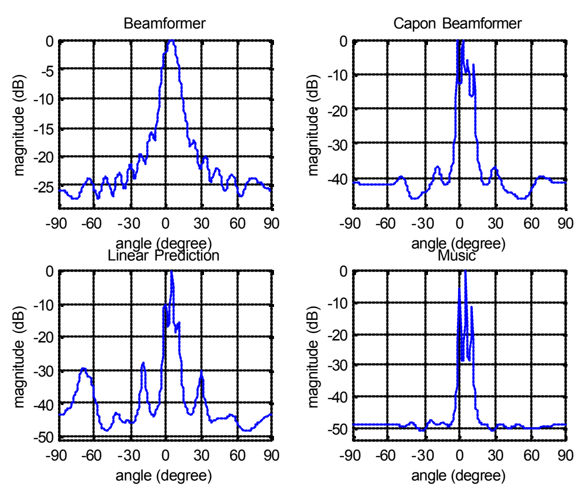

Fig. 5: SNR=20, M=15, θ= [0° 5° 10°]

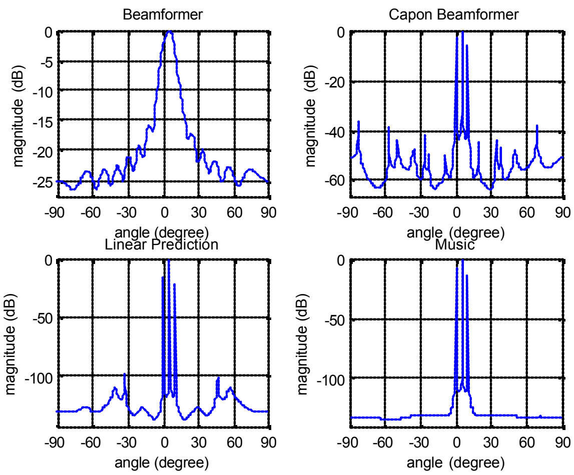

Fig. 6: SNR=100, M=15, θ= [0° 5° 10°]

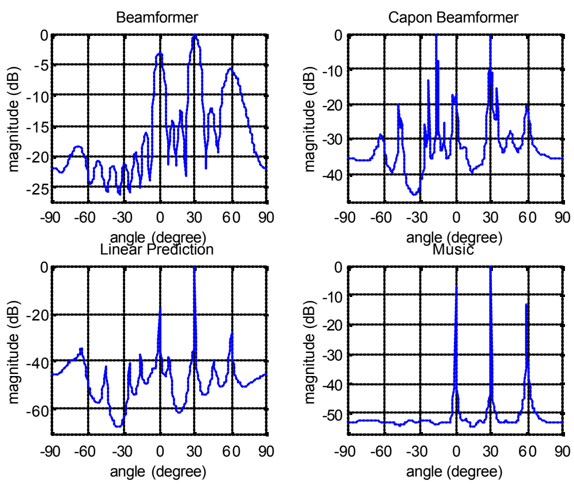

Fig. 7: SNR=20dB, M=15, θ= [0° 30° 60°]

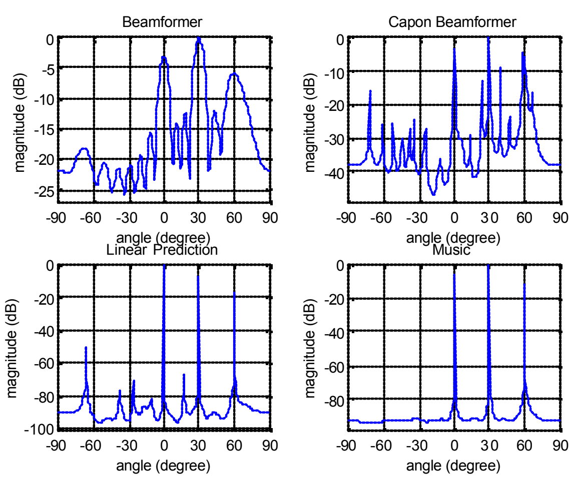

Fig. 8: SNR=100dB, M=15, θ= [0° 30° 60°]

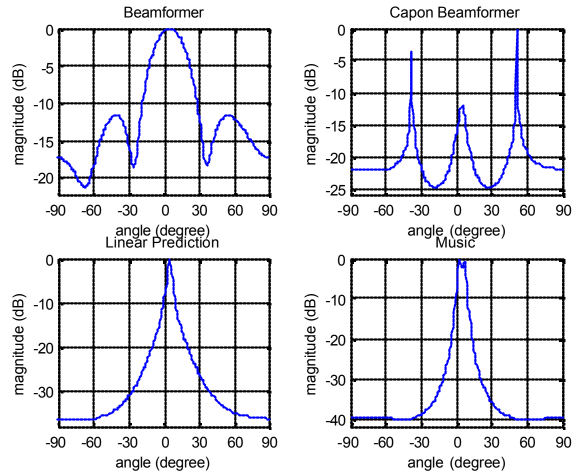

Fig. 9: SNR=20dB, M=4, θ= [0° 5° 10°]

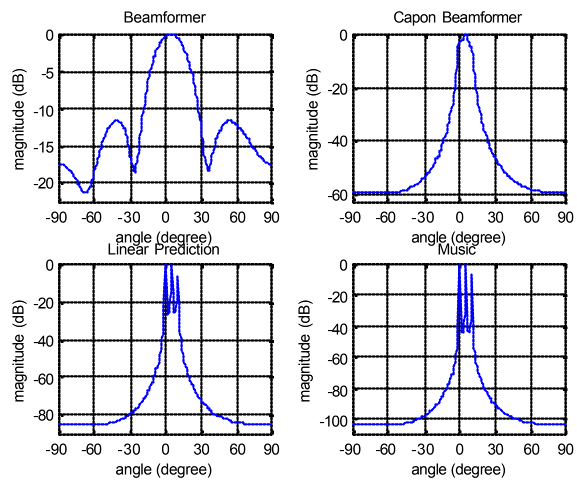

Fig. 10 : SNR=100dB, M=4, θ= [0° 5° 10°]

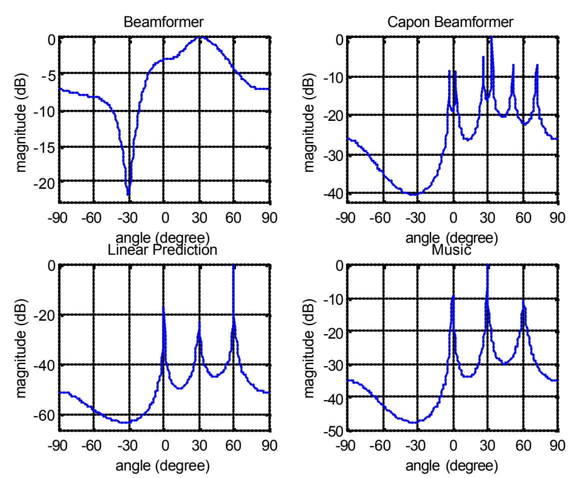

Fig. 11: SNR=20dB, M=4, θ= [0° 30° 60°]

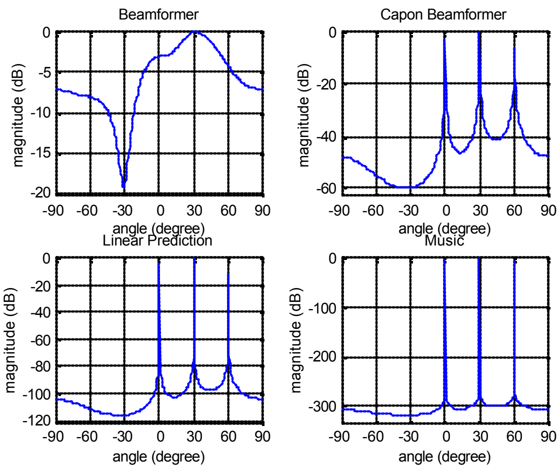

Fig. 12: SNR=100, M=4, θ= [0° 30° 60°]

The performance of the various estimators is illustrated in Fig. 5-12. The performance improvement of the MUSIC estimator was so significant.

CONCLUSION

In this study we have studied different methods of DOA estimation. The performance improvement of the MUSIC estimator was so significant that it became an alternative to most existing methods. However, some investigations must be done when we have coherent signals and non-gaussian noise.

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