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New Method for Decreasing the Number of Quantum Dot Cells in QCA Circuits

[1]*Razieh Farazkish,* [2] *Mostafa Rahimi Azghadi,* [2]*Keivan Navi and* [1]*Majid Haghparast*

[1]Department of Computer Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
[2]Faculty of Electrical and Computer Engineering, Shahid Beheshti University, Tehran, Iran

Abstract: A method for decreasing the number of Quantum Dot Cells in Quantum Dot Cellular Automata (QCA) circuits is presented. The proposed method is based on physical relation and computing physical forces between electrons. Correctness of our method is proved using some simple physical proofs. Our method is useful when the QCA circuit has many inverter gates. It should be noted that in order to achieve more stability, electrons of QCA arrange in such a manner that their potential energy reaches the minimum level. From physical terms it could be simply proven that calculation of resultant of forces and moment of forces and also calculation of potential energy will have equal result. Therefore one can claim that the final inverter has the minimum potential energy and therefore is in its most stable state.

Key words: Quantum cellular automata . Nanoelectronic circuits . QCA . QCA inverter . Logical Simplification . Electron . Resultant of forces . Moment of forces

INTRODUCTION

Quantum-dot Cellular Automata (QCA) has revolutionized nano-level computing technologies [1]. Logical states of zero or one can be represented by two stable configuration of a pair of electrons, which can occupy four dots diagonally. Circuits implemented using such devices does not require traditional interconnections and have extremely low power dissipation. The QCA cells, as well as circuits utilizing these cells, have been fully fabricated and tested by many researchers [2-4]. Moreover, extensive research has been performed towards implementation of QCA's based on molecular structures, which can operate at room temperature [5-7].

The basic boolean primitive in QCA is the majority gate. Hence, construction of efficient QCA circuits using the majority gates has attracted a lot of many [8-11]. M. Rahimi *et al*. [10] has introduced a novel method for the construction of a simplified QCA circuits based on five-pin majority gates. Besides, another important component in constructing QCA circuits is inverter, because any QCA circuit can be efficiently built using only majority gates and inverters. Hence efficient constructing inverter in QCA is of great interest.

In this paper, we present a new method for decreasing the number of QCA cells in a QCA inverter gate. The presented method is based on physical relations and our proofs have been prepared in the paper. The reduction in cell counts of a QCA inverter result in simpler construction of QCA circuits and decreases the QCA circuit complexity.

MATERIALS AND METHODS

Background: A QCA cell in Fig. 1 (a), four quantum dots positioned at the corner of a square and two electrons that can move to any quantum dot within the cell through electron tunneling [12] is illustrated. Due to Coulombic interactions, only two configurations of the electron pair exist those are stable. Assigning a polarization P of-1 and +1 to distinguish between these two configurations leads to a binary logic system.

Any QCA circuit can efficiently be built using the only majority gates and inverters. As shown in Fig. 1 (b), the majority function implemented using an ordinary QCA gate is as follows:

Assume the inputs are A, B and C, then the logic function of a majority gate is given as:

$$M(A, B, C) = AB+BC+AC \quad (1)$$

Each QCA majority gate as illustrated in Fig. 1 (b) requires only five QCA cells and also every QCA inverter gate can be implemented by 11 quantum cells (Fig. 1 (c)).

As we mentioned before, the majority gate constitutes complete gate beside an inverter. Therefore by reducing the number of inverter cells, particularly in

Corresponding Author: Majid haghparast, Department of Computer Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

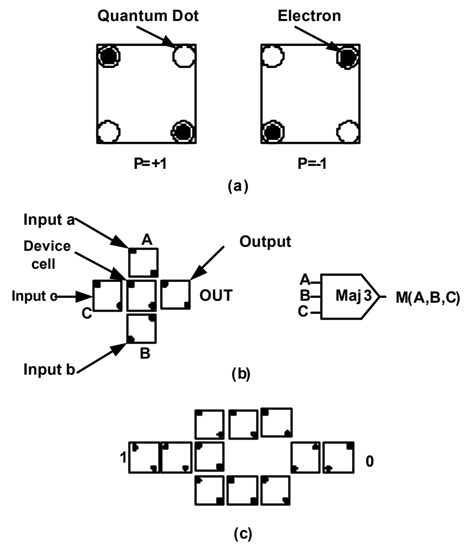

Fig. 1: (a) Basic QCA cell and binary encoding, (b) A QCA majority gate (c) A QCA inverter

big and complicated QCA circuits, the volume of circuit decreases drastically and we will need less hardware, which in addition to economization of costs, leads to less complexity.

New Method plus calculations for decreasing the number of Inverter Logic Gate Cells: For basic calculation, at first we assume a simpler model of standard inverter (Fig. 2) and then generalize it. For this purpose, at first we calculate the forces bring by all the electrons available in cell (2). As we know, the electrons of cell (2) can be only located at diameter position [12]. Consequently we calculate the resultant of the forces and moment of forces in two situations of (1) and (2) which have been shown in Fig. 2.

In all figures, rectangle shows a QCA cell and the circles inside show the electrons inside that cell. These circles contain numbers that show the electron number and are used in calculation of the force on the electrons of cell 2. Arrays also show the forces on the electrons of cell 2.

- In all of the calculations the following postulates are assumed:
- The forces bring from cell (1) are deniable against other forces.
- The force which is bringing by the electrons of cell (2) can be deniable.
- Electrons are set at the corner points of quadrangular.

All the cells are similar with the length of a (a = 18 nm) which are set close to each other without any space.

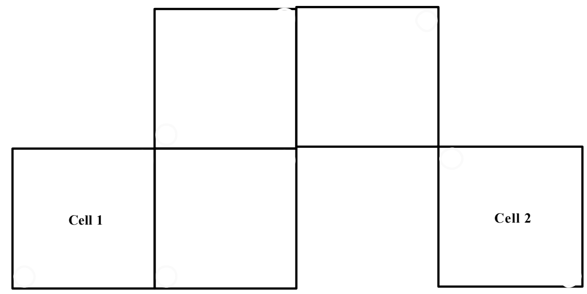

Fig. 2: Simple model of a QCA inverter

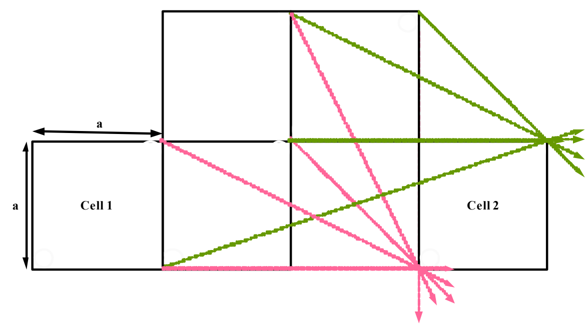

Fig. 2(a): Forces on the electrons of cell 2 in state1

It is worth mentioning that the angle between vectors has been calculated by Pythagoras' theorem in right angle trapezoids [17, 18].

The force that two electric charges put on each other is calculated from relation (2). In this relation, F is force, k is fixed colon, q_1 is the first electric charge, q_2 the 2[nd] electric charge and r is the distance of two electric charges from each other. By putting the amounts of k, q and a we obtain relation (3). F_T is the resultant of forces that are calculated from relation (4) in which in addition to forces; we need the angle between two vectors. Moment of forces is obtained from relation (5). In this relation, the relationship is between F (force), R (moment arm) and φ (the angle between force vector and moment arm). Moment of forces can be internal or external according to right hand law. In this paper for simplicity purposes, we have considered internal moment negative and external moment positive [14-16].

$$F=\frac{kq_1q_2}{r^2}=\frac{kq^2}{r^2}\left(q_1=q_2=1.6*10^{-19}c\right),\left(k=9*10^9\right) \tag{2}$$

$$\begin{aligned}\frac{kq^2}{a^2}&=\frac{9*10^9*(1.6)^2*10^{-38}}{324*10^{-18}}\\&=\frac{9*(1.6)^2*10^{-11}}{324}=7.11*10^{-13}=A=cte\end{aligned} \tag{3}$$

$$F_T=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\phi} \tag{4}$$

$$\tau = F * R * \cos\varphi \quad R = \frac{\sqrt{2}}{2}a = \frac{\sqrt{2}}{2} * 18 = 9\sqrt{2} = 12.73(\mathrm{nm}) \tag{5}$$

In all figures, F_i is the force entered by electron i to the favorite electron in cell 2 and r_i is the distance of these two electrons which is calculated according to relation (2). F_i' is the total of F_i and F_j forces. F_T is the total resultant of forces entered to the considered electron in cell 2. φ_T is the angle that the total of forces put with the positive direction of x axis and τ is the moment of forces that are calculated from relation (5).

Method of proving: First the force entering by all electrons on e_1 and e_2 in cell 2 are obtained from relation (2). Then the resultant of forces are calculated two by two from relation (4) and finally the total resultant of forces and its angel is obtained with positive direction of x axis. After that the moment of forces is obtained according to right hand law in relation (5). Comparison between the moment of forces in state 1 and 2 in each figure shows that in which state electrons are more stable.

Whereas the method of proving is similar in all figures, therefore we do the proving for Fig. 2 completely and in the rest of figures, only write the final result.

$$F_1 = F_2 = \frac{kq^2}{r_1^2} = \frac{kq^2}{\left(\sqrt{2}a\right)^2} = \frac{kq^2}{2a^2} = \frac{1}{2}A$$

$$F_3 = \frac{kq^2}{r_2^2} = \frac{kq^2}{(2a)^2} = \frac{kq^2}{4a^2} = \frac{1}{4}A$$

$$F_4 = \frac{kq^2}{r_3^2} = \frac{kq^2}{\left(\sqrt{5}a\right)^2} = \frac{kq^2}{5a^2} = \frac{1}{5}A$$

$$F_5 = \frac{kq^2}{r_4^2} = \frac{kq^2}{\left(\sqrt{5}a\right)^2} = \frac{kq^2}{5a^2} = \frac{1}{5}A$$

$$F_6 = \frac{kq^2}{r_5^2} = \frac{kq^2}{(2a)^2} = \frac{kq^2}{4a^2} = \frac{1}{4}A$$

$$F_1' = \sqrt{(0.2A)^2 + (0.2A)^2 + 2(0.2A)(0.2A)\cos(39.43°)} = \left[\sqrt{0.14}\right]A \cong 0.37A$$

$$F_2' = \sqrt{(0.25A)^2 + (0.25A)^2 + 2(0.25A)(0.25A)\cos(45°)} = \left[\sqrt{0.3}\right]A \cong 0.54A$$

$$F_3' = A + 0.37A + 0.54A = 1.91A$$

$$F_T = 1.91A = 13.58*10^{-13}(\mathrm{N})$$

$$\phi_T = -45°$$

$$\tau_1 = 12.73 * 13.58 * 10^{-13} * \cos(90°) = 0$$

Fig. 2 State1 (e_2):

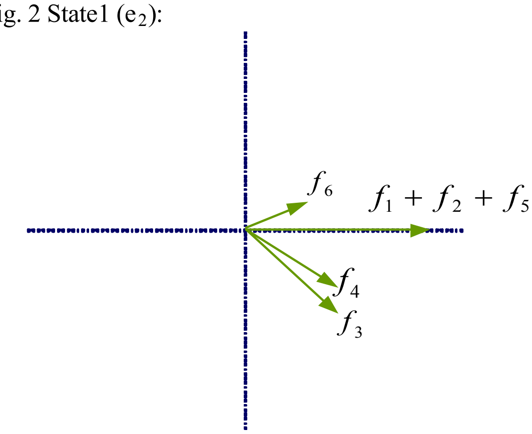

Fig. 2: (a2) Displaying the forces entering electron e2 on the coordinates system

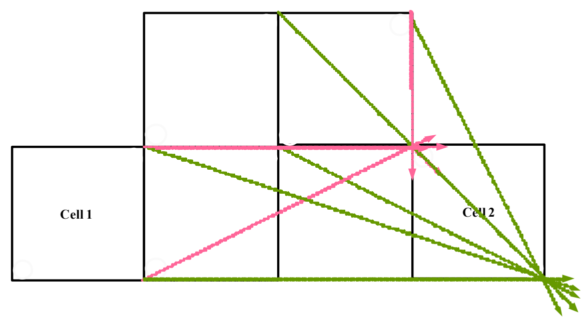

Fig. 2(b): Forces on the electrons of cell 2 in state2

$$F_1 = F_2 = \frac{kq^2}{r_1^2} = \frac{kq^2}{(2a)^2} = \frac{kq^2}{4a^2} = \frac{1}{4}A$$

$$F_3 = \frac{kq^2}{r_2^2} = \frac{kq^2}{\left(\sqrt{2}a\right)^2} = \frac{kq^2}{2a^2} = \frac{1}{2}A$$

$$F_4 = \frac{kq^2}{r_3^2} = \frac{kq^2}{\left(\sqrt{5}a\right)^2} = \frac{kq^2}{5a^2} = \frac{1}{5}A$$

$$F_5 = \frac{kq^2}{r_4^2} = \frac{kq^2}{(3a)^2} = \frac{kq^2}{9a^2} = \frac{1}{9}A$$

$$F_6 = \frac{kq^2}{r_5^2} = \frac{kq^2}{\left(\sqrt{10}a\right)^2} = \frac{kq^2}{10a^2} = \frac{1}{10}A$$

$$F_1' = \sqrt{(0.2A)^2 + (0.5A)^2 + 2(0.2A)(0.5A)\cos(19.44°)} = \left[\sqrt{0.83}\right]A \cong 0.91A$$

$$F_2' = \sqrt{(0.1A)^2 + (0.91A)^2 + 2(0.1A)(0.91A)\cos(73.15°)} = \left[\sqrt{0.89}\right]A \cong 0.94A$$

$$F_3' = 2F_1 + F_5 = 0.5A + 0.11A = 0.61A$$

$$F_4' = \sqrt{(0.61A)^2 + (0.94A)^2 + 2(0.61A)(0.94A)\cos(18.14°)} = \left[\sqrt{0.89}\right]A \cong 1.5A$$

Fig. 2 State2 (e_1):

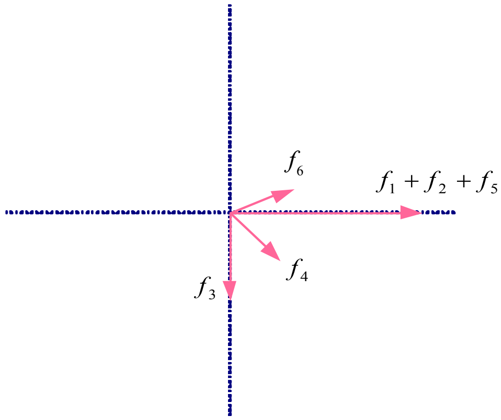

Fig. 2: (b1) Displaying the forces entering electron e1 on the coordinates system

$$F_T = 1.5A = 10.66*10^{-13}(N)$$
$$\phi_T = -9.07°$$
$$\tau_2 = -12.73*10.66*10^{-13}*\cos(80.93°) = -21.71*10^{-13}$$
$$\tau_t = -21.71*10^{-13}$$

$$F_1 = F_2 = \frac{kq^2}{r_1^2} = \frac{kq^2}{a^2} = A$$
$$F_3 = \frac{kq^2}{r_2^2} = \frac{kq^2}{a^2} = A$$
$$F_4 = \frac{kq^2}{r_3^2} = \frac{kq^2}{\left(\sqrt{2}a\right)^2} = \frac{kq^2}{2a^2} = \frac{1}{2}A$$
$$F_5 = \frac{kq^2}{r_4^2} = \frac{kq^2}{(2a)^2} = \frac{kq^2}{4a^2} = \frac{1}{4}A$$
$$F_5 = \frac{kq^2}{r_5^2} = \frac{kq^2}{\left(\sqrt{5}a\right)^2} = \frac{kq^2}{5a^2} = \frac{1}{5}A$$

$$F_1' = \sqrt{A^2 + (0.5A)^2 + 2(A)(0.5A)\cos(45°)} = 1.4A$$
$$F_2' = \sqrt{(1.4A)^2 + (0.2A)^2 + 2(1.4A)(0.2A)\cos(93.06°)}A \cong 1.36A$$
$$F_3' = \sqrt{(2.25A)^2 + (1.36A)^2 + 2(2.25A)(1.36A)\cos(46.53°)} \cong 3.55A$$

$$F_T = 3.55A = 25.24*10^{-13}(N)$$
$$\phi_T = -23.26°$$

$$\tau_1 = -12.73*25.24*10^{-13}*\cos(21.74) = -295.6*10^{-13}$$

Fig. 2 State2 (e_2):

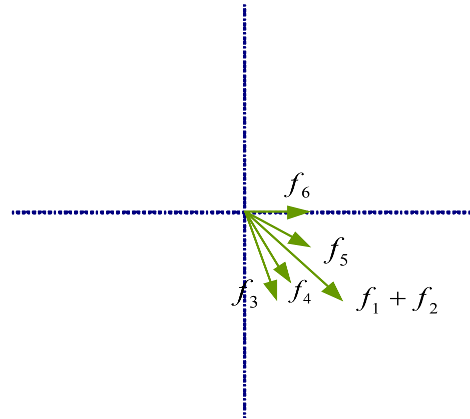

Fig. 2: (b2) Displaying the forces entering electron e2 on the coordinates system

$$F_1 = F_2 = \frac{kq^2}{r_1^2} = \frac{kq^2}{\left(\sqrt{5}a\right)^2} = \frac{kq^2}{5a^2} = \frac{1}{5}A$$
$$F_3 = \frac{kq^2}{r_2^2} = \frac{kq^2}{\left(\sqrt{5}a\right)^2} = \frac{kq^2}{5a^2} = \frac{1}{5}A$$
$$F_4 = \frac{kq^2}{r_3^2} = \frac{kq^2}{\left(2\sqrt{2}a\right)^2} = \frac{kq^2}{8a^2} = \frac{1}{8}A$$
$$F_5 = \frac{kq^2}{r_4^2} = \frac{kq^2}{\left(\sqrt{10}a\right)^2} = \frac{kq^2}{10a^2} = \frac{1}{10}A$$
$$F_6 = \frac{kq^2}{r_5^2} = \frac{kq^2}{(3a)^2} = \frac{kq^2}{9a^2} = \frac{1}{9}A$$

$$F_1' = \sqrt{(0.2A)^2 + (0.12A)^2 + 2(0.2A)(0.12A)\cos(19.44°)} = 0.3A$$
$$F_2' = \sqrt{(0.11A)^2 + (0.1A)^2 + 2(0.11A)(0.1A)\cos(18.43°)} = 0.2A$$
$$F_3' = \sqrt{(0.4A)^2 + (0.3A)^2 + 2(0.4A)(0.3A)\cos(29.16°)} \cong 0.68A$$
$$F_4' = \sqrt{(0.2A)^2 + (0.68A)^2 + 2(0.2A)(0.68A)\cos(30.94°)} \cong 0.85A$$
$$F_T = 0.85A = 6.04*10^{-13}(N)$$
$$\phi_T = -20.07°$$
$$\tau_2 = 12.73*6.04*10^{-13}*\cos(87.57) = 3.07*10^{-13}$$
$$\tau_t = -292.53*10^{-13}$$

Based on the above mentioned calculations, since the moment of forces is more in state 2; therefore, the electrons set in state 1 which is more stable. This state acts as a wire and is not favorable for us. In order to make an inverter logic gate, we change Fig. 2 as follows using the above structure (Fig. 3). As you can see in Fig. 3, in this state we have a space equal to one QCA cell.

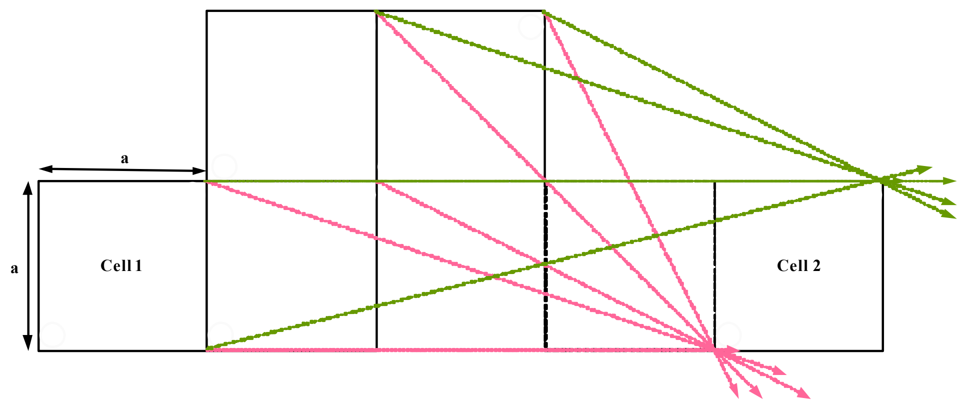

Fig. 3(a): Forces on the electrons of cell 2 in state1

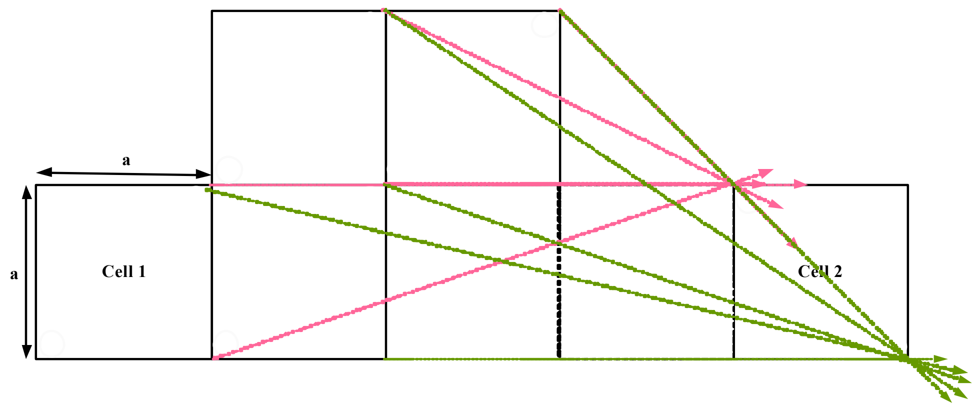

Fig. 3(b): Forces on the electrons of cell 2 in state1

Fig. 3 State2 (e_1):

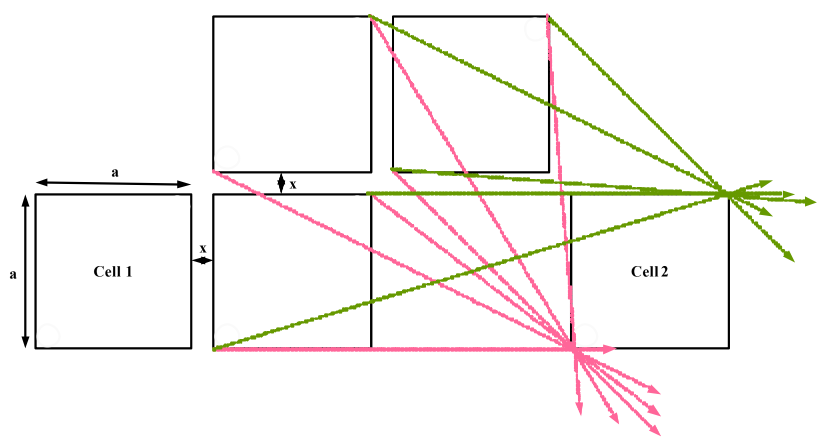

Fig. 4(a): Forces on the electrons of cell 2 in state1

Fig. 4 State1 (e_1):

Fig. 3 State1 (e_1):

$F_T = 0.85A = 6.04*10^{-13}(N)$

$\phi_T = -20.07°$

$\tau_1 = 3.07*10^{-13}$

In these state calculations are similar to Fig. 2. state 2 for e2.

Fig. 3 State1 (e_2):

$F_T = 0.6A = 4.27*10^{-13}(N)$

$\phi_T = 1.5°$

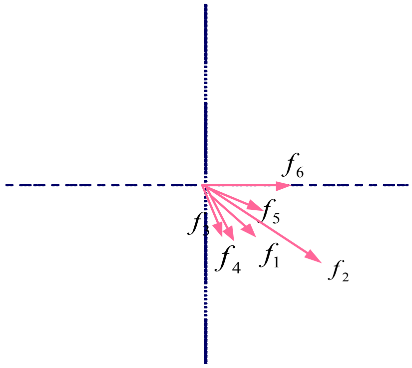

Fig. 4: (a2) Displaying the forces entering electron e1 on the coordinates system

$$\tau_2 = -12.73 * 4.27 * 10^{-13} * \cos(69°) = -19.57 * 10^{-13}$$
$$\tau_t = -16.5 * 10^{-13}$$
$$F_T = 1.5A = 10.66 * 10^{-13}(N)$$
$$\phi_T = -9.07°$$
$$\tau_1 = -21.71 * 10^{-13}$$

$$F_T = 0.47A = 3.34 * 10^{-13}(N)$$
$$\phi_T = -17.5°$$
$$\tau_2 = 12.73 * 3.34 * 10^{-13} * \cos(85°) = 3.83 * 10^{-13}$$
$$\tau_t = -17.88 * 10^{-13}$$

Based on the above mentioned calculations, it seems that the electrons set in state 1 which is more stable; but the calculation approximately shows the same result in both states and therefore, it is not completely trustable.

By increasing the space, the force become lower and the results became better; however it is not favorable for us because electrons are set in state 1 again. Consequently, we assume a new hypothesis: The cells have a space of x (x= 2 nm) with each other. Consequently, Fig. 2 changed to 4.

With due regard to the importance of results of proving Fig. 4, this physical proof is made completely.

$$F_1 = \frac{kq^2}{r_1^2} = \frac{kq^2}{(2a+2x)^2} = \frac{23.04*10^{-29}}{16*10^{-16}} \cong 10^{-13}$$
$$F_2 = \frac{kq^2}{r_2^2} = \frac{kq^2}{2a^2+4x^2+4ax} = \frac{23.04*10^{-29}}{808*10^{-18}} \cong 3*10^{-13}$$
$$F_3 = \frac{kq^2}{r_3^2} = \frac{kq^2}{4a^2+2x^2+4ax} = \frac{23.04*10^{-29}}{1448*10^{-18}} \cong 10^{-13}$$
$$F_4 = \frac{kq^2}{r_4^2} = \frac{kq^2}{5a^2+5x^2+8ax} = \frac{23.04*10^{-29}}{1928*10^{-18}} \cong 10^{-13}$$
$$F_5 = \frac{kq^2}{r_5^2} = \frac{kq^2}{5a^2+5x^2+10ax} = \frac{23.04*10^{-29}}{2*10^{-15}} \cong 10^{-13}$$
$$F_6 = \frac{kq^2}{r_6^2} = \frac{kq^2}{(2a+2x)^2} = \frac{23.04*10^{-29}}{16*10^{-16}} \cong 10^{-13}$$

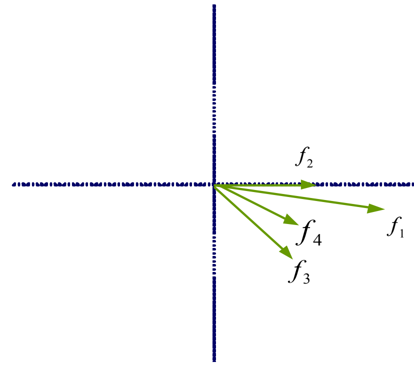

Fig. 4: (a2) Displaying the forces entering electron e2 on the coordinates system

$$F_1' = \sqrt{(10^{-13})^2 + (10^{-13})^2 + 2(10^{-13})(10^{-13})\cos(20.81°)}$$
$$\cong 1.96*10^{-13}$$
$$F_2' = \sqrt{(1.96*10^{-13})^2 + (10^{-13})^2 + 2(1.96*10^{-13})(10^{-13})\cos(25.35°)}$$
$$\cong 2.89*10^{-13}$$
$$F_3' = \sqrt{(2.89*10^{-13})^2 + (10^{-13})^2 + 2(2.89*10^{-13})(10^{-13})\cos(9.24°)}$$
$$\cong 7.17*10^{-13}$$
$$F_4' = \sqrt{(7.17*10^{-13})^2 + (10^{-13})^2 + 2(7.17*10^{-13})(10^{-13})\cos(31.17°)}$$
$$\cong 8.05*10^{-13}$$
$$F_T = 8.05*10^{-13}(N)$$
$$\phi_T = -15.58°$$
$$\tau_1 = 12.73*8.05*10^{-13}*\cos(60.58°) = 50.21*10^{-13}$$
$$F_T = 8.05*10^{-13}(N)$$
$$\phi_T = -15.58°$$
$$\tau_1 = 12.73*8.05*10^{-13}*\cos(60.58°) = 50.21*10^{-13}$$

Fig. 4 State1 (e_2):

$$F_1 = \frac{kq^2}{r_1^2} = \frac{kq^2}{2a^2+2x^2+4ax} = \frac{23.04*10^{-29}}{808*10^{-18}} \cong 3*10^{-13}$$
$$F_2 = \frac{kq^2}{r_2^2} = \frac{kq^2}{(2a+2x)^2} = \frac{23.04*10^{-29}}{16*10^{-16}} \cong 10^{-13}$$
$$F_3 = \frac{kq^2}{r_3^2} = \frac{kq^2}{(2a+2x)^2} = \frac{23.04*10^{-29}}{16*10^{-16}} \cong 10^{-13}$$
$$F_4 = \frac{kq^2}{r_5^2} = \frac{kq^2}{5a^2+5x^2+10ax} = \frac{23.04*10^{-29}}{2*10^{-15}} \cong 10^{-13}$$
$$F_5 \cong 0; \quad F_6 \cong 0$$

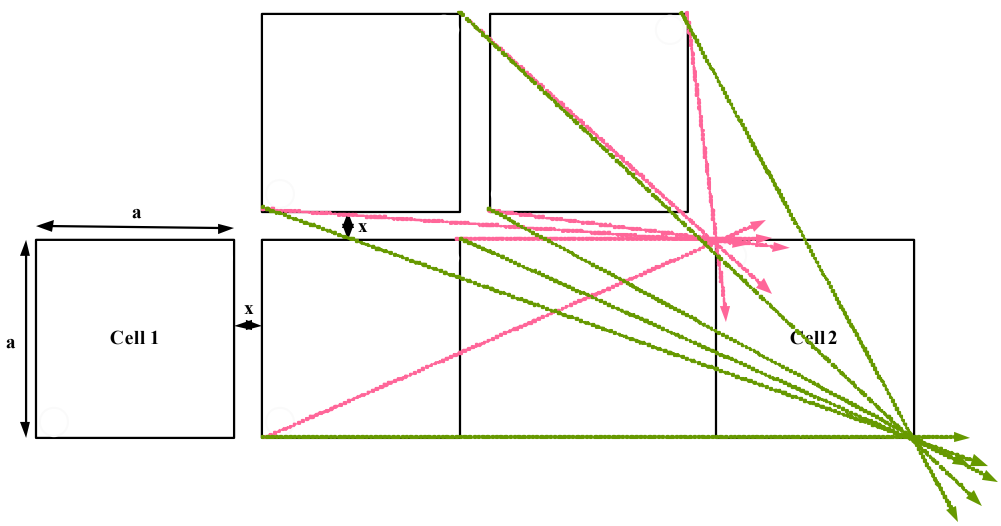

Fig. 4(b): Forces on the electrons of cell 2 in state2

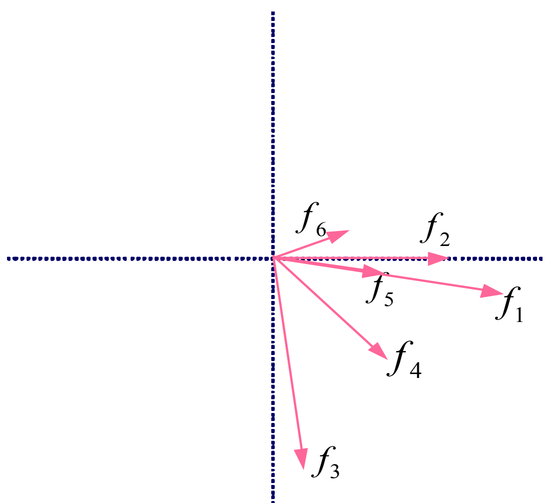

Fig. 4: (b1) Displaying the forces entering electron e1 on the coordinates system

$$F_1' = \sqrt{\left(3*10^{-13}\right)^2 + \left(10^{-13}\right)^2 + 2\left(3*10^{-13}\right)\left(10^{-13}\right)\cos(17.33°)}$$
$$\cong 3.96*10^{-13}$$

$$F_2' = \sqrt{\left(3.96*10^{-13}\right)^2 + \left(10^{-13}\right)^2 + 2\left(3.96*10^{-13}\right)\left(10^{-13}\right)\cos(17.9°)}$$
$$\cong 4.92*10^{-13}$$

$$F_3' = \sqrt{\left(4.92*10^{-13}\right)^2 + \left(10^{-13}\right)^2 + 2\left(4.92*10^{-13}\right)\left(10^{-13}\right)\cos(36.05°)}$$
$$\cong 5.76*10^{-13}$$

$$F_T = 5.76*10^{-13}\,(N)$$
$$\phi_T = -18.02°$$
$$\tau_2 = -12.73*5.76*10^{-13}*\cos(71.98°) = -22.73*10^{-13}$$
$$\tau_t = 27.48*10^{-13}$$

Fig. 4 State2 (e_1):

$$F_1 = \frac{kq^2}{r_1^2} = \frac{kq^2}{a^2+2x^2+2ax} = \frac{23.04*10^{-29}}{404*10^{-18}} \cong 6*10^{-13}$$

$$F_2 = \frac{kq^2}{r_2^2} = \frac{kq^2}{(a+2x)^2} = \frac{23.04*10^{-29}}{484*10^{-18}} \cong 5*10^{-13}$$

$$F_3 = \frac{kq^2}{r_3^2} = \frac{kq^2}{a^2+2x^2+2ax} = \frac{23.04*10^{-29}}{404*10^{-18}} \cong 6*10^{-13}$$

$$F_4 = \frac{kq^2}{r_4^2} = \frac{kq^2}{2a^2+5x^2+6ax} = \frac{23.04*10^{-29}}{884*10^{-18}} \cong 2*10^{-13}$$

$$F_5 = \frac{kq^2}{r_5^2} = \frac{kq^2}{4a^2+5x^2+8ax} = \frac{23.04*10^{-29}}{1604*10^{-18}} \cong 10^{-13}$$

$$F_6 = \frac{kq^2}{r_6^2} = \frac{kq^2}{5a^2+4x^2+8ax} = \frac{23.04*10^{-29}}{1924*10^{-18}} \cong 10^{-13}$$

$$F_1' = \sqrt{\left(6*10^{-13}\right)^2 + \left(2*10^{-13}\right)^2 + 2\left(6*10^{-13}\right)\left(2*10^{-13}\right)\cos(39.28°)}$$
$$\cong 7.65*10^{-13}$$

$$F_2' = \sqrt{\left(7.65*10^{-13}\right)^2 + \left(6*10^{-13}\right)^2 + 2\left(7.65*10^{-13}\right)\left(6*10^{-13}\right)\cos(58.92°)}$$
$$\cong 11.93*10^{-13}$$

$$F_3' = \sqrt{\left(5*10^{-13}\right)^2 + \left(10^{-13}\right)^2 + 2\left(5*10^{-13}\right)\left(10^{-13}\right)\cos(24.22°)}$$
$$\cong 5.92*10^{-13}$$

$$F_4' = \sqrt{\left(5.92*10^{-13}\right)^2 + \left(11.93*10^{-13}\right)^2 + 2\left(5.92*10^{-13}\right)\left(11.93*10^{-13}\right)\cos(47.29°)}$$
$$\cong 16.24*10^{-13}$$

$$F_T = 16.54*10^{-13}\,(N)$$
$$\phi_T = -5.48°$$
$$\tau_1 = -12.73*16.54*10^{-13}*\cos(39.52°) = -162.13*10^{-13}$$

Fig. 4 State2 (e_2):

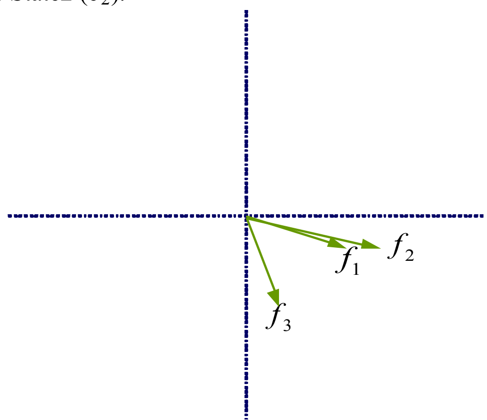

Fig. 4: (b2) Displaying the forces entering electron e2 on the coordinates system

$$F_1 = \frac{kq^2}{r_1^2} = \frac{kq^2}{(a+x)^2+(2a+x)^2} = \frac{23.04*10^{-29}}{1844*10^{-18}} \cong 10^{-13}$$

$$F_2 = \frac{kq^2}{r_2^2} = \frac{kq^2}{5a^2+4x^2+8ax} = \frac{23.04*10^{-29}}{1924*10^{-18}} \cong 1.2*10^{-13}$$

$$F_3 = \frac{kq^2}{r_3^2} = \frac{kq^2}{(a+x)^2+(2a+x)^2} = \frac{23.04*10^{-29}}{1844*10^{-18}} \cong 10^{-13}$$

$$F_4 \cong 0$$
$$F_5 \cong 0$$
$$F_6 \cong 0$$

$$F_1' = \sqrt{\left(1.2*10^{-13}\right)^2+\left(10^{-13}\right)^2+2\left(1.2*10^{-13}\right)\left(10^{-13}\right)\cos(3.3°)} \cong 2.14*10^{-13}$$

$$F_2' = \sqrt{\left(2.14*10^{-13}\right)^2+\left(10^{-13}\right)^2 +2\left(2.14*10^{-13}\right)\left(10^{-13}\right)\cos(36.15°)} \cong 3*10^{-13}$$

$$F_T = 3*10^{-13}(N)$$
$$\phi_T = 60.6°$$
$$\tau_2 = 12.73*3*10^{-13}*\cos(60°) = 19.1*10^{-13}$$
$$\tau_t = -143.03*10^{-13}$$

By comparisons between Fig. 4 and Fig. 2, we found that the moment of the forces bring to electrons of cell (2) became lower in Fig. 4 in both states and electrons have a more stability, but state 1 is more stable again. Therefore, we consider a compound form for Fig. 5 which is made of Fig. 3 and 4 and based on the following calculations:

Fig. 5 State2 (e_1):

$$F_T \cong 2*10^{-13}(N)$$
$$\phi_T = -13.07°$$
$$\tau_1 = -12.73*2*10^{-13}*\cos(58.07°) = -13.49*10^{-13}$$

Fig. 5 State1 (e_2):

$$F_T \cong 10^{-13}(N)$$
$$\phi_T = -9.5°$$
$$\tau_2 = 12.73*10^{-13}*\cos(35.5°) = 10.31*10^{-13}$$
$$\tau_t = -3.18*10^{-13}$$

When the electrons set in state 1, we gained a result similar to the arrangement of electrons in state 1 of Fig. 4.

Fig. 5 State2 (e_1):

$$F_T = 5.76*10^{-13}(N)$$
$$\phi_T = -27.86°$$
$$\tau_1 = 12.73*5.76*10^{-13}*\cos(17.14°) = 70.39*10^{-13}$$

Fig. 5 State2 (e_2):

$$F_T \cong 10^{-14}(N)$$
$$\tau_2 \cong -10^{-14}$$
$$\tau_t \cong 70.39*10^{-13}$$

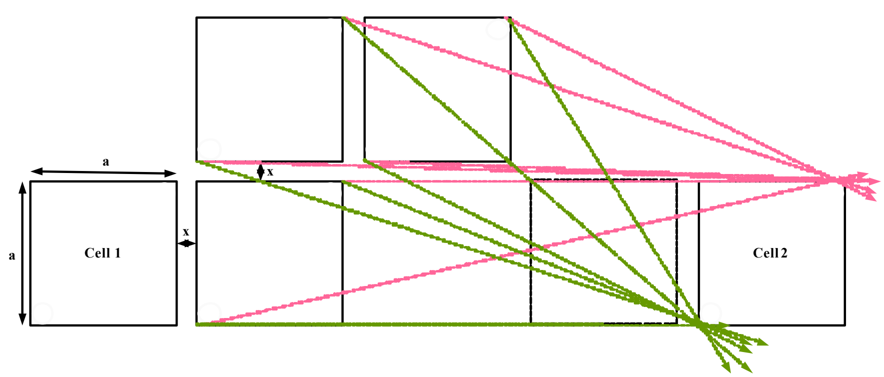

Fig. 5(a): Forces on the electrons of cell 2 in state1

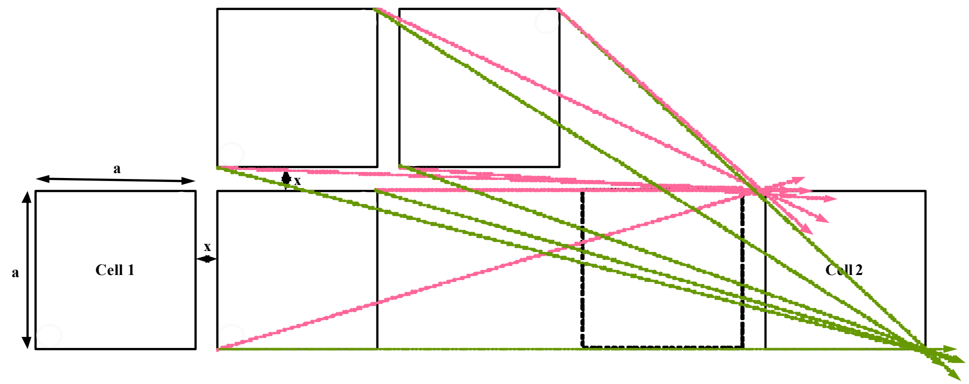

Fig. 5(b): Forces on the electrons of cell 2 in state2

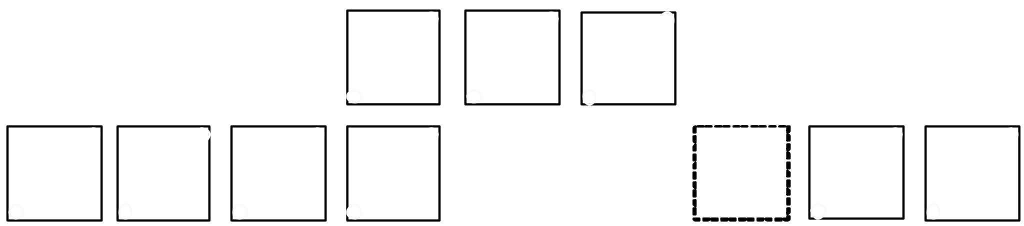

Fig. 6: A QCA inverter

The resultant of forces and the moment of forces calculated in Fig. 5. State 2 for e2 is very small in comparison to other forces and consequently it will be very more stable. So, we can say that if QCA cells set as Fig. 5, the electrons of cell (2) are set in state 2 which act as an inventor.

All the functions, which have been performed up to now, can be generalized to standard inverter and finally the number of standard inverter cells which is 13 or less decreases to 9 or less cells (Fig. 6). So, we can decrease the number of cells of all QCA circuits, because as already mentioned, the inverter and majority function form a complete gate together.

As we observed, in this paper, we tried to use some formulae and physical proofs to remarkably reduce the number of standard QCA inverter cells. Regarding the fact that majority gate forms a complete gate beside the inverter, therefore by reducing the number of inverter cells we can improve making QCA logical gates and minimize the size of these gates in attempting to reduce the complexity of circuits. This is of particular importance in big and sophisticated circuits.

CONCLUSION

Each QCA circuit can be built only using majority and inverter gates. Hence constructing each of these gates in a more efficient form can be valuable. This study proposes a method for decreasing the number of Quantum Dot Cells in Quantum Dot Cellular Automata (QCA) circuits based on a new technique for reducing the number of QCA cells in an inverter gate. The proposed method is based on physical rules and relations. Through physical and mathematical calculation we prove that our method is completely true. This method can be useful in decreasing complexity of QCA design as it works toward removing some cells in a QCA inverter gate, without any effect on the inverter functionality

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