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Hybrid Fuzzy Capacitated Hub Center Allocation Problem with Both Qualitative and Quantitative Variables

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Abstract: Hubs are facilities which are used to switch and transfer commodities between terminals (supplies and demands) in many-to-many distribution systems. The p-hub center allocation problem is concerned with allocating demand and supply nodes to hubs in order to route the traffic between origin-destination pairs in minimum time. The capacities of the hubs are given. Most of the researches have discussed this subject considering the quantitative parameters such as cost, capacity and time, in this paper we aim to consider qualitative aspects besides quantitative variables like service agility, zone traffic, capability for development in future and other aspects which DMs are interested in. Two models are presented, in first model we want to locate hub facilities based on qualitative variables. We had to decide in a fuzzy environment according to the linguistic terms about the future market demands, zone traffic and the other criteria. We will use fuzzy TOPSIS in order to find the location of p hubs then use an IP formulation to allocate terminal nodes to hubs. In second model a hybrid formulation performs both location and allocation phases with qualitative and quantitative criteria simultaneously. A numerical example is presented to illustrate the application of the proposed models and a comparison is described.

Key words: Hub location problem . Fuzzy TOPSIS . multi criteria decision making

INTRODUCTION

Hub location problems have many applications in computer networks, transportation [23], airlines [15, 9] telecommunications [21], Civil Aeronautics Board studies [15], postal delivery services, emergency services and rejuvenating processes [7]. Instead of connecting a direct link for every O-D route, a hub network provides indirect connection via a specified set of facilities called hubs which are used to switch, sort and transfer commodities. In some literatures like Thomadsen's work on hub location [22] it is assumed to have a complete graph for hub nodes and here we have the same assumption. There are several kinds of hub location problems based on characteristics of a particular hub network. One classification index is the way terminal nodes are allocated to hubs so that each non-hub node can be allocated either to one (single allocation scheme) or to more hubs (multiple allocation scheme). Ebery [10] and Costa *et al* [6] discussed single allocation and Ebery and Krishnamoorthy [11] and Camargo [3] considered multiple allocation problem. Another classification is derived from capacity constraint and hub problems may involve different kinds of capacity constraints in a network; it may be on the load of commodity a hub facility can handle or on the transfer arcs between hubs and terminals or on both of them. Fixed hub locating costs may also be assumed. A general survey of different hub location problems can be found in [2]. Table 1 presents a review on recent researches on hub location problem outlining principal characteristics of the researches.

This paper considers p-hub/D/SA, cap/•/minimax t which according to Drezner and Hamacher's classification pattern [7] means Capacitated Single Allocation p-Hub Center Problem. The first part p-hub says the problem is in p-hub class and second part D stands for Distance which means the cost or travel time has direct relation with distance. The third part is for problem constrains and other characteristics of the problem. Here we have Single Allocation and also capacity constrain is considered to match the real world problems. Final part shows the objective function of the problem, our objective is to minimize the maximum flow time in a network which ensures a time for service delivery. This objective function is useful for time sensitive services and transportation networks such as express mail services in which one can ensure a maximum time (for example 48 hours in Iran) to deliver letters from anywhere to any destination. To solve this problem we make the following assumptions:

- The objective function is *Minimax*.
- The solution space is *discrete* and *finite*.

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Table 1: A review on recent researches and various versions of hub location problem

		Decision variables			Capacitated		Objective function		Decision variable			
Researches	Complete graph	Hub location	Terminal allocation	Multiple allocation	Hub	Arcs	Minimax	Minisum	Cost	Time	Fuzzy	Qualitative var.
Rodri´guez-Marti´n (2008)	-	√	√	-	√	√	√	-	√	-	-	-
Campbell *et al*. (2007)	-	-	√	√	√	√	√	-	-	√	-	-
Yaman *et al*. (2007)	√	√	√	-	√	√	√	-	-	√	-	-
Thomadsen and Larsen (2007)	√	√	√	-	-	-	-	√	√	-	-	-
Abdinnour-Helm (2000)	√	√	√	-	-	-	-	√	√	-	-	-
Camargo *et al*. (2008)	√	√	√	√	-	-	-	√	√	-	-	-
Graça Costa *et al*. (2008)	√	√	√	-	√	√	√	√	√	√	-	-
Ebery *et al*. (2000)	√	√	√	√	√	√	-	√	√	-	-	-
Ebery (2001)	√	-	√	-	-	√	-	√	√	-	-	-
This research	√	√	√	-	√	-	√	-	-	√	√	√

- All of the hub nodes are connected to one another
- Each non-hub node is connected to exactly a single hub.
- The set of candidate hub locations are known (set H).
- To travel between two non-hub nodes, one or two hubs have to be passed, i.e. direct connection between non hub nodes is not allowed.
- There is no fixed cost for establishing a hub.
- The capacities of the hub nodes are known.
- All decision variables of the model are binary variables (0-1).

First we should mention that our model yields the minimum latest delivery and if DMs are interested in minimizing the total cost simultaneously, there would be a conflict between service quality and total cost [23]. In fact there is a tradeoff between these two criteria so that solution that yields optimum in former, does not generally yields in latter. As reviewed in Table 1, the works of some authors like Campbell *et al* (2007) and Yaman *et al* (2007) fall in first group and those of Rodri´guez-Marti´n (2008) and Camargo *et al* (2008) are in second group.

This paper mainly deals with solving hub location problem with a hybrid approach of both qualitative and quantitative variables. In previous literatures quantitative aspects are well considered and many algorithms and solutions are presented. But there is a lack of a qualitative approach to deal with situations where DMs desire to consider such aspects. Cost and time are often accepted as key decision criteria for hub location ignoring qualitative criteria such as zone traffic, capability for future development and availability. These criteria are expressed in linguistic variables and here we aim to use an extended version of TOPSIS in fuzzy environment to select best hub locations among the available number of candidate locations then allocate non-hub nodes to hubs based on time criteria. This model is very suitable even if we have more than one DM and have to solve the problem using group decision-making under fuzzy environment. Then an IP formulation is used to allocate non-hub nodes to hub nodes with objective function of minimizing the maximum travel time between any O-D pair (section 2.2). The p-hub center allocation problem is addressed as a subproblem of the location problem, where number of hubs is known and hub locations are given. Campbell *et al* (2007) presented a general formulation for this problem. Sometimes in real-world problems location of hubs are not specific and should be determined. There is a lack of formulation in hub location allocation in previous literatures and we presented a new formulation for hub/D/SA, cap/•/minimax t in section 2.3. Ernest and Krishnamoorthy [8] considered median version of this problem and presented new formulation with fewer variables. A major drawback in formulations and solutions presented in previous papers on hub location is that with a minor increase in number of candidate hub location, set H, the time required to solve the problem increases so that it cannot be solved in polynomial time. Presented approach could als o be used in other class of hub locations like in work of Rodri´guez *et al*. [18] to reduce the solution space of candidate hub locations. Many heuristic [11, 18, 22] and methaheuristic [1, 12, 16, 13] solutions are published for different problems in hub location because of solution time constrains for NP-hard property of hub location problems. Kara and Tansel (2000) prove that the uncapacitated single allocation p-hub center location problem is NP-complete. So reducing the solution space using a systematic model which satisfies both quantitative and qualitative needs of DMs is useful to solve the problem in a reasonable time. Our MADM approach could be used to decrease

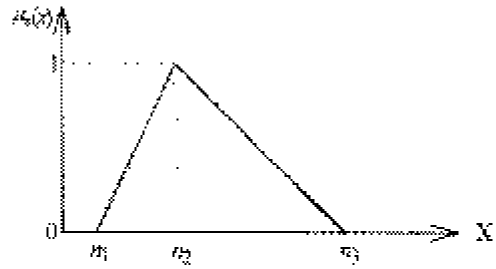

Fig. 1 : Triangular fuzzy number ñ

members of set H using fuzzy TOPSIS and then using an IP formulation to gain the final solution. The full procedure is described in section 2.

Due to inherent property of qualitative variables, the crisp value is inadequate to model real-life situations. In fuzzy TOPSIS, the rating of each alternative and the weight of each criterion are described by linguistic terms which can be expressed in triangular fuzzy numbers. A linguistic variable is a variable whose values are words or sentences in natural or artificial language (Zadeh, 1965). By using hedges like 'more', 'many', 'few', etc. and connectors like AND, OR and NOT with linguistic variables, an expert can form rules, which will govern the approximate reasoning.

Definition 1.1. A triangular fuzzy number ñ can be defined by a triplet (n_1, n_2, n_3) shown in Fig. 1. The membership function $\mu_{\tilde{n}}(x)$ is defined as:

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, x < n_1, \\ \dfrac{x-n_1}{n_2-n_1}, n_1 \le x \le n_2, \\ \dfrac{x-n_3}{n_2-n_3}, n_2 \le x \le n_3, \\ 0, x > n_3. \end{cases} \quad (1)$$

We use fuzzy triangular number for linguistic criteria in fuzzy TOPSIS proposed in this paper.

The design of hub networks usually consists of the selection of which nodes of the network will be selected to establish a hubs and how the other non-hubs will be allocated to the hubs. In next section two proposed models are presented that fuzzy TOPSIS is prerequisite for both of them and is described in section 2.1. Then first approach which includes two phase for hub location and allocation is explained in section 2.2. it is an IP formulation based on Campbell's (2007) revised model to allocate non-hub nodes to selected hub nodes in section 2.1. Second model that presents a hybrid approach to the problem is described in section 2.3. Section 3 provides a numerical example to illustrate the application of proposed model and comparison of two model with primitive capmbell's model.

PROPOSED MODELS

In these models DMs present a set of nodes that are candidate for establishing hub facilities and DM's information about the characteristics of those locations is not crisp. Travel time between nodes and capacity of hubs are given.

Figure 2 shows the overall procedure that is divided to four zones. ILP model zone is used in certain situations with crisp data. Fuzzy TOPSIS zone has several steps to rank and/or select the *p* best locations to establish hub facilities among candidate locations set *H*. Based on DMs approach to the problem, the function of this zone varies. If they use model I (described in section 2.2) fuzzy TOPSIS will rank all candidate hub location and select *p* best hubs according to qualitative and quantitative criteria they wish to apply (as shown in Fig. 2) and then we enter IP model zone to allocate non-hub nodes to *p* hubs using our proposed model based on Campbell's model [4] using available data and fuzzy TOPSIS results. This model is useful when qualitative variables are of high importance and we want to locate hubs on best-fitted nodes. This procedure is illustrated in Fig. 3(a) shows the initial solution space with five candidate hub locations denoted by squares *H1* through *H5* and some terminal nodes (demand or supply node) denoted by small circles, (b) shows the location phase in which the *p* best locations for hubs are selected using fuzzy TOPSIS and (c) is the final result and both location and allocation are completed and the network is established.

But if we prefer to involve maximum travel time on locating hub facilities and reinforce the model with qualitative specifications, we recommend selecting model II (described in section 2.3). If this model is used, fuzzy TOSIS will only rank all candidate hubs and give attribute CC_i to each of them. As illustrated in Fig. 2, in this model both location and allocation steps are simultaneously presented in this model.

Note that the network connecting hubs is a complete graph and each non-hub node is allocated to single hub. Hereby to transfer commodity from an origin to destination, two hubs should be passed utmost.

Fuzzy TOPSIS method: When we deal with qualitative variables in real life linguistic terms is best way to express data and we have to use fuzzy mathematics. Youssef *et al.* [8] used fuzzy logic to find optimum topology for switched network. In this section we will use a systematic approach to extend the TOPSIS to the fuzzy environment which C. T. Chen [5] has developed. In IP formulation of both models we needed a score for each candidate hub location based on qualitative criteria and TOPSIS is a popular method that

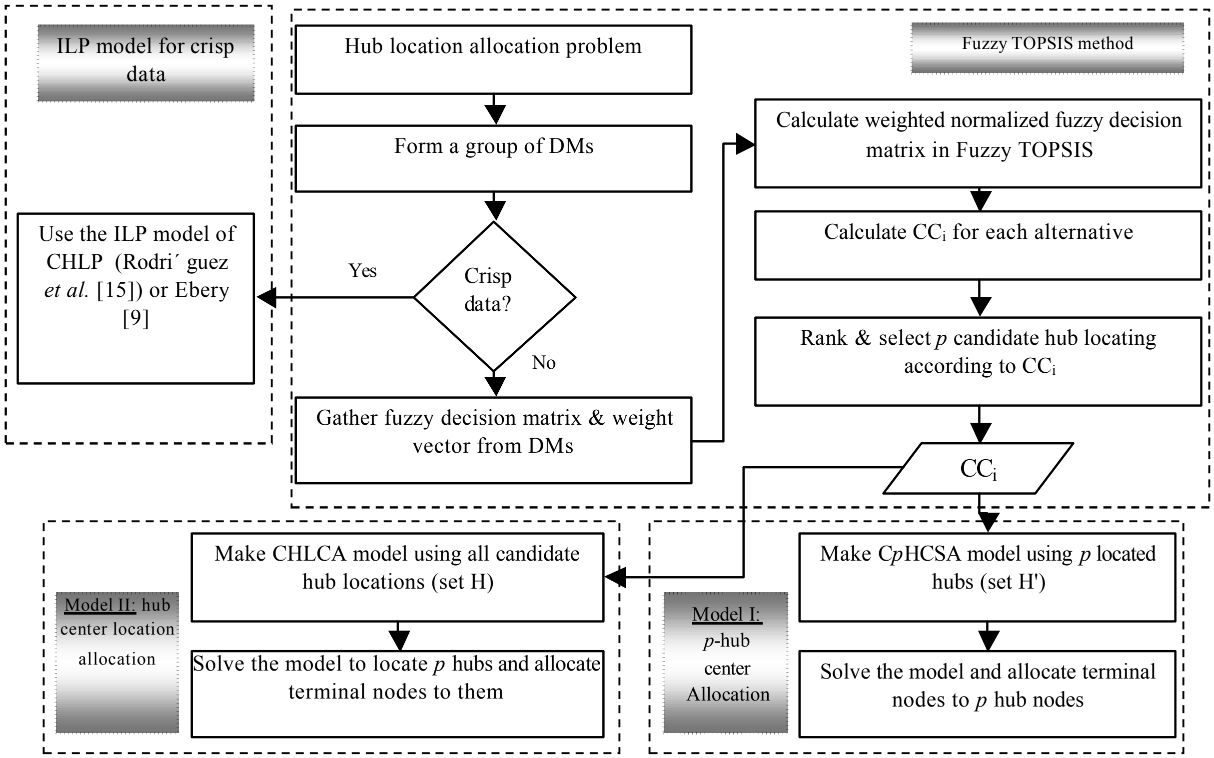

Fig. 2: The overall procedure of proposed model

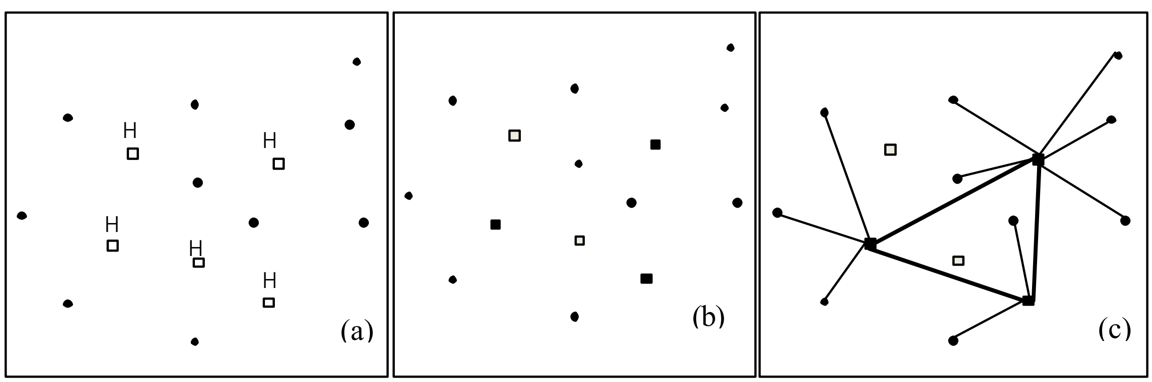

Fig. 3: Steps of hubs location and allocation of terminals. (a) Solution space, (b) hub location using fuzzy TOPSIS (c) allocation of terminals to located hubs using Campbell's revised model

provides this score based on positive-ideal and negative-ideal solution.

In this paper we assumes the DM to have already adopted the triangular fuzzy number to represent the fuzzy future market demands, zone traffic organizational agility, capacity for future development and etc. in hub location problem. In practice, the DMs are familiar to work with triangular distribution pattern and can easily estimate optimistic, pessimistic and most likely parameters. The pessimistic value that has a very low likelihood of belonging to the set of available values (membership degree = 0 if normalized); the most possible value that definitely belongs to the set of available values (membership degree = 1 if normalized); and the most optimistic value that has a very low likelihood of belonging to the set of available values (membership degree = 0 if normalized). Additionally, when there is a lack of knowledge of distribution, triangular distribution is appropriate for representing a fuzzy number (Rommelfanger, 1996). First we review some preliminary properties and delimitations that are used in this paper.

Property 1: If both $\tilde{m}$ and $\tilde{n}$ are real numbers; then the distance measurement d $(\tilde{m},\tilde{n})$ is identical to the Euclidean distance [5].

Property 3: Let $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ be three triangular fuzzy numbers. The fuzzy number $\tilde{B}$ is closer to fuzzy number $\tilde{A}$ than the other fuzzy number $\tilde{C}$ if and only if $d(\tilde{A},\tilde{B}) < d(\tilde{A},\tilde{C})$.

Definition 2.1.1: Let $\tilde{m}$ = (m1, m2, m3) and $\tilde{n}$ = (n1, n2, n3) be two triangular fuzzy numbers, then the vertex method is defined to calculate the distance between them as

$$d(\tilde{m},\tilde{n}) = \sqrt{\frac{1}{3}[(m_1-n_1)^2+(m_2-n_2)^2+(m_3-n_3)^2]} \quad (2)$$

Definition 2.1.2: Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers. The fuzzy number $\tilde{A}$ is closer to fuzzy number $\tilde{B}$ as $d(\tilde{A},\tilde{B})$ approaches 0.

Definition 2.1.3: Some basic operations between two fuzzy numbers are as

$$(\tilde{m}(+)\tilde{n})^\alpha = [m_l^\alpha + n_l^\alpha, m_u^\alpha + n_u^\alpha] \quad (3)$$

$$(\tilde{m}(-)\tilde{n})^\alpha = [m_l^\alpha - n_l^\alpha, m_u^\alpha - n_u^\alpha] \quad (4)$$

$$(\tilde{m}(.)\tilde{n})^\alpha = [m_l^\alpha . n_l^\alpha, m_u^\alpha . n_u^\alpha] \quad (5)$$

In this paper, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. These linguistic variables can be expressed in positive triangular fuzzy numbers as Table 1 and 2.

Assume that a decision group has *K* persons, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as [5]:

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1(+)\tilde{x}_{ij}^2 + \cdots + \tilde{x}_{ij}^K] \quad (6)$$

$$\tilde{w}_j = \frac{1}{K}[\tilde{w}_j^1(+)\tilde{w}_j^2 + \cdots + \tilde{w}_j^K] \quad (7)$$

Where $\tilde{x}_{ij}^k$ and $\tilde{w}_j^k$ are the rating and the importance weight of the *k*th decision maker. Another expression to calculate consensus weights is presented in [17]. According to the classification for group decision making (Rigopoulos *et al.*), this expression is aggregated individual solution that makes a model for entire team.

Table 1: Linguistic variables for the importance weight of each criterion

Very low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very high (VH)	(0.9, 1.0, 1.0)

Table 2: Linguistic variables for the ratings

Very poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

Step 0: Form a group of stakeholder decision makers.

Step 1: Get the fuzzy multicriteria group decision-making information for hub location problem in matrix format for decision matrix and weight vector as

$$\widetilde{DM} = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix} \quad (8)$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n] \quad (9)$$

Where $\tilde{x}_{ij}, \forall I,j$ and $\tilde{w}_j, j = 1,2,\ldots,n$ are the linguistic variables. These linguistic variables can be described by triangular fuzzy numbers, $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$.

Step 2: Normalize the *DM* matrix using linear scale transformation as follows

$$\tilde{R} = [\tilde{r}_{ij}]_{m.n} \quad (10)$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), j \in B;$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), j \in C;$$

$$c_j^* = \max_i c_{ij}, if\ j \in B;$$

$$a_j^- = \min_i a_{ij}, if\ j \in C.$$

where $\tilde{R}$ is normalized fuzzy decision matrix and B and C are the set of benefit criteria and cost criteria, respectively

Step 3: Construct the weighted normalized fuzzy decision matrix $\tilde{V}$ as:

$$\tilde{V} = [\tilde{v}_{ij}]_{mn}, i = 1,2,\dots,m, j = 1,2,\dots,n \tag{11}$$

Where $\tilde{v}_{ij} = \tilde{r}_{ij}(.)\tilde{w}_j$ and we know that all elements $\tilde{v}_{ij}, \forall i,j$ are positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1].

Step 4: Define the fuzzy positive-ideal solution (FPIS, A^*) and fuzzy negative-ideal solution (FNIS, A^-) as

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-)$$

Where $\tilde{v}_j^* = (1,1,1)$ and $\tilde{v}_j^- = (0,0,0)$ $j=1,2,\dots n$.

Step 5: Calculate the distance of each alternative from A^* and A^- as

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), i = 1,2,\dots,m \tag{12}$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1,2,\dots,m \tag{13}$$

Where d (-,-) is the distance measurement between two fuzzy numbers.

Step 6: Calculate closeness coefficient for each alternative A_i (i=1, 2,…, m) as

$$CC_i - \frac{d_i}{d_i^* + d_i^-}, i - 1,2,\dots,m \tag{14}$$

Step 7: Determine the ranking order of all alternatives and select the best p locations from among a set of feasible alternatives, obviously an alternative A_i is closer to the FPIS (A^*) and farther from FNIS(A^-) as CC_i approaches to 1.

Step 8: Allocate non-hub nodes to selected hub nodes using model CpHCSA algorithm presented in section 2.2.

Model I: C*p*HCSA: We consider capacity restriction only on the volume of traffic entering a hub via collection. The notations used in the model are as follows:

x_{ik}	A binary variable that equals 1 if node i is allocated to hub k and 0 otherwise.
t_{ik}	Transfer time for commodity to travel from node i to hub k
t_{km}	Transfer time for commodity to travel from hub k to hub m
a	Discount factor for trips between two hub nodes
H'	The set of hubs which are selected as hub locations from fuzzy TOSIS
N	The set of terminal nodes
O_i	Total commodity to be transferred from node i
C_k	Capacity restriction on hub k
y_k	A binary variable that equals 1 if node k is an opened hub and 0 otherwise.

The formulation for the Capacitated p-Hub Center Single Allocation (CpHCSA) proposed in this section is Campbell's revised model.

$$Min \max_{i,j\in N; k,m\in H'} X_{ik}X_{jm}\left(\frac{1}{CC_k}t_{ik} + \alpha t_{km} + \frac{1}{CC_m}t_{jm}\right) \tag{15}$$

s.t.:

$$\sum_{k\in H'} x_{ik} = 1, \qquad \forall i \in N \tag{16}$$

$$X_{kk} = 1, \qquad k \subset H' \tag{17}$$

$$\sum_{i\in N} O_i X_{ik} < C_k, \qquad \forall k \in H' \tag{18}$$

$$X_{ik} \subset \{0,1\}, \qquad \forall i \subset N, k \subset H' \tag{19}$$

where CC_i is the closeness criterion factor obtained from fuzzy TOPSIS algorithm. Adding this factor compensates inordinate travel time from non-hub nods to a qualitatively good hub. The objective of the formulation is quadratic. Only if X_{ik}= 1 and X_{jm}= 1 (i.e., i is assigned to hub k and j is assigned to hub m), the travel time on the path i-k-m-j equals $t_{ik}+ at_{km}+ t_{jm}$ and equals 0 otherwise. The objective minimizes the maximum travel time between any O-D pair. Constraints (17) and (19) ensure that every node is assigned to exactly one hub. Constrain (17) is technically redundant for solving procedure and means putting the hubs in their locations predefined with fuzzy TOPSIS so that if there is a hub at node k, then $X_{kk} = 1$. Constrain (18) sets the total flow into hub k via collection less than maximum capacity.

Model II: CHLCA: In this model deciding on location of hubs is left to Capacitated Hub Location Center Allocation (CHLCA) formulation which is a modified version of C*p*HCSA. The intense of qualitative variables is detracted and CC_i amplifies probability of selecting good-quality nodes as hub. We replace constraint (17) with following constraint:

$$\sum_{k \in H} x_{kk} = p \qquad (20)$$

which means exactly *p* hubs should be selected. We can show the constraint (20) as:

$$\sum_{i \in N} x_{ik} \geq y_k,\ k \in H \qquad (21)$$

$$y_k \leq X_{ik},\ \forall i \in N, k \in H \qquad (22)$$

$$\sum_{k \in H} y_k = p \qquad (20)$$

These constraints assure that number of opened hubs are exactly *p* and non-hub node will be allocated to them. The modified formulation is:

$$\text{Min} \max_{i,j \in N;\, k,m \in H} X_{ik} X_{jm} \left(\frac{1}{CC_k} t_{ik} + a t_{km} + \frac{1}{CC_m} t_{jm} \right)$$

s. t.:

$$\sum_{k \in H} x_{ik} = 1, \quad \forall i \in N$$

$$\sum_{i \in N} x_{ik} \geq y_k,\ k \in H$$

$$y_k \leq X_{ik},\ \forall i \in N, k \in H$$

$$\sum_{k \in H} y_k = p$$

$$\sum_{i \in N} O_i X_{ik} \leq C_k \quad \forall k \in H$$

$$X_{ik} \in \{0,1\}, \quad \forall i \in N, k \in H$$

Switching from set H' to H, the problem size increases and time required to solve the problems will increases incrementally. Model I, as mentioned in introduction, can decrease solving time with cutting out undesirable hub nodes.

NUMERICAL EXAMPLE

In this section a numerical example is presented to illustrate the procedure and application of the proposed models in real problems. This problem can arise in cargo delivery system, airline transportation, postal service, supply chain distribution and etc.

Table 3: The importance weight of criteria

	D1	D2
C1	H	M
C2	VH	H
C3	L	ML
C4	ML	H

Table 4: The ratings of the five candidates by two decision makers under all criteria

Criteria	Candidate hub location	Decision-makers D1	D2
C1	H1	P	F
	H2	MG	G
	H3	VG	MG
	H4	MP	MG
	H5	G	G
C2	H1	MP	F
	H2	F	MG
	H3	VG	G
	H4	F	MG
	H5	VG	P
C3	H1	P	MP
	H2	F	MG
	H3	MG	G
	H4	VG	G
	H5	MP	VG
C4	H1	F	MP
	H2	VP	P
	H3	P	G
	H4	MG	F
	H5	F	MP

Suppose [|*N*|, |*H*|, |*D*|] is Number of source and/or destination nodes, hub nodes and DMs in group respectively. We have a hub location problem [3, 5, 2] (Fig. 4). There are 4 main qualitative criteria that DMs want to consider including future market demand, availability, zone traffic and capacity for future development. Note that the third criterion is a cost criterion i.e. the less zone traffic the better location to establish hub. We track the forgoing procedure to solve the example and locate two hubs out of 5 candidate locations and allocate 3 terminal nodes to them.

Step 0: We form a group consists of two DMs.

Step 1: DMs present the information about hub locations for each criterion in Table 3 and rating of all alternative hub locations in Table 4.

Table 5: The fuzzy decision matrix and fuzzy weights of five alternatives

	C1	C2	C3*	C4
H1	(1.5, 3, 5)	(2, 4, 6)	(0.5, 2, 4)	(2, 4, 6)
H2	(6, 8, 9.5)	(4, 6, 8)	(4, 6, 8)	(0, 0.5, 2)
H3	(7, 8.5, 9.5)	(8, 9.5, 10)	(6, 8, 9.5)	(3.5, 5, 6.5)
H4	(3, 5, 7)	(4, 6, 8)	(8, 9.5, 10)	(4, 6, 8)
H5	(7, 9, 10)	(4.5, 5.5, 6,5)	(5, 6.5, 7.5)	(2, 4, 6)
Weight	(0.5, 0.7, 0.85)	(0.8, 0.95, 1)	(0.05, 0.2, 0.35)	(0.85, 0.6, 0.75)

*Cost Criterion

Table 6: The fuzzy normalized decision matrix

	C1	C2	C3	C4
H1	(0.15, 0.3, 0.5)	(0.2, 0.4, 0.6)	(0.013, 0.025, 0.1)	(0.25, 0.5, 0.75)
H2	(0.6, 0.8, 0.95)	(0.4, 0.6, 0.8)	(0.006, 0.008, 0.013)	(0, 0.06, 0.25)
H3	(0.7, 0.85, 0.95)	(0.8, 0.95, 10)	(0.005, 0.006, 0.008)	(0.44, 0.63, 0.81)
H4	(0.3, 0.5, 0.7)	(0.4, 0.6, 0.8)	(0.005, 0.005, 0.006)	(0.5, 0.75, 1)
H5	(0.7, 0.9, 1)	(4.5, 5.5, 6,5)	(0.007, 0.008, 0.01)	(0.25, 0.5, 0.75)

Table 7: The weighted normalized fuzzy decision matrix

	C1	C2	C3	C4
H1	(0.075, 0.021, 0.425)	(0.17, 0.38, 0.6)	(0.001, 0.005, 0.035)	(0.213, 0.5, 0.75)
H2	(0.3, 0.56, 0.808)	(0.32, 0.57, 0.8)	(0.000, 0.002, 0.005)	(0, 0.06, 0.25)
H3	(0.35, 0.595, 0.808)	(0.48, 0.903, 1)	(0.000, 0.001, 0.003)	(0.374, 0.63, 0.81)
H4	(0.15, 0.35, 0.595)	(0.32, 0.57, 0.8)	(0.000, 0.001, 0.002)	(0.425, 0.75, 1)
H5	(0.35, 0.63, 0.85)	(0.36, 0.523, 0.65)	(0.001, 0.002, 0.006)	(0.625, 0.5, 0.75)

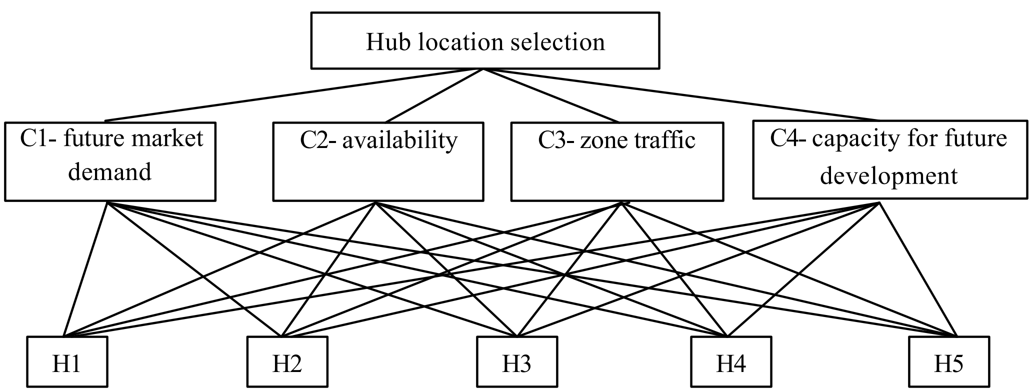

Fig. 4: Example [3, 5, 2] (The hierarchical structure)

Calculate $\tilde{D}\tilde{M}$ and $\tilde{W}$ matrices using equations (6) and (7) as Table 5.

Step 2: Normalized $\tilde{D}\tilde{M}$ matrix $\tilde{R}$ is calculated in Table 6.

Step 3: Calculate weighted normalized fuzzy decision matrix $\tilde{V}$ as Table 7.

Step 4: Determine FPIS and FNIS as:

$$A^* = [(1, 1, 1); (1, 1, 1); (1, 1, 1); (1, 1, 1)];$$
$$A^- = [(0, 0, 0); (0, 0, 0); (0, 0, 0); (0, 0, 0)].$$

Step 5: Calculate the distance of each alternative from A^* and A^- as Table 8.

Step 6: Calculate closeness coefficient for each alternative A_i (i=1, 2, 3, 4, 5) as Table 9.

Step 7: Now we can determine the ranking order of all alternatives as

$$H_3\ (CC_i = 0.3577),\ H_5\ (CC_i = 0.3128),\ H_4\ (0.265186),$$
$$H_2\ (CC_i = 0.254697),\ H_1\ (CC_i = 0.157485).$$

Now if DMs are firm on qualitative variables and use model I, algorithm proceeds to step 8 and if they

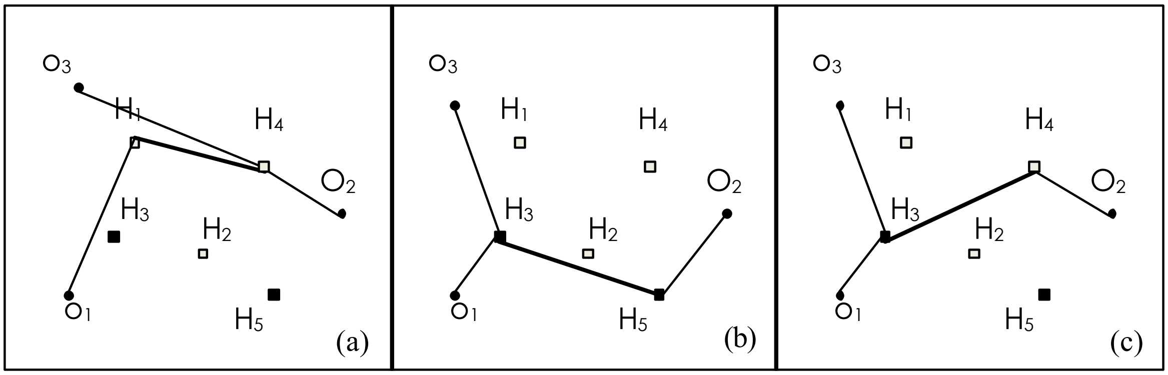

Fig. 5: Results for numerical example (a) Cambell's model (b) CpHCSA (c) CHLCA

Table 8: The distance measurement

	A^*	A^-
H1	4.359342	0.814856
H2	3.684605	1.259174
H3	2.976003	1.657123
H4	3.466393	1.25098
H5	3.367028	1.532566

Table 9: The Closeness Criterion of each location

	CC
H1	0.157485
H2	0.254697
H3	0.357668
H4	0.265186
H5	0.312794

Table 10: Model parameters

Transfer time (non-hub to hub)		Transfer time (hub to hub)	O_i		C_k	
t_{13}	15	15	O_1	3	C_1	5
t_{23}	40		O_2	8	C_2	10
t_{33}	10		O_3	4	C_3	11
t_{15}	35				C_4	12
t_{25}	20				C_5	15
t_{35}	25					

Table 11: Comparison between results

Model	Solution	o.f.	A.D.T
Campbell	$x_{11} = x_{24} = x_{34} = 1$	175	31.8
CpHCSA	$x_{13} = x_{33} = x_{25} = 1$	128	47
CHLCA	$x_{13} = x_{24} = x_{33} = 1$	105	37

decided to reinforce location step by quality of hubs and use model II we should skip to step 10.

Step 8: Selecting H_3 and H_5 with highest CC, we have 2 hubs to allocate terminal nodes to them in next steps.

Step 9: Use CpHCSA model to allocate 5 non-hub nodes to 2 hub nodes. Table 10 shows given model parameters which are obtained from a universal table including information about all candidate hub locations and non-hub nodes. Transfer time and capacities are given based on the nodes and their location and total flow from each terminal node is also known. With a = 0.8 the results are $x_{13} = x_{33} = x_{25} = 1$ with optimum objective function 128 which means non-hub nodes 1 and 3 are allocated to hub H_3 and no-hub node 2 is allocated to hub H_5 and latest delivery time is 47 between node 1 and 2 (Fig. 5).

Step 10: Use CHLCA model to locate 2 hubs and allocate non-hub nodes to them. Here transfer time between terminals and hubs may affect the solution and compensate low CCs and yields better travel time. The results are x_{13}=x_{25}=x_{32}=1 with optimum objective function 80 which is remarkably less then Model I and latest delivery time is 77. Figure 5 displays solution of each model.

Table 11 compares results from two models with Campbell's model (i.e. CC_i=1). Column *o.f.* is objective function of model I that calculated with each solution and assured delivery time is in *A.D.T* column. Results shows minimum time often increases by considering qualitative variables but this is not always true. But we always get less o.f respectively in model I and II than primitive Campbell's model.

CONCLUSIONS

In this paper, we have used a novel approach to consider hub location problem from both qualitative and quantitative point of view so that DMs are able to apply parameters like zone traffic, availability, capability for future development and etc to location problem. It is supposed to deal with fuzzy data to help the model to be very applicable due to lack of certainty and crisp data in real word situations especially about qualitative variables. We used fuzzy TOPSIS to deal with linguistic variables and presented two models to locate hub facilities. The p-hub center allocation problem is a subproblem of the p-hub center location problem to allocate supply and demand nodes to established hubs and it is of considerable interest on its own for time-sensitive delivery and transportation systems. These models efficiently help DMs to locate hub facilities with a strategic view for future developments and more aspect other than time and cost. As presented in Table 11 by moving from Campbell's model to hybrid models, it is possible to observe accretion in minimum time but *o.f.* reduces and hubs with good quality are selected. This approach can be applied on models with cost objective function like Rodriguez (2008) model.

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