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# Development of a New Heuristic for Three Machine Flow-Shop Scheduling Problem with Transportation Time of Job

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**Abstract:** This paper is pertains to heuristic technique which obtain an optimal or near optimal scheduling of job for tree-machine flow-shop scheduling problem where in trans-plantation from one machine to another machine is taken into account.

**Key words:** Tree Machine Flow-Shop Scheduling with Transportation of Time

## INTRODUCTION

In a general "*i*-Job, 3machine", *I*= 1,...,*n* flow-shop scheduling theory is that the moving time for a Job from one machine to the other machines in the processing course of Jobs is ignored. In this paper we consider there are practical situations when creation times are required by Jobs for their transplantation from one machine to the other machines.

This situation can be considered when the machines on which Jobs are to be processed are planted at different places and these Jobs require forms of loading–time of Jobs and then unloading-time of Jobs. Consider flow-shop consisting of *i*-Jobs and three machines A, B, C. All Jobs are to be processed on these machines according to the order A, B, C. Each Job can be processed at a time on one machine and each machine can process only one Job at a time. Associated with each *i*-Job are processing times $t_{ix}$ on machines X = A, B, C and they are known prior to making scheduling decisions.

One of the earliest results in flow-shop scheduling theory is an algorithm given by Johnson [1] for scheduling Jobs in a two machine flow-shop to minimize the time at which all Jobs are completed. In this model an ordering of Jobs found time at which all Job processed by first machine and then by second and third machine in the prescribed order. This flow-shop problem due to Johnson can be described in the following manner. Consider a set $I$ = 1,...,*n* of *i*-Jobs where Job-*i* is defined by processing time $A_i > 0$, $B_i > 0$, $C_i > 0$ on machine A, B and C respectively. Each *i*-Job must complete on machine A before processing on machine B or C. The objective is to Schedule the Jobs on the machine in order that the maximum Job completion time is minimized.

## Main Results

### In this Section I Want to Prove the Basic Theorem as Follow As:

**Theorem 3.1:** Consider flow-shop consisting of n-Jobs and three machines A, B, C. All Jobs are to be processed on these machines according to the order A, B, C each Job can be processed at a time on one machine and each machine can process only one Job at a time. Associated with each Job I are processing times $t_{ix}$ on machines $x$ =A, B, C and they are known prior to making scheduling decisions.

**Proof:** Let $t_i$ and $g_i$ be the transportation times of Job I from machines A to B and B to C, respectively. Let the following structural relationship hold:

$$Min\{t_{iA} + t_i\} \geq Max\{t_{iB} + t_i\}$$

Then optimal schedule minimizing the total elapsed time is given by the following rule. Then optimal schedule minimizing the total elapsed time is given by the following rule. Job *i* precedes Job i+1, if.

$$Min\ (t_{iA} + t_i + t_{iB} + g_i, t_{i+1} + t_{i+1B} + g_{i+1} + t_{i+1B}) <$$

$$Min\ (t_{i+1A} + t_{i+1} + t_{i+1B} + g_{i+1}, t_i + t_{iB} + g_i + t_{iB})$$

To prove above theorem, we first prove the following lemma. Let

$$Min\ (t_{iA} + t_i) \geq Max(t_{iB} + t_i)$$

Then

$$Y_{pA} + U_P \geq Y_{p-1B}$$

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Here, under assumptions of Theorem 3.1, I consider the problem for $p = 2,3,\ldots,n$.

Let statement $P(q)$ for an arbitrary numbers q be defined as

$$P_{(q)} : Y_{q+1A} + U_{q+1} \geq Y_{qB}$$

Now for any arbitrary natural number q

$$\begin{aligned} Y_{1A} &= U_{1A} \\ Y_{1B} &= U_{1A} + U_1 + U_{1B} \\ Y_{2A} + U_2 &= U_{1A} + U_{2A} = U_2 \end{aligned}$$

We have

$$Y_{2A} + U_2 \geq Y_{1B}$$

Hence $p(q)$ is true for q = 1. Let statement $p(q)$ be true for q = m *i*, e

$$Y_{m+1A} + U_{m+1} \geq Y_{mB}$$

Now

$$\begin{aligned} Y_{m+1B} &= Max(Y_{m+1A} + U_{m+1}, Y_{mB}) + U_{m+1B} \\ &= Y_{m+1A} + (U_{m+1} + U_{m+1B}) \end{aligned}$$

$$Y_{m+2A} + U_{m+2} = Y_{m+1A} + (U_{m+2A} + U_{m+2}).$$

Now

$$U_{m+2A} + U_{m+2} \geq U_{m+1} + U_{m+1B}$$

Hence

$$Y_{m+2A} + U_{m+2} \geq Y_{m+1B}$$

Therefore, statement $P(q)$ is true for q = m+1. Hence statement $P(q)$ is true by induction hypothesis for every value of q.

**Note1:** If $Min\{t_{iA} + t_i\}) \geq Max\{t_i + t_{iB}\}$, then we can easily prove on the similar line as above

$$Y_{iB} \geq Y_{i+1A} + U_{i+1}$$

We now proceed to the proof of theorem.
Consider the Job-schedules S and S', defined by

$$\begin{aligned} S &= (j_1, j_2, \ldots, j_{i-1}, j_i, j_{i+1}, j_{i+2}, \ldots, j_n) \\ S' &= (j_1, j_2, \ldots j_{i-1}, j_i, j_{i+1}, j_{i+2}, \ldots, j_n) \end{aligned}$$

Let $U_{px}$, denote the processing and completion time of $q$th Job on machine X in the schedule S Similarly Let $U'$, $Y'$ denote the processing and completion respectively. Similarly, let $U'_p$ and $V'_p$ denote the transportation times of $P$th Job in the schedule S' from machines A to B and B to C, respectively.

$$\begin{aligned} Y_{pC} &= Max(Y_{pB} + U_p, Y_{p-1B}) + U_{pB} \\ &= Y_{pA} + U_p + U_{pB} \end{aligned}$$

Also, it is obvious that

$$\begin{aligned} Y_{pC} &= Max(Y_{pB} + V_p, Y_{p-1C}) + U_{pC} \\ &= Max(Y_{pA} + U_p + U_{pB} + V_p, Y_{p-1C}) + U_{pC} \end{aligned} \tag{1}$$

Now schedule $S$ is preferable to $S'$, if

$$Y_{nC} \leq Y'_{nC} \tag{2}$$

$$\begin{aligned} &Max(Y_{nA} + U_n + U_{nB} + V_n, Y_{n-1C}) + U_{nC} \leq \\ &Max(Y'_{nA} + U' + U'_{nB} + V'_n, Y'_{n-1C}) + U'_{nC} \end{aligned}$$

Now

$$\begin{aligned} Y_{nA} + U_n + U_{nB} + V_n &= U'_{nA} + U'_n + U'_{nB} + V'_n \\ U_{nC} &= U'_{nC} \end{aligned}$$

Therefore equation (2) is true, if

$$Y_{n-1C} < Y'_{n-1C} \tag{3}$$

Continuing in the same manner, one can get

$$Y_{pC} \leq Y'_{pC}, \qquad p = i+2, i+3, \ldots, n$$

$$Y_{i+1C} < Y'_{i+1C} \tag{4}$$

Now we calculate the values of, as $Y_{i+1C}, Y'_{i+1C}$ follows

$$Y_{i+1C} = Max(Y_{i+1B} + V_{i+1}, Y_{iC}) + U_{i+1C} =$$

$$Max(Max(Y_{i+1A} + U_{i+1}, Y_{iB}) + U_{i+1B} + V_{i+1}, Y_{iC}) + U_{i+1C} =$$

$$Max(Y_{i+1A} + U_{i+1} + U_{i+1B} + V_{i+1}, Y_{iC}) + U_{i+1C} =$$

$$\begin{aligned} &Max(Y_{i+1A} + U_{i+1} + U_{i+1B} + V_{i+1}, \\ &Max(Y_{iB} + V_i, Y_{i-1C}) + U_{iC}) + U_{i+1C} = \end{aligned}$$

$$\begin{aligned} &Y_{i+1C} = Max(Y_{i+1A} + U_{i+1} + V_{i+1} + U_{i+1B}, \\ &Y_{iB} + V_i + U_{iC} + Y_{i-1C} + U_{iC}) + U_{i+1C} \end{aligned}$$

$$\begin{aligned} &= Max(Y_{i+1A} + U_{i+1} + V_{i+1} + U_{i+1B}, \\ &Max(Y_{iA} + U_i, Y_{i-1B}) + U_{iB} + V_i + U_{iC}, \\ &Y_{i-1C}) + U_{iC}) + U_{i+1C} \end{aligned}$$

$$\begin{aligned} &= Max(Y_{i+1A} + U_{i+1} + V_{i+1} + U_{i+1B}, \\ &Y_{iA} + U_i + U_{iB} + V_i + U_{iC}, Y_{i-1C} + U_{iC}) + U_{i+1C} \end{aligned}$$

$$= Max(Y_{i-1A} + U_{iA} + U_{i+1A} + U_{i+1} + V_{i+1} + U_{i+1B} + U_{i+1C}, Y_{i-1A} + U_{iA} + U_i + U_{iB} + V_i + U_{iC} + U_{i+1C}, Y_{i-1C} + U_{iC} + U_{i+1C}).$$

Similarly We Have:

$$Y'_{i+1C} = Max(Y'_{i-1A} + U'_{iA} + U'_{i+1A} + U'_{i+1} + V'_{i+1} + U'_{i+1B} + U'_{i+1C}, Y'_{i-1A} + U'_{iA} + U'_i + U'_{iB} + V'_i + U'_{iC} + U'_{i+1C}, Y'_{i-1C} + U'_{iC} + U'_{i+1C})$$

Now comparing $S$ and $S'$ one can easily have:

$$Y_{i+1A} = Y'_{i-1A}$$

$$Y_{i-1C} = Y'_{i-1C}$$

$$U_{ix} = U'_{i+1} = t_{ix}, U_i = U'_{i+1} = t_i, V_i + V'_{i+1} = g_i$$

$$U_{i+1x} = U'_{ix} = t_{i+1x}, U_{i+1} = U_{ix} = t_{i+1}, V_{i+1} = V' = g_{i+1}$$

Hence Using above in (4) Gives

$$Max(Y_{i-1A} + t_{iA} + t_{i+1A} + t_{i+1} + g_{i+1} + t_{i+1B} + t_{i+1C}, Y_{i-1A} + t_{iA} + t_i + t_{iB} + g_i + t_{iC} + t_{i+1C}, Y_{i-1C} + t_{iC} + t_{i+1C})) <$$

$$Max(Y_{i-1A} + t_{i+1A} + t_{iA} + t_i + g_i + t_{iB} + t_{iC}, Y_{i-1A} + t_{i+1A} + t_{i+1} + t_{i+1B} + g_{i+1} + t_{i+1C} + t_{iC}, Y_{i-1C} + t_{i+1C} + t_{iC})$$

Or

$$Max(Y_{i-1A} + t_{iA} + t_{i+1A} + t_{i+1} + g_{i+1} + t_{i+1B} + t_{i+1C}, Y_{i-1A} + t_{iA} + t_i + t_{iB} + g_i + t_{iC}) <$$

$$Max(Y_{i-1A} + t_{i+1A} + t_{iA} + t_i + g_i + t_{iB} + t_{iC}, Y_{i-1A} + t_{i+1A} + t_{i+1} + t_{i+1B} + g_{i+1} + t_{i+1C} + t_{iC})$$

Subtract this on Both the Side:

$$Y_{i-1A} + t_{iA} + t_{i+1A} + t \;\; + t_{i+1} + t_{iB} + t_{i+1B} + g_i + g_{i+1} + t_{iC} + t_{i+1C}.$$

$$Max(-t_i - g_i - t_{iB} - t_{iC}, -t_{i+1A} - t_{i+1} - t_{i+1B} - g_{i+1}) <$$

$$Max(-t_{i+1} - g_{i+1} - t_{i+1B} - t_{i+1C}, -t_{iA} - t_i - t_{iB} - g_i)$$

$$Min(t_i + g_i + t_{iB} + t_{iC}, t_{i+1A} + t_{i+1} + t_{i+1B} + g_{i+1}) <$$

$$Min(t_{i+1} + g_{i+1} + t_{i+1B} + t_{i+1C}, t_{iA} + t_i + t_{iB} + g_i)$$

$$Min(t_{iA} + t_i + t_{iB} + g_i, t_{i+1} + t_{i+1B} + g_{i+1} + t_{i+1C}) <$$

$$Min(t_{i+1A} + t_{i+1} + t_{i+1B} + g_{i+1}, t_i + t_{iB} + g_i + t_{iC}).$$

**Note 2:** Now if the structural relationship in the theorem instead is taken as

$$Min(g_i + t_{iC}) \geq Max(t_{iB} + t_{iC})$$

Then also the theorem can be shown to hold good.

**Decomposition Algorithm:** The utility of above theorem can be summarized into following steps to give us decomposition algorithm that is numerical method to obtain optimal schedule minimizing total elapsed time for a `3-machine , n-Job` sequencing problem where in transportation times are taken into account . Let the given `3– machine, n-Job`.

Sequencing problem satisfying a given structural relationship be stated in the tableau form as follows:

| Job | machine A | $t_i$ | machine B | $g_i$ | machine C |
|---|---|---|---|---|---|
| 1 | $A_1$ | $t_1$ | $B_1$ | $g_1$ | $C_1$ |
| 2 | $A_2$ | $t_2$ | $B_2$ | $g_2$ | $C_2$ |
| 3 | $A_3$ | $t_3$ | $B_3$ | $g_3$ | $C_3$ |
| . | . | . | . | . | . |
| . | . | . | . | . | . |
| .. | . | . | . | . | . |
| N | $A_n$ | $t_n$ | $B_n$ | $g_n$ | $C_n$ |

Where $A_i$, $B_i$, $C_i$ denote the processing times of $i$-Job on machines $A,B,C,$ and $t_i$ and $g_i$ Denotes the transportation times of Job $i$ from machines A to B and B to C, respectively satisfying the structural relationship.

$$\min(A_i + t_i) \geq \max(B_i + t_i)$$

or

$$Min(C_i + g_i) \geq Max(B_i + g_i)$$

The rule to obtain optimal schedule minimizing the total elapsed time is decomposed into following steps:

**Step 1:** Convert the given problem into two machine problem.

Let G and H be the fictitious machines with the respective processing times $G_I$ and $H_I$ defined by

$$G_i = A_i + t_i + B_i + g_i$$

$$H_i = t_i + B_i + g_i + C_i$$

**Step 2:** Find the optimal sequence due to Johnson's [1954] procedure.

**Step 3:** Optimal sequence obtained in step 2 gives the required optimal sequence for the given problem.

**Particular Case:** If we set $t_i$= 0, $g_i$= 0 in the theorem, an alternate proof of the algorithm due to Johnson's [1954] discussed in 1.2 Chapter 1 becomes evident.

**Numerical Example:** Consider the following" 3- machine, 5-Job, sequencing flow-shop problem in the tableau form be given as.

Tableau Form as Follows:

| Job | machine A | $t_i$ | machine B | $g_i$ | machine C |
|---|---|---|---|---|---|
| 1 | 8 | 5 | 2 | 8 | 9 |
| 2 | 10 | 4 | 5 | 4 | 8 |
| 3 | 4 | 6 | 3 | 2 | 7 |
| 4 | 9 | 3 | 6 | 5 | 8 |
| 5 | 5 | 7 | 3 | 8 | 11 |

Where $t_i$ is the transportation time of Job *i* from machine A to B and $A_i$ and Bi are the processing times of Job *i* on machines A and B respectively. Solution As per step 1, let G and H is fictitious machines representing respectively. Then reduced problem as per step 2 is the tableau form is as follows.

| Job | machine G | machine H |
|---|---|---|
| | $G_i = A_i + t_i + B_i + g_i$ | $H_i = t_i + B_i + g_i + C_i$ |
| 1 | 20 | 21 |
| 2 | 23 | 19 |
| 3 | 15 | 18 |
| 4 | 23 | 22 |
| 5 | 23 | 29 |

Now using Johnson's (1954) procedure for the above reduced problem, the optimal sequence is (3, 1, 5, 4 and 2).

As per step (3) now this S is also optimal for our problem. If T(S) denotes total elapsed time for the schedule S then T(S) is calculated as in the following Tableau:

Tabular

| Job i | machine (A) in-out | $t_i$ | machine (B) in-out | $g_i$ | machine (C) in-out |
|---|---|---|---|---|---|
| 3 | 0.4 | 6 | 10-13 | 2 | 15-22 |
| 1 | 4-12 | 5 | 17-19 | 5 | 24-33 |
| 5 | 12-17 | 7 | 24-27 | 8 | 36-47 |
| 4 | 17-26 | 3 | 29-35 | 5 | 47-55 |
| 2 | 26-36 | 4 | 40-45 | 4 | 55-61 (T =61) |

## CONCLUSION

Hence (3, 1, 5, 4, 2) is obviously optimal as per our algorithm's claim.

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