# Numerical Simulation of Composite Plates to be used for Optimization of Mobile Bridge Deck

<sup>1</sup>M. Naghipour, <sup>2</sup>H.M. Daniali and <sup>2</sup>S.H.A. Hashemi Kachapi

<sup>1</sup>Department of Civil Engineering, Babol University of Technology, P.O. Box 484, Babol, Iran <sup>2</sup>Department of Mechanical Engineering, Babol University of Technology, P.O. Box 484, Babol, Iran

**Abstract:** In this study, for optimization and decreasing of the weight of Military Mobile Bridges (MMB), numerical simulations of laminated composite plate, which is more applicable in manufacturing of deck of MMB were done. The maximum deflection and stress are of the major parameters that are taken in to account in the plates design. Final purpose of this study is to obtain suitable mechanical property of composite plates. To reach to this purpose, the behavior of laminated composite plates under pressure loading is studied by using two numerical simulations. In Primary stage, Sightly plates have been solved by finite element simulation using the ANSYS software which applies the First order Shear Deformation Theory (FSDT) and the results have been compared with the results of finite difference method using the MATLAB software which applies the Classical Laminate Plate Theory (CLPT). The plates have been evaluated with actual condition of problem such as distributed load, under two clamp boundary conditions with different layers. The effects of fiber orientation, number of layers and stiffness ratio on the displacement and stress response of symmetric and anti-symmetric laminated composite plates subjected to uniformly pressure loads are presented. The deflection of central point of plates becomes minimum for 90/0° fiber orientation with 5 layers. The thickness of individual layers plays an important role in the response of the plate and stiffness ratio of 20 which it's for composite material of Carbon Reinforce Fiber Polymer (CFRP). At result, application the composite material of CARBON/Epoxy is suitable for bridge deck where it has been extreme of strength apposite minimum density.

**Key words:** Military Mobile Bridge (MMB) . Finite element method . Finite difference method . Laminate . Composite materials

## INTRODUCTION

The project aims at the development of easily and rapidly deployable mobile Bridging system for MLC70 load for the tactical sites for the advancement of troops [1-4]. Mobile bridge is bridge which used in critical condition Quid Pro Quo constant bridge. Mobile bridges are frequently used at military industry and Armed forces world over in assault and other roles. Such bridges are often mechanized and are carried on suitable trucks having mobility parameters matching with other accompanying equipment. While, addition of length is Sightly at this kind bridge, it's weighted and mechanism of bridge installation can't set up and bridge has been not possible of install. In this field, for addition of length and optimization of mobile bridge are following decrease of bridge weight. For this purpose, we apply composite materials in bridge deck for decrease of bridge weight.

In recent years, the use of laminated plates as structural members has increased considerably. Due to their high stiffness and high strength to weight ratios and high rigidity [3], they have been used widely in many engineering applications such as military industry, mobile bridge aircraft, missile, shipbuilding, auto industries and building construction. The correct and effective use of such laminates requires more complex analysis in order to predict accurately the elastic response (such as deflection, stress analysis) of these structures under external loading. A considerable amount of research work has been carried out on the elastic behavior of laminated plates (particularly thin plates). Among the published works, von Karman plate theory has gathered the most attention for the non-linear responses of plates going under large deflection. A number of studies have been carried out concerning the large deflection analysis of plates and a comprehensive list of papers published in the area is given in Refs [5-8].

Composite structures can be analyzed by using analytical and numerical methods. Generally, when a composite structure is modeled, some assumptions and simplifications have to be made. For the solution of coupled, non-linear partial differential equations, many procedures (such as Galerkin method, Fourier series and Rayleigh-Ritz have been used. The Finite-Element Method (FEM) has been widely utilized in the analysis of composite structures, based First-Order Shear Deformation Theory (FSDT). However, the FEM has some difficulties when the composite structure investigated is quite thick [9-15]. Commercial finiteelement analysis programs have special composite elements to be used in the analysis of composite structures. These composite elements can have only one or two elements through the thickness of structures [16]. This limitation weakens the modeling of a multilayer structure. In our analysis, we do not apply any requirement on the element numbers. A multilayer composite structure may have so many elements through the thickness.

Hence, the pursuit of corresponding mechanical and mathematical models for predicting their behavior of static and dynamic characteristics, which is of great significance in modeling many mechanical parts in the afore-mentioned employment aspects besides other systems, has been an intensive research focus for several decades.

In the classical thin plate theory [6, 7], extended to classical laminated plate theory (CLPT) by Reissner and Stavsky [8] and Whitney [17], among others, the straight lines normal to the mid-surface before deformation are assumed to remain straight and normal to the mid-surface after deformation. That is, the transverse normal and shearing effects are negligible. As is known, this theory over-predicts deflection and stresses in that laminated composite materials are always of high flexibility in shear which necessitates that the transverse shearing strains must be accounted for to obtain an accurate evaluation of the mechanical behavior. To this end, numerous numerical models on the bases of the first-order shear deformation theory (FSDT) considering transverse shear deformations were successively reported in literature [18-20], in which free vibration studies are involved with the hypotheses that the inplane stresses or displacements vary linearly along the thickness direction and the rotations normal to the midsurface are independent on the transverse deflection. The first-order shear deformation theories (FSDTs) for bending plates proposed by Reissner (1945) and Mindlin (1951) have been used extensively in the analysis of shear flexible plates and shells (Noor and Burton) [19, 21-23]. However, the FSDT needs shear correction coefficients to rectify the constant

states of the transverse shear stress or strain through the thickness direction to approximately determine the shear strain energy. In order to model the mechanical behavior of laminated composite plates more adequately, every endeavor was made to derive refined, higher-order shear deformation theories (HSDTs) [24-28]. Another types of laminated plate theories, termed as the layerwise theories (LWTs), Three-dimensional theory (3D), 3D layerwise theory and the 3D elasticity theory [29-32] etc.

### FINITE DIFFERENCE APPROACH

**Classical plate equation:** The small transverse displacement w of a thin plate is governed by the Classical Plate Equation [6, 7],

$$\nabla^2 D \nabla^2 w = p \tag{1}$$

Where, p is the distributed pressure load acting in the same direction as z (and w) and D is the bending/flexural rigidity of the plate defined as follows:

$$D = \frac{Eh^3}{12(1 - v^2)}$$
 (2)

In which E is the Young's modulus, v is the Poisson's ratio of the plate material and h is the thickness of the plate. Furthermore, the differential operator  $\nabla^2$  is called the Laplacian differential operator  $\Delta$ ,

$$\Delta \equiv \nabla^2 = \begin{cases} \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{r} \frac{\partial}{\partial r} & \text{Cylindrical coordinate [circular plate]} \\ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} & \text{Cartesian coordinate [rectangular plate]} \end{cases}$$
(3)

If the bending rigidity D is constant throughout the plate, the plate equation can be simplified to,

$$\nabla^4 \mathbf{w} = \frac{\mathbf{p}}{\mathbf{D}} \tag{4}$$

Where,  $\nabla^4 = \nabla^2 \nabla^2 = \Delta \Delta$  is called the biharmonic differential operator. This small deflection theory assumes that w is small in comparison to the thickness of the plate h and the strains and the midplane slopes are much smaller than 1. A plate is called thin when its thickness h is at least one order of magnitude smaller than the span or diameter of the plate.

Classical Laminate Plate Theory (CLPT): The plate is assumed to be constructed by isotropic material and

subjected to transverse loading. Also, the Cartesian coordinate system is used. The classical laminate plate theory is summarized in following [6, 7]:

$$u(x,y,z) = u_0(x,y) - z \frac{\partial W}{\partial X}$$

$$v(x,y,z) = v_0(x,y) - z \frac{\partial W}{\partial Y}$$

$$w(x,y,z) = w(x,y)$$
(5)

Where u, v, w are the normal displacements in x, y, z directions, respectively. Term of strain-displacement:

Which  $\epsilon_1^{(0)}$ ,  $\epsilon_2^{(0)}$  and  $\epsilon_6^{(0)}$  are in-plan stretch and shear strains of middle surface and also  $k_x$ ,  $k_y$ ,  $k_{xy}$  are deflexions;

$$\begin{aligned} k_{x} &= \frac{\partial^{2} w}{\partial x^{2}}, \quad k_{y} &= -\frac{\partial^{2} w}{\partial y^{2}}, \quad k_{xy} &= -\frac{2\partial^{2} w}{\partial x \partial y}, \\ \epsilon_{1}^{(0)} &= \frac{\partial u}{\partial x} \quad \epsilon_{2}^{(0)} &= \frac{\partial v}{\partial y} \quad \epsilon_{6}^{(0)} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{aligned} \tag{7}$$

Equilibrium equations in composite plate:

$$\begin{cases} \frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} = 0\\ \frac{\partial N_y}{\partial x} + \frac{\partial N_y}{\partial y} = 0\\ \frac{\partial^2 M_x}{\partial x^2} + \frac{2\partial^2 M_{xy}}{\partial y \partial x} + \frac{\partial^2 M_y}{\partial y^2} + P = 0 \end{cases}$$
(8)

Which N and M are following:

$$\begin{cases}
N_{x} \\
N_{y} \\
N_{xy}
\end{cases} = \int_{-h/2}^{h/2} \begin{cases}
\sigma_{x} \\
\sigma_{y} \\
\tau_{xy}
\end{cases} dz, \quad
\begin{cases}
M_{x} \\
M_{y} \\
M_{xy}
\end{cases} = \int_{-h/2}^{h/2} \begin{cases}
\sigma_{x} \\
\sigma_{y} \\
\sigma_{xy}
\end{cases} zdz \qquad (9)$$

And term of stress-strain;

$$\begin{cases}
\sigma_{x} \\
\sigma_{y} \\
\sigma_{xy}
\end{cases}_{k} = \begin{bmatrix} \overline{Q} \end{bmatrix}_{k} \begin{cases}
\varepsilon_{x} \\
\varepsilon_{y} \\
\varepsilon_{xy}
\end{cases}_{k}, \quad
\begin{cases}
\sigma_{1} \\
\sigma_{2} \\
\sigma_{6}
\end{cases} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix} \begin{cases}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{6}
\end{cases} (10)$$

Which

$$\begin{split} &Q_{11} = \frac{E_{1}}{1 - v_{12}v_{21}}, \ \ Q_{12} = \frac{v_{12}E_{1}}{1 - v_{12}v_{21}}, \ \ Q_{22} = \frac{E_{2}}{1 - v_{12}v_{21}}, \\ &Q_{66} = G_{12}, \ \ v_{12} = \frac{E_{2}}{E_{1}}v_{12}, \epsilon_{x} = \frac{\partial u}{\partial x}, \ \ \epsilon_{y} = \frac{\partial v}{\partial y}, \\ &\epsilon_{z} = \frac{\partial w}{\partial z}, \ \ \gamma_{xy} = \epsilon_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \left[\overline{Q}\right]_{k} = \left[T\right]_{k}^{-1} \left[Q\right]_{k} \left[T\right]_{k}^{-T} \end{split}$$

Using Eqs. (6, 7) and term of stress-strain, yields;

$$\begin{cases}
N_{x} \\
N_{y} \\
N_{xy}
\end{cases} = \begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix} \begin{cases}
\mathcal{E}_{1}^{(0)} \\
\mathcal{E}_{2}^{(0)} \\
\mathcal{E}_{6}^{(0)}
\end{cases} + \begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix} \begin{bmatrix}
k_{x} \\
k_{y} \\
k_{xy}
\end{bmatrix},$$

$$\begin{cases}
M_{x} \\
M_{y} \\
M_{xy}
\end{cases} = \begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix} \begin{bmatrix}
\mathcal{E}_{1}^{(0)} \\
\mathcal{E}_{2}^{(0)} \\
\mathcal{E}_{6}^{(0)}
\end{bmatrix} + \begin{bmatrix}
D_{11} & D_{12} & D_{16} \\
D_{12} & D_{22} & D_{26} \\
D_{16} & D_{26} & D_{66}
\end{bmatrix} \begin{bmatrix}
k_{x} \\
k_{y} \\
k_{xy}
\end{bmatrix},$$
(12)

Which

$$A_{ij} = \sum_{k=1}^{N} (\overline{Q_{j}})_{k} (z_{k} - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\overline{Q_{j}})_{k} (Z_{k}^{2} - Z_{k-1}^{2})$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\overline{Q_{ij}})_{k} (Z_{k}^{3} - Z_{k-1}^{3})$$
(13)

Substituted Eqs. (6, 7) in Eq. (10), then, substituted of this equation in Eq. (9) and using Equilibrium equations, yields;

$$\begin{split} &D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} \\ &+ 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} - B_{11}\frac{\partial^4 w}{\partial x^4} - 3B_{16}\frac{\partial^3 u}{\partial x^2 \partial y} \\ &- (B_{12} + 2B_{66})\frac{\partial^3 u}{\partial x \partial y^2} - B_{26}\frac{\partial^3 u}{\partial y^3} + B_{16}\frac{\partial^3 v}{\partial x^3} \\ &- (B_{12} + 2B_{66})\frac{\partial^3 v}{\partial x^2 \partial y} - 3B_{26}\frac{\partial^3 v}{\partial x \partial y^2} - B_{22}\frac{\partial^3 v}{\partial y^3} = P(x,y) \end{split}$$

For isotropic condition rewrite:

$$\begin{split} D_{11} \frac{\partial^4 w}{\partial x^4} + 6D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} \\ + 6D_{26} \frac{\partial^4 w}{\partial x \partial y^3} + D_{22} \frac{\partial^4 w}{\partial y^4} = P(x,y) \end{split} \tag{15}$$

Where

$$D_{11} = D_{22} = D, \ D_{12} + 2D_{66} = 2D, \quad \frac{E_1}{E_2} = \frac{v_{12}}{v_{21}}$$
 (16)

By solving of Eq. (15) the deflection of composite plate can be gained.

#### FINITE ELEMENT APPROACH

The finite element formulation used in this study is based on the first-order shear deformation theory (FSDT), thereby making it applicable to thin as well as thick laminated plates. The geometry of a laminated plate is shown in Fig. 1. In Fig. 1, x, y, z are plate axes while 1, 2, 3 are the principal material directions [7, 18].

## First-order shear deformation theory

Assumptions:

$$\begin{split} & \epsilon_{z} = 0 \\ & \epsilon_{x} = \frac{\partial u}{\partial x}, \quad \epsilon_{y} = \frac{\partial v}{\partial y} \\ & \epsilon_{xz} = \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \quad \epsilon_{yz} = \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ & \epsilon_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \end{split} \tag{17}$$

Displacement variable field:

$$u(x,y,z) = u(x,y) + z\overline{\alpha}(x,y)$$

$$v(x,y,z) = v_0(x,y) + z\overline{\beta}(x,y)$$

$$w(x,y,z) = w(x,y)$$
(18)

Which

$$\overline{\beta} = -(\partial w/\partial y), \ \overline{\alpha} = -(\partial w/\partial x)$$

Substitute Esq. (18) to Eq (17), which yields:

$$\begin{split} & \epsilon_{x} = \frac{\partial u_{0}}{\partial x} + z \frac{\partial \overline{\alpha}}{\partial x}, \quad \epsilon_{y} = \frac{\partial v_{0}}{\partial y} + z \frac{\partial \overline{\beta}}{\partial y}, \quad \epsilon_{z} = 0 \\ & \epsilon_{xz} = \frac{1}{2} \left( \overline{\alpha} + \frac{\partial w}{\partial x} \right), \quad \epsilon_{yz} = \frac{1}{2} \left( \overline{\beta} + \frac{\partial w}{\partial y} \right) \\ & \epsilon_{xy} = \frac{1}{2} \left( \frac{\partial u_{0}}{\partial y} + \frac{\partial v_{0}}{\partial x} \right) + \frac{z}{2} \left( \frac{\partial \overline{\alpha}}{\partial y} + \frac{\partial \overline{\beta}}{\partial x} \right) \end{split} \tag{19}$$

Mid plane strain:

$$\varepsilon_{x_0} = \frac{\partial u_0}{\partial x}, \quad \varepsilon_{y_0} = \frac{\partial v_0}{\partial y}, \quad \varepsilon_{xy_0} = \frac{1}{2} \left( \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right)$$
 (20)

Curvatures:

$$k_x = \frac{\partial \overline{\alpha}}{\partial x}, \quad k_y = \frac{\partial \overline{\beta}}{\partial y}, \quad k_{xy} = \frac{1}{2} \left( \frac{\partial \overline{\alpha}}{\partial y} + \frac{\partial \overline{\beta}}{\partial x} \right)$$
 (21)



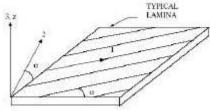


Fig. 1: Geometry of a laminated plate

Substitute upon equations in equation of stress-strain and using thin plate theory, yield:

$$\begin{cases}
\sigma_{x} \\
\sigma_{y} \\
\sigma_{yz} \\
\sigma_{xz} \\
\sigma_{xy}
\end{cases} = \left[\overline{Q}\right]_{k} \begin{cases}
\varepsilon_{x_{0}} + zk_{x} \\
\varepsilon_{y_{0}} + zk_{y} \\
\varepsilon_{yz} \\
\varepsilon_{xz} \\
\varepsilon_{xy_{0}} + k_{xy}z
\end{cases}_{k}$$
(22)

To Specify Quantity of deflection corresponding by upon (part 2) and determine of right side Quantity, can be to attain stress values in layers.

Finite element simulation: To test the correctness of the finite element formulation, several numerical case of the corresponding to actual conditions (bridge rectangular deck at size 24×4×0.02 m, in direction x, y, z, respectively) which results are available are considered. All laminates considered are rectangular in shape and simply supported on all edges, corresponding Fig with one thickness (b/h = 200). The first stage considered of 5 layer cross-ply  $(0/0^{0}, 45/-45^{0}, 60/-60^{0})$  $90/0^{\circ}$ ) laminate subjected to a pressure loading p = 10 N/m<sup>2</sup> where shown in Fig. 2 and then considered for 10, 20, 30 and 50 layers at upon condition. The total thickness of all layers is the same and layers of same symmetric orientation have equal thickness. The results of the present analysis agree well with the available results to finite difference method.

Applied element in ANSYS software is SHELL99. SHELL99 may be used for layered applications of a structural shell model. It usually has a smaller element formulation time. SHELL99 allows up to 250 layers. If more than 250 layers are required, a user-input

Table 1: Mechanical and numeral properties of models using in numerical approaches

Dimensional property	Number of layer	Orientation	Loading	Mechanical property
Length = 24 m Width = 4 m Thickness = 0.02 m	, 10, 20, 30, 50	0/0,45/-45, 60/-60, 90/0	$P_{const} = 10^{\frac{N}{m^2}}$	$\begin{split} \frac{E_1}{E_2} &= 5,10,20,30,50,  E_2 = E_3 \; , \; E_1 = 100e + 9 \\ G_{12} &= G_{23} = G_{13} = 0 \; .5 \; E_2,  v_{12} = v_{23} = v_{13} = 0.25 \\ \text{Failure criteria} \; \downarrow \\ & \text{xTenStrs} = 8.3E + 8,  \text{yTenStrs} = 2.6E + 7 \\ & \text{zTenStrs} = 1E + 9 \\ & \text{xComStrs} = -7.79E + 8,  \text{yComStrs} = -1.24E + 8 \\ & \text{zComStrs} = -1E + 9 \\ & \text{xyShStrs} = 4.1E + 7,  \text{yzShStrs} = 4.1E + 7 \\ & \text{xyShStrs} = 4.1E + 7 \end{split}$

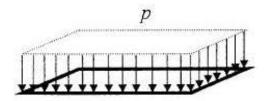


Fig. 2: Uniform pressure loading on composite plate

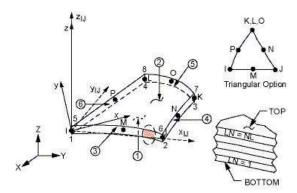


Fig. 3: Shell99 Geometry

constitutive matrix is available. The element has six degrees of freedom at each node: Translations in the nodal x, y and z directions and rotations about the nodal x, y and z-axes. See shell99 geometry in Fig. 3. Mechanical and numeral properties are tabulated in Table 1.

## NUMERICAL RESULTS AND DISCUSSION

Having validated the finite element formulation, the same has been used to study the influence of various parameters on the static behavior of laminated composite plates.

Effect of stiffness ratio, fiber orientation and number of layer on deck composite plate: Multi-layer rectangular laminated calm on two edges and subjected to uniformly distributed pressure load with b/h = 200

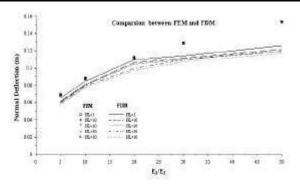


Fig. 4: Variation of central deflection w<sub>Nmax</sub> at fibers oriented of 0/0

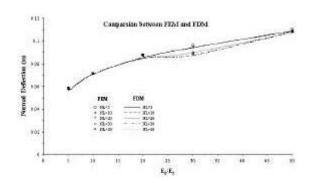


Fig. 5: Variation of central deflection w<sub>Nmax</sub> at fibers oriented of 45 /-45

and the thickness of 20 mm which is the same as thickness of the bridge deck, are tested using FEM and FDM to evaluate the effect of fiber orientation, number of layers, stiffness ratio and boundary conditions to be used for the MMB deck.

To reach this purpose, variable layers 1, 10, 20, 30, 50 and different fiber orientations  $0/0^0$ ,  $45/-45^0$ ,  $60/-60^0$  and  $90/0^0$  with different stiffness ratios  $E_1/E_2 = 5$ , 10, 20, 30, 50 were considered and the deflection principle normal stress and shear stress was obtained. The results caused a composite plate with minimum density and maximum strength was optimized.

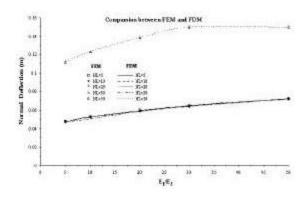


Fig. 6: Variation of central deflection  $w_{Nmax}$  at fibers oriented of 60 / -60

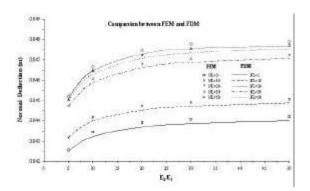


Fig. 7: Variation of central deflection  $w_{Nmax}$  at fibers oriented of 90 /0

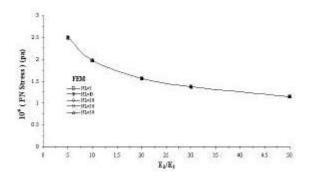


Fig. 8: Variation of Principle normal stress  $\sigma_{PNS}$  at fibers oriented of 0/0

Figure 4-15 show the outcome result from deflection, normal stress and shear stress from different stiffness ratios in various layers and several fiber orientations. Minimum number of layers against maximum advantage is the most important factors in optimization, design and manufacturing of composite materials to decrease the deflection and the existing stresses.

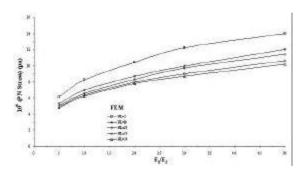


Fig. 9: Variation of Principle normal stress  $\sigma_{PNS}$  at fibers oriented of 45/-45

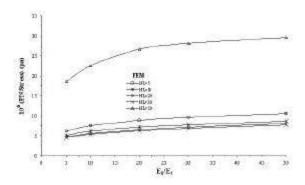


Fig. 10: Variation of Principle normal stress  $\sigma_{PNS}$  at fibers oriented of 60/-60

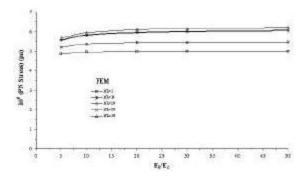


Fig. 11: Variation of Principle normal stress  $\sigma_{PNS}$  at fibers oriented of 90/0

As it is illustrated, increasing the number of layers does not have much of an effect in fiber orientations 0/0, 45/-45 and 60/-60, however in fiber orientation 90, increasing the layers, to some extend, concludes to a higher deflection, although it still has obvious difference against other orientations, which is because of the increasing stiffness in the D matrix.

This discrepancy is more seen in the increasing of the fewer layers. For instance, the discrepancy is more in layer increase from 5 to 10, rather than the increasing after the 30<sup>th</sup> layer.

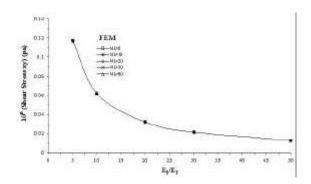


Fig. 12: Variation of transverse shear stress $\tau_{xy}$  at fibers oriented of 0/0

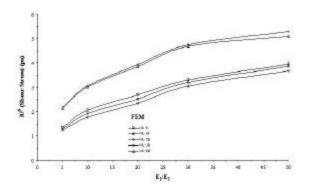


Fig. 13: Variation of transverse shear stress $\tau_{xy}$  at fibers oriented of 45/-45

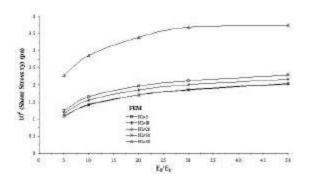
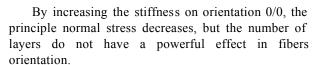


Fig. 14: Variation of transverse shear stress $\tau_{xy}$  at fibers oriented of 60/-60



In one unique case however (Fig. 10), increasing the number of layers result to lower number of layers, which in orientation 90/0 is exactly opposite, but it has a higher overall normal stress than the other two cases.

Shear stress is an important parameter and plays an important role in delamination between the plates and

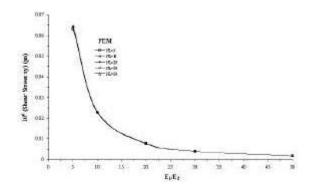


Fig. 15: Variation of transverse shear stress  $\tau_{xy}$  at fibers oriented of 90/0

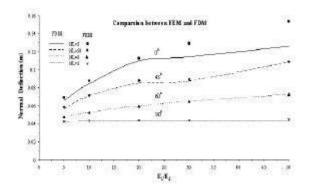


Fig. 16: Optimize central deflection w<sub>Nmax</sub> for various fibers orientation and number of layers

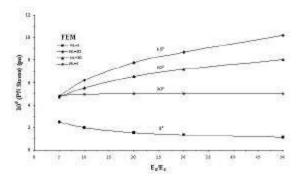


Fig. 17: Optimize principle normal stressσ<sub>PNS</sub> for various fibers orientation and number of layers

the coupling plate with the space structure. The 0/0 and 90/0 orientations cause the shear stress to decrease, while the stiffness ratio  $E_1/E_2$  increases and the number of layers did not have an observable effect in this case.

In the other two orientations however, increasing the number of layers did have a visible effect and increased the shear stress.

Considering the things mentioned above and the shown figures, we can clearly see that the number of layers do not have a powerful effect in the 90/0

Table 2: Maximum deflection, principle normal stress and shear stress in fiber orientation 90/0

		WNmax			σ <sub>PNS</sub>	$\tau_{xy}$
Number of layer	$E_1/E_2$	FSDT	CLPT	Error	FSDT	FSDT
NL = 5		0.042548	0.042542	0.0006E-3	4.86 E+6	0.063073 E+6
		0.043893	0.043711	0.1820E-3	4.99 E+6	0.007816 E+6
		0.044195	0.043896	0.2990E-3	5.01 E+6	0.001803 E+6
NL = 20		0.045185	0.045176	0.0009E-3	5.57 E+6	0.064205 E+6
		0.047484	0.047313	0.1710E-3	5.96 E+6	0.007716 E+6
		0.047892	0.047685	0.2070E-3	6.05 E+6	0.001755 E+6
NL = 50		0.045044	0.045037	0.0007E-3	5.68 E+6	0.064624 E+6
		0.04726	0.047105	0.1550E-3	6.12 E+6	0.007786 E+6
		0.047725	0.047521	0.2040E-3	6.22 E+6	0.00178 E+6

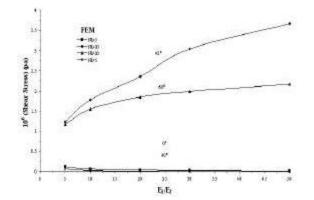


Fig. 18: Optimize transverse shear stresss  $\tau_{xy}$  for various fibers orientation and number of layers

orientation (which is the optimum orientation) and the minimum number of layers, which is 5, can be used. This much decrease of the number of layer, eases a lot of problems in manufacturing the plates and it is also easier to control the delamination. Also by decreasing the stiffness ratio ( $E_1/E_2$ ), makes the number of layers less effective, which is because of the fact that the material become close to a homogeneous condition and the number of layers have less effect in this kind of materials.

It should be mentioned that the fibers used in the laminates can have different orientations which can effect on the strength of its surface.

The shear stress and the normal stress are analyzed in the 5 layer condition which is its optimum condition and according to the results we can see that in 90/0 orientation the maximum deflection and maximum shear stress and in 0/0 orientation, the normal stress reach their minimum value.

The results are shown in optimum condition in Figure 16-19 and prove that using the 90/0 orientation, 5 layers and the stiffness ratio  $(E_1/E_2)$  of 20, is quite

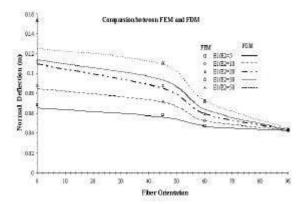


Fig. 19: Variation of central deflection with fiber orientation angle in 5 layer

suitable for the composite plate deck. It is also better to use symmetric layers, rather than none symmetric layers.

Figure 19, demonstrates the effect of different orientations with the matrix for 5 layers and different stiffness ratios.

Comparison of the numerical results is shown in Table 2.

## **CONCLUSION**

The finite element formulation presented in this study, which was based on shear deformation theory (FSDT) and predicts reasonably good results for the laminated plate of mobile bridge deck with different stacking sequence and fiber orientation. The results have also been compared with the result of finite difference method, based classic plat theory (CLPT). This comparison was done to determine the maximum deflection. The principle normal stress and shear stress of the central point was also determined using the finite element formulation. The studies reveal

the influence of various parameters and show the following facts:

- The central deflection is a minimum for 0° fiber orientation
- The central deflection decreases with increase in stiffness ratio.
- The normal stress and in-plane shear stress with increase in stiffness ratio.
- The central deflection decreases with increase in number of layers, but the rate of decrease is negligible beyond 20 layers.
- The normal stress is found to decrease and in-plane shear stress is found to increase with increase in the number of layers for plate of arrangements in all cases.
- The variation of deflection, normal stress and inplane shear stress with stiffness ratio follow the same pattern for both simply supported and clamped conditions. But the variation of transverse shear stress with stiffness ratio for clamped plates is different from that for simply supported plates.

Result's shown, using stiffness ratio,  $E_1/E_2 = 30$  which is for composite material CFRP (Carbon/Epoxy), farther, number of 20 layers and fiber orientation of 0° and apply clamped conditions, can be effective substance for composite plate of deck.

#### **ACKNOWLEDGEMENTS**

The authors gratefully acknowledge for financial support of Defense Technology Research Center of IRAN. The authors also would like to thank to Amin Naghipour for editing the paper.

#### REFERENCES

- Amit Manglik, Goutam Kumar Sarkar and N.B. Vijayakumar, 2006. Mechanised launching system for long single span mobile bridge. Advances in Bridge Engineering.
- Naresh Kumar and N.B. Vijayakumar, 2006.
   Design of a long single span mobile bridge for defence applications. Advances in Bridge Engineering.
- John B. Kosmagka, 1999. Structural testing of DARPA/BIR composite army bridge tread way composite structural. Elsevier, 4: 99-115.
- 4. Th. Keller and M. Schollmayer, 2003. Plate bending behavior of a pultruded GFRP bridge deck system. Composite structure, Elsevier.

- Sylvester, R.J., 1962. Buckling of sandwich cylinders under axial load. J Aeronaut Sci., 29: 863-872.
- Timoshenko, S.P. and S. Woinowsky-Krieger, 1959. Theory of plates and shells. New York: McGraw Hill.
- Szilard, R., 1974. Theory and analysis of plates (classical and numerical methods). New Jersey: Prentice-Hall.
- 8. Reissner, E. and Y. Stavsky, 1961. Bending and stretching of certain types of heterogeneous aelotropic elastic plates. ASME J. Appl. Mech., 28: 402-408.
- 9. Desai, Y.M., G.S. Ramtekkar and A.H. Shah, 2003. Dynamic analysis of laminated composite plates using a layer-wise mixed finite element model. Compos Struct., 59: 237-249.
- Srinivas, S., C.V.J. Rao and A.K. Rao, 1970. An exact analysis for vibration of simply-supported homogeneous and laminated thick rectangular plates. J Sound Vibr., 12: 187-199.
- 11. Zou, G.P., M. Naghipour and F. Taheri, 2003. A nondestructive method for evaluating natural frequency of glued-laminated beams reinforced with GRP. Nondestructive Testing and Evaluation, 19 (1-2): 53-65.
- Noor, A.K. and W.S. Burton, 1990. Threedimensional solutions for antisymmetrically laminated anisotropic plates. ASME J. Appl. Mech., 57: 182-188.
- 13. Noor, A.K. and W.S. Burton, 1990. Assessment of computational models for multilayered anisotropic plates. Compos Struct., 14: 233-265.
- 14. Fan, J.R. and J.Q. Ye, 1990. An exact solution for the statics and dynamics of laminated thick plates with orthotropic layers. Int. J. Solids Struct., 26: 655-662.
- 15. Chen, W.H. and W.W. Jiang, 1991. Resonant failure analysis of general composite laminates by using a mixed finite element method. J. Sound Vibr., 146: 479-490.
- 16. ANSYS Help Files.
- Whitney, J.M., 1987. Structural analysis of laminated anisotropic plates. Lancaster: Technomic.
- 18. Reddy, J.N., 1979. Free vibration of antisymmetric angle ply laminated plates including transverse shear deformation by the finite element method. J. Sound Vibr., 4: 565-576.
- 19. Noor, A.K. and W.S. Burton, 1989. Stress and free vibration analysis of multilayer composite plates. Compos Struct., 11: 183-204.

- Bert, C.W. and M. Malik, 1997. Differential quadrature: A powerful new technique for analysis of composite structures. Compos Struct., 39: 179-189.
- 21. Noor, A.K., W.S. Burton and C.W. Bert, 1996. Computational models for sandwich panels and shells. Appl Mech. Rev., 49 (3): 155-199.
- 22. Kant, T. and K. Swaminathan, 2001. Estimation of transverse/interlaminar stresses in laminated composites-a selective review and survey of current developments. Compos Struct., 49 (1): 65-75.
- 23. Kant, T., 1982. Numerical analysis of thick plates. Comput Methods Appl. Mech. Eng., 31: 1-18.
- Nelson, R.B. and D.R. Lorch, 1974. A refined theory for laminated orthotropic plates. ASME J. Appl. Mech., 41: 177-183.
- 25. Khdeir, A.A. and L. Librescu, 1988. Analysis of symmetric cross-ply laminated elastic plates using a higher-order theory. Part II: Buckling and free vibration. Compos Struct., 9: 259-277.
- Zenkour, A.M., 2000. Natural vibration analysis of symmetrical cross-ply laminated plates using a mixed variational formulation. Eur. J. Mech. A/Solids, 19: 469-485.

- Matsunaga, H., 2000. Vibration and stability of cross-ply laminated composite plates according to a global higher-order plate theory. Compos Struct., 48: 231-244.
- 28. Aydogdu, M. and T. Timarci, 2003. Vibration analysis of cross-ply laminated square plates with general boundary conditions. Compos Sci. Technol., 63: 1061-1070.
- 29. Sun, C.T. and J.M. Whitney, 1973. Theories for the dynamic response of laminated plates. AIAA J., 11: 178-183.
- 30. Reddy, J.N., 1987. A generalization of two-dimensional theories of laminated composite plates. Commun Appl. Numer Meth., 3: 173-180.
- 31. Cho, K.N., C.W. Bert and A.G. Striz, 1991. Free vibrations of laminated rectangular plates analyzed by higher order individual-layer theory. J. Sound Vibr., 145: 429-442.
- 32. Nosier, A., R.K. Kapania and J.N. Reddy, 1993. Free vibration analysis of laminated plates using a layerwise theory. AIAA J., 31: 2335-2346.