

Mathematical Processes: A Viewpoint-oriented Manipulation Perspective

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Abstract: View-point oriented manipulation of concepts can be helpful for generating new ideas in basic sciences and in the meantime, justifying the processes that are principally meaningful to the related disciplines. Mathematics, as a major ground for basic sciences, seems to be an appropriate exemplar to show how such a claim can be valid. The processes that are particularly attractive in case of mathematics are the projections from one space onto another, the transformations from one formalism into another and finally the critiques that are made to show how far two formalisms can be mutually consistent. In this paper, having reviewed the principles of view-point oriented manipulation, mathematical processes will then be discussed in this regard and an example will be presented showing the way it works for transforming a formalism into a new one.

Key words: Viewpoint-oriented manipulation . Mathematical process . Projection . Transformation . Formalism

INTRODUCTION

There are three distinct schools of thought in mathematical philosophy, belonging to Hilbert, Brouwer and Russell. Hilbert, as a major proponent of formalism believes that mathematics is nothing more than meaningless symbols and in this sense it can be considered as the investigation of formal axioms systems [1], while, Brouwer as the founder of movement in intuitionism held that there are no non-experienced mathematical truths and mathematical objects in this sense arise from the a-priori forms of the volitions that inform the perception of empirical objects [2]. In the meantime, Russell as the founder of mathematical logicism quoted that mathematics is reducible to logic and hence nothing but a part of logic [3]. In all these schools, a major concern seems to be the inference mechanisms which are essential to check the status of truth, validity or consistency for the intermediate facts/ propositions as the main mathematical arguments. Although validity or truth is the ultimate goal to be pursued in mathematics, there are areas of concern, for which viability of a postulate makes definitely more sense. For example, the way a new idea is formed or a projection/ transformation is achieved, is something whose viability is of more concern compared to its truth or falseness. What we intend to elaborate in this paper, is the point that these projections/ transformations and besides that, the

mental processes that lead to new mathematical ideas/ concepts, can in reality be justified in terms of a process of view-point oriented manipulation according to which a target concept is modified into a new structure based on a view-point.

A BRIEF REVIEW ON VIEWPOINT-ORIENTED MANIPULATION PROCESS

By manipulating a concept/ issue from the view-point of another concept/issue we mean to see how far a concept/issue, is capable of pursuing a certain purpose within the realm of another concept/issue. Some of these purposes are improving/ enhancing the capability of a method, justifying the role/ utility of an entity regarding another entity and anticipating the impact/ prospect of an issue or a theory regarding another issue or theory. Taking this point into account, a systematized medium is to be designed wherein the interaction essential to this manipulation can come about in a plausible manner. This may be realized through an inter-play between the frames of issues in which their predicates and arguments become subject to matching. The output of viewpoint oriented manipulation would therefore highly depend on the structure of systematicness in the way this matching is to be performed. To represent the concept of an issue, a frame structure [4] including a variety of attributes can be used [5-7]. For instance, Major Characteristics, Basic Constituents,

Advantage and Disadvantage can respectively show the impacts of using an issue, the way an issue can be achieved, the context within which an issue can be used successfully and finally the context within which an issue can not be used successfully. Obviously, the value of each of these attributes can be described in terms of a number of predicates whose arguments can be regarded as the prime items with respect to the related issue. By a prime item, we mean an item which deserves to become focus of attention, once view-point oriented manipulation of the target is considered. Having represented the target and the view-point in terms of a number of predicates consisting of non-action type entities as their arguments, the first attempt would be to see as hypotheses, which prime items, respectively in the target and the view-point, can be regarded as corresponding to each other. Here, prime items, not only include the arguments of the predicates describing the value of an attribute, but also include the functions, tools, or means that are essential to performing these predicates. To rank the hypotheses, semantic similarity between the alternative prime items in the target and those in the view-point are taken into account [8, 9]. To validate the hypotheses, predicates should be found for the arguments in the target that can match those discussed for the corresponding arguments in the view-point. To approach this, the arguments in the target themselves should be regarded as new frames consisting of the predicates applicable to them. The predicates obtained through building these frames are then cross-checked with those already existing in the view-point, to see: (i) whether or not they can be mutually consistent, or if not (ii) which predicates would in reality contradict each other. Taking this point into account, the process of view-point oriented manipulation is regarded to be hierarchical in nature and should therefore continue until the stage where appropriate results can be obtained regarding the status of consistency between the predicates in the view-point and those in the target.

MATHEMATICAL PROCESSES IN TERMS OF VIEW-POINT ORIENTED MANIPULATION

A variety of questions regarding mathematics are concerned with the way entities like space,

form/shape/topology or function/formalism are defined or may become subject to restructuring. Based on the way an entity is manipulated, the role of mathematical knowledge to fulfill it would be different (Fig. 1). As it is seen from the figure, both pure mathematicians and philosophers can take the responsibility of defining new spaces, while restructuring a formalism can be done by an applied mathematician or an engineer as well. It is seen that, for all these processes a kind of view-point oriented manipulation can exist, according to which a target frame is modified based upon the influence of a viewing frame which somehow reflects the mental requirements/ expectations of the specialist. For instance, when a new space is defined by a mathematician (in his/her pure mode) or a philosopher, the influence of these mental requirements/ expectations on a previous structure preferably with abstract characteristics, would yield a new structure standing for the defined space. In the same way, under the influence of an applied mathematician's (or engineers) requirements, a previous formalism can be modified into a new version that can be capable of fulfilling these requirements.

AN EXAMPLE

To show how view-point manipulation works, let consider an example where a function ($y = (x^2 - 1)/2$) is to be modified under the influence of a view-point, which in this case is

$$\text{Matrix} \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}.$$

As it is known from basic linear algebra, result of modification would be " $X + 3Y = Y^2 - 1$ ". As it is seen, our view-point consists of two partial viewpoints, each to be applied separately to the target vector that stands for the target function. Each partial view-point can in turn be regarded in terms of a number of partial view-points each being applied to an independent component of the target vector. Once it was seen that a partial view-point could not be decomposed into further components, local results of applying them to the components in the target vector would be integrated (or fused) in some way to produce the final components in

Layer	Level	
	Definition	Restructuring
Space	Pure mathematician, philosopher	Pure mathematician, sociologist, psychologist
Topology	Pure mathematician, artist	Applied mathematician, designer
Formalism	Applied mathematician, computerist	Applied mathematician, engineer

Fig. 1: Inter-relations between layers and levels

View-point Target Modified Target

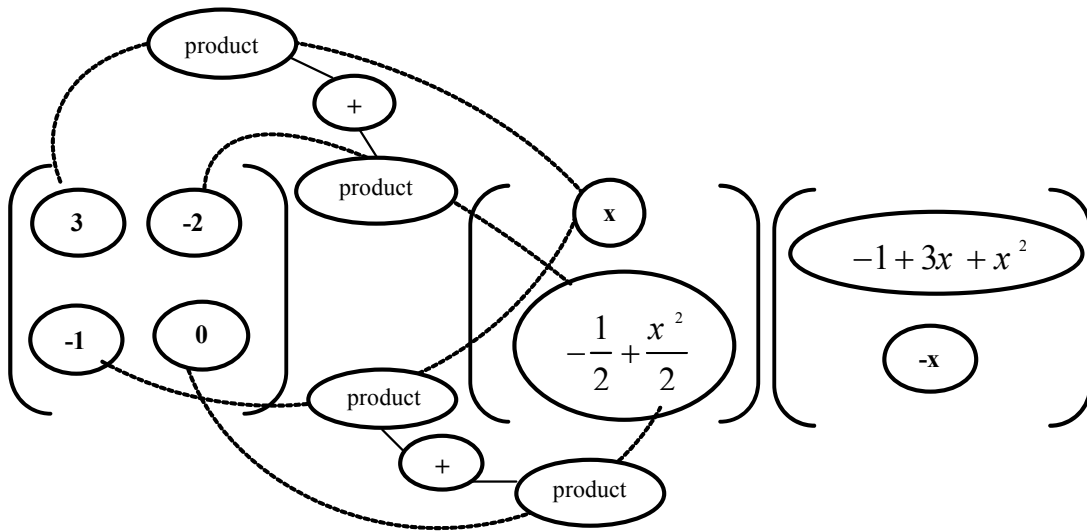


Fig. 2: The process of applying the viewpoint to the target

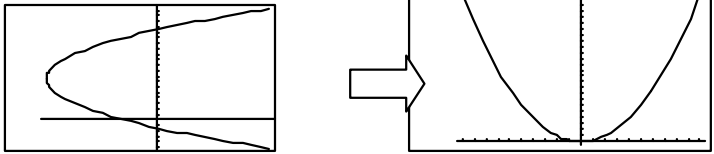
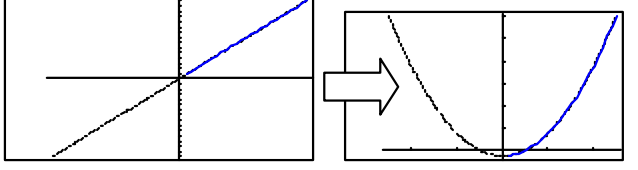
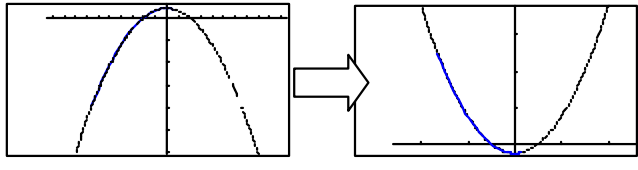
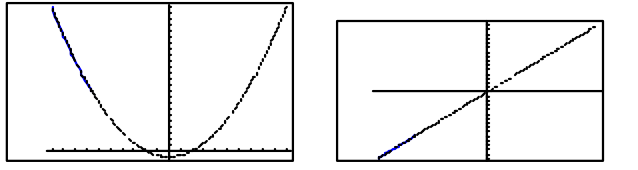
Type of integration operator	Status of modification for the target
Sum	
Max	$x \geq 0.3$ max{ 
	max{ $-3.3 \leq x \leq 0.3$ } 
	max{ $x \leq -3.3$ } 

Fig. 3: Role of integration operator in transforming the target

the modified target vector. As it is seen, a simple summation is used as an operator in linear algebra to perform this integration.

It should however be noted that, based upon the type of non-linearity considered for the modification, different operators such as Min, Max,... can be equally taken into consideration. This would allow us to rely on view-point manipulation as a method for applying systematically the specific constraints that may exist regarding the view-point. Let say, considering for instance Min or Max instead of Sum as the operator for integrating the local results (of applying partial viewpoints), would enable us to produce a wide range of variations for "change in position", "change in angle", "change in curvature", "change in scale",... with respect to the original target. In this manner, the operators used for integration or other types of constraints, themselves can be viewed as part of the view-point itself which would later participate in the process of target modification. This will yield a promising medium for explaining the possible mathematical processes in a unified way (Fig. 2 and 3).

CONCLUDING REMARKS

It was discussed in the paper how view-point oriented manipulation of concepts can justify the mathematical processes such as transforming a formalism into a new one. It was seen that the operator used for integration or any other type of constraint necessary for transformation, can be viewed as a part of the view-point itself, which would later may participate in the process of

target modification. This would enable us to produce a wide range of variations for qualities such as "change in position", "change in angle", "change in curvature", "change in scale", etc.. Taking this point into account, view-point oriented manipulation of mathematical objects can act as an effective means for qualitative simulation of the mathematical processes and can therefore achieve a promising role in both mathematics education and research as well.

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