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A Fast Algorithm for Connectivity-Based Computations in Dynamic Networks

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Abstract: Connectivity graphs are widely used in different branches of engineering, especially in ad hoc network analysis, decentralized control of Unmanned Air Vehicles (UAV) and robot control. In large network simulations, the extensive cost and the huge amount of memory required for performing the calculations is a major problem. Connectivity probability and propagation-based link reliability approaches are some other calculations which use connectivity-based calculations. Therefore, methods should be developed that are fast and require less memory to determine the connectivity-based calculations in order to expedite such calculations. In this paper, we focus on applications which use distance between nodes as a base in calculations. Dynamic networks in which moving agents produce varying connectivity graphs in time increase this type of calculations. We used mean degree distribution versus radio range instead of degree distribution for a specified radio range to investigate congestion/interference of networks when the approximate method is used. We introduce a modification factor for connectivity distance, R in the approximate Manhattan measure to improve accuracy in a wide range of R. The results for two random distributions of agents based on Monte Carlo simulations are compared to the present real methods to show the superiority of our approach.

Key words: Dynamic networks . connectivity-based calculations . approximate connectivity . link reliability . Monte Carlo simulation

INTRODUCTION

Computation of connectivity graphs as a connectivity-based calculation has developed as a key problem in various engineering branches especially in communication and control. For example, control of Unmanned Air Vehicles (UAV) based on decentralized control has developed in recent years. Decentralized control approach is a result of studying swarms of insects which offer clear examples of self-organized, emergent behavior [1]. In the problem of coordinating multiple robots, a representation of the configuration space appears naturally, by using graph-theoretic models to describe local interactions in the formation [2]. In these cases, graph-based models serve as an interface between the discrete and the continuous when trying to manage the design complexity associated with formation control problems. Notable results for these problems have been presented in [3-6].

Network connectivity is considered as a network reliability measure and there are various measures of connectivity. Two-terminal connectivity measures the ability of the network to satisfy the communication needs of a specific pair of nodes. Two-terminal reliability is defined as the probability that there exists at least one path in the network between a specific pair of nodes.

Reliability is widely used in engineering. In networks, connectivity reliability means the probability that there exists at least one feasible link between two nodes under predefined conditions. Reliability problems become more and more important as modern systems become more and more complex. This motivates the study of network reliability, a topic which has been extensively studied in the past few decades [7].

The minimum overall two-terminal values can be interpreted as the reliability level guaranteed by the network to all users. The average overall two-terminal value provides a measure of the resilience of the network [8]. Connectivity probability can also be considered as a network reliability measure. This measure provides the probability that a randomly distributed network is connected.

In simulating large mobile networks, a main problem is the high cost of storage requirements. It may even be impossible to model such networks due to the huge extent of storage requirements. The key to modeling such networks to determine link reliability and connectivity probability is to dynamically determine the connectivity graph. Therefore, the

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connectivity matrix must be computed over and over again. As the number of nodes in the network is increased, computational time required to determine the connectivity graph grows quadratically. Therefore, it is necessary to compute the connectivity matrix for connectivity-based calculations over and over again. Thus for the simulation of large dynamic networks it is important to develop faster connectivity-based determination techniques with a reasonable degree of accuracy [9, 10].

In this paper, we discuss the problem of determining connectivity graphs of dynamic networks using approximate methods and introduce a new efficient algorithm for approximating the connectivity graph. This method has a good accuracy while it requires less computation and is much faster than previous methods. Results are presented based on Monte Carlo simulations and may be generalized for computation of link reliability and connectivity probability. Moreover, we used mean degree distribution versus radio range (MDDVRR) instead of degree distribution for a specified radio range (DDSRR) to investigate congestion/interference of network when approximate methods are used. We compared the performance of these measures for two different distributions of nodes.

The outline of this paper is as fallows: In section 2 we introduce graph generation using real and approximate methods. In section 3 the strategy for comparing the graphs and some connectivity metrics and measures are discussed. In section 4 connectivity probability is discussed. Link and network reliability objectives are introduced in section 5. In section 6 the experimental results in static and dynamic cases including a comparison of our approximate and present real methods for different problem are presented. Finally in the concluding section the results are discussed.

GRAPH GENERATION USING REAL AND APPROXIMATE METHODS

The most realistic model of connectivity of agents which Barrett *et al*. in [10] label real connectivity is based on calculations using Euclidian distance between nodes and comparing the obtained results with radio distance. This model is based on two-ray ground propagation model where the received power is inversely proportional to the square of the distance up to a limiting distance and is inversely proportional to the fourth power of the distance beyond that threshold. In this model we do not consider radio interference effects. Therefore the real model inherently has some approximation. For two nodes $N_i(x_i,y_i)$ and $N_j(x_j,y_j)$, the Euclidian distance is as follows:

$$D_E\ (N_i,N_j) = [(x_i-x_{j)}^2 + (y_i-y_{j)}^2]^{1/2} \quad (1)$$

Radio distance R, is defined as the connectivity range for every node which is compared by Euclidian distance between a pair of nodes N_i and N_j.

$$D_E\ (N_i,N_j) \le R \Rightarrow N_i \text{ is connected to } N_j \quad (2)$$

$$D_E\ (N_i,N_j) > R \Rightarrow N_i \text{ is not connected to } N_j \quad (3)$$

In the remainder of the paper we name this method as the real method and use it for comparison with our proposed algorithms. In order to save computational time, R^2 is used instead of R for comparison with the square of Euclidian distance. This saves the computation of root square for computing the distance between every pair of nodes. Computationally this model includes two subtractions, two multiplications, an addition and a comparison which must be carried out for each pair of nodes. Therefore, the total amount of computational time, T_E, for n nodes will be:

$$T_E = (2T_s + 2T_m + T_a + T_c) * \binom{n}{2} \quad (4)$$

where T_s, T_m, T_a and T_c are the computational times of a subtraction, a multiplication, an addition and a comparison respectively.

Using the number of clock cycles required for the execution of instructions on an Intel Pentium 4 processor [11] results in:

$$T_E = (2*3+2*5+3+1)* \binom{n}{2} = 10\ (n^2+n) \text{ clock cycles} \quad (5)$$

This is the case for an undirected network since radio range is equal for all nodes in undirected networks. Barrett *et al*. have investigated the performance of some approximate measures like Manhattan distance metric for connectivity, k-means cluster connectivity and box connectivity for determining connectivity graphs. They have shown that the Manhattan distance is one of the best approximate measures for computing the connectivity graph of undirected networks [10]. We have based our method upon this metric and have implemented a fast algorithm which reduces the computational time in dynamic networks. The Manhattan distance for two nodes N_i and N_j is simply the sum of component-wise distances:

$$R_{Mij} = |x_i-x_j| + |y_i-y_j| \quad (6)$$

The computations for this metric include two subtractions, an addition and three comparisons being two comparisons for the absolute values and one for comparing R_{Mij} with R for each pair of nodes [10]. Therefore the total amount of computational time T_M, for n nodes will be:

$$T_M = (2T_s + T_a + 3T_c) * \binom{n}{2} \qquad (7)$$

which on a Pentium 4 Intel processor will require

$$T_M = (2*3+3+3*1)*\binom{n}{2} = 6\,(n^2+n) \text{ clock cycles} \qquad (8)$$

Comparing the results of equations (5) and (8) shows that by using Manhattan measure instead of Euclidian measure the computational time reduces by 40%. This reduction can be appreciable when the number of nodes is very large because the computational time is proportional to the square of n..

COMPARING CONNECTIVITY GRAPHS: METRICS AND MEASURES

In order to evaluate the efficiency of our method well-established techniques for comparing radio connectivity graphs were used and the connectivity matrices (real and approximate) in both static and dynamic networks were compared. In each case after determining the connectivity matrices, they were compared with each other. Two different approaches were used for comparing the graphs. The first comparison approach which is more statistical is based on degree of nodes. In the first approach two different methods are used. In the first method DDSRR of nodes for a network are determined using real and approximate measures. DDSRR shows the number of nodes which have different possible degrees in the network. The results are compared to investigate congestion of network when approximate methods are used. Here, two graphs are considered to be similar if they have the same DDSRR. Also MDDVRR is used as a measure. In this method mean degree of nodes for different connectivity distances is determined using real and approximate methods. Two graphs are considered to be similar if they have the same MDDVRR. In the second approach the resultant connectivity matrices are compared. In other words, links are compared one by one and Hamming distance between their connectivity matrices is used. A connectivity matrix for each graph is set up where every 1 in the matrix shows the connection between the nodes corresponding to the related row and column nodes. Hamming distance from one binary vector to another is the number of corresponding elements which differ. For example H ([1101], [0110]) = 3.

$$\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \qquad\qquad \begin{array}{cccc} 0 & 1 & \boxed{1} & 1 \\ 1 & 0 & 1 & 0 \\ 1 & \boxed{1} & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}$$

Real Graph Approximate Graph

Fig. 1: An illustration of the percent Hamming of two graphs as well as the weighted percent Hamming distance. In the approximate graph the misses are circled and the false positives are boxed

An example of comparing two connectivity matrices is shown in Fig. 1. Obviously the size of graphs to be compared must be the same. A deficiency of the Hamming distance is that it is not invariant with respect to graph size and one cannot draw any sound conclusions from it when comparing networks of various sizes. If we divide the Hamming distance by the number of elements of the connectivity matrix we will have a metric that is invariant with respect to the size of the graphs. Barrett and *et al*. in [10] called this metric the percent Hamming distance and denoted it %H.

$$\%H = H * 100 / n^2 \qquad (9)$$

The Hamming distance between the two graphs is H(r,a) = 6. For matrices in Fig. 1:

$$\%H\,(r, a) = (4+2) * 100/ 4^2 = 37.5\,\% \qquad (10)$$

CONNECTIVITY PROBABILITY: DEFINITION AND ANALYSIS

At first we describe some of the concepts related to the problem of network connectivity in a static environment where the nodes do not move. Then we define connectivity when the nodes are mobile as in an ad hoc network and introduce connectivity probability as a measure of network connectivity in a dynamic network.

Let D be a bounded domain on the Euclidean plane $R^2 = \{(x, y) \mid x, y \in R\}$ with piecewise smooth borders. Assume that there are n nodes inside the domain. At t = 0 the nodes are somehow placed using a random distribution and then they start moving around. Let $d_i = (x_i, y_i)$ be the radius vector of node i. Further, we assume that every node has a transceiver with communication range d_R: If the distance between two nodes is larger than d_R, they cannot establish a direct communication link. Nodes can transmit information by using multi-hop connections.

By definition a network is connected (or fully connected) if for every pair of nodes there exists a path between them. On the other hand, a network is connected if there exists a path, i.e., a sequence of distinct nodes such that consecutive nodes are adjacent, between any two nodes in the network [12].

Note that for a network to be connected we require existence of a path from source to destination at any moment in time. Connectivity probability is used to quantify the network connectivity. In the static environment (when the positions of nodes are fixed), the probability for a network to be connected depends on the density of the nodes and their connectivity range. To evaluate the connectivity probability in a typical static simu lation scenario, a number of nodes are placed randomly in the simulation area. A random variable is introduced which is equal to one if the network is found connected and zero if it is disconnected. The average of this random variable over several trials gives the connectivity probability [13, 14]. Madsen *et al*. [13] showed how to extend this approach to the case of a dynamic network.

Under motion of nodes a semi-axis of time R^+ is divided into intervals $t_1^{\pm}$, $t_2^{\pm}$, $t_3^{\pm}$, … where t_k^+ (t_k^-) denotes a time interval during which the network is connected (disconnected). Let's introduce the function $f^+(t)$ such that $f^+(t) = 1$ if $t \varepsilon t_k^+$ and $f^+(t) = 0$ if $t \varepsilon t_k^-$ .Time intervals $t_k^{\pm}$ can be considered as randomly distributed; thus, $f^+(t)$ is a stochastic process [13].

In a dynamic network, connectivity probability CP^+ is defined by

$$CP^+ = E\,[f^+(t)] \tag{11}$$

Where E [.] stands for the expected value (if it exists). The function $f(t)$ can be introduced in a similar fashion:

$$f\cdot(t) = \begin{cases} 1 & t \in t_k \\ 0 & t \in t^+_k \end{cases} \tag{12}$$

The probability that a network is disconnected is given by

$$CP- = E\,[f\text{-}(t)]$$

Since

$$f^+(t) = 1 - f\text{-}(t)$$

then

$$CP^+ + CP- = 1.$$

One can see that in general the probability CP^+ is time-dependent: $CP^+ = CP^+(t)$. For stationary stochastic processes $CP^+(t)$ = constant. If the stationary process is ergodic, then equality (11) can be substituted with the following:

$$\lim_{T\to\infty} \int_0^T f^+(t)dt = CP^+ \tag{13}$$

By definition, the system is ergodic if a measure of any invariant sub-domain of the phase space is either equal to zero or is equal to the measure of the whole space. This equality is equivalent to the following:

$$CP^+ = \lim_{\tau\to\infty} \frac{2}{\tau} \mathrm{mes}(T^+ \cap [0,\tau]) \tag{14}$$

where $T^+ = U\ t_k^+$ and mes stands for measure of a domain. In this case it is the total length of the intervals T^+ n [0, t]. That is, the probability that a network is connected can be defined as a density of set T^+ on the semi-axis $R^+ = \{t \mid t > 0\}$. Therefore, the problem of connectivity of a network consisting of moving nodes is reduced to the problem of existence and estimation of the expected value in equation (11). If the mobility model is stationary and ergodic, then formula (14) can be used to estimate connectivity probability as in [14, 15].

Dynamical systems theory can be used for modeling the movement of nodes in an ad hoc mobile network. In homogeneous ad hoc mobile networks, where the properties and specification of the nodes are the same, we can reasonably assume that the movement of nodes can be described by the same system of differential equations. If we introduce some form of randomization in the movement pattern of nodes, then it would require construction of stochastic differential processes. In the theory of dynamical systems, a phase flow that is a group of shifts along the trajectories is introduced which generates a dynamical system. The system can be described by differential equations of the following form [13]:

$$\dot{x} = g(x), \quad x \in \Pi \tag{15}$$

where Π is the phase space, x is a set of coordinates in Π (usually it is position and velocity) and dot means time differentiation. Let n be the number of nodes and $x^{(1)}, \ldots, x^{(n)}$ are their phase coordinates. Then these coordinates satisfy the following differential equations:

$$\dot{x}^{(k)} = g(x^{(k)}), \quad k = 1,\ldots,n \tag{16}$$

Its phase space = Π x … x Π = $(\Pi)^n$ is a direct product of n copies of initial phase space and phase coordinates = ($x^{(1)}$, …, $x^{(n)}$) are a set of coordinates of individual nodes. Note that if the system (15) has an invariant measure μ in Π, then the system (16) also has an invariant measure in, that is a direct product = μ_1 x …x μ_n.

The problem of estimation of the connectivity measure can be simplified significantly if dynamical system (16) is ergodic in . The key result of the ergodic theory is Birkhoff ergodic theorem. Then for almost all solutions of the ergodic system (15) the following equality holds:

$$\lim_{T\to\infty}\frac{1}{T}\int f(\hat{x}(t))dt = \int_{\Pi}\frac{f(\hat{x})d\hat{\mu}}{\text{mes}\,\Pi} \qquad (17)$$

where

$$\text{mes}\,\hat{\Pi} = \int_{\Pi} d\hat{\mu} = \hat{\mu}(\hat{\Pi}) \qquad (18)$$

is the measure of the whole phase space [13].

Let, for example, f be a characteristic function of a measurable domain D: f ( ) = 1 if ε D and f ( ) = 0 if ε. Due to the fact that f is limited and D is measurable, function f:$\to$R is integrable. In this case, the left-side of (17) is equal to the fraction of time interval 0=t=T when (t) lies in the domain D. Then the connectivity probability of a network will be equal to the right-side of (17) that in this case is:

$$CP^{+} = \text{mes } D/\text{ mes} \qquad (19)$$

Formula (19) is computationally more efficient than evaluating the expected value as shown by Madsen *et al.* in [13] and it can easily be implemented easily using Monte Carlo simulations.

LINK AND NETWORK RELIABILITY

In network reliability analysis, a telecommunication network with unreliable components can be modeled as an undirected network N (V, L) with node set V={v_1,…,v_n} and link set L={l_1,…,l_m} under the following assumptions [16]:

- Nodes are perfectly reliable; however, links fail randomly.
- Each link $l_i \in$ L, independently of other links, can be in either of two states, that is operative or failed, with respective probabilities p_i and q_i=1-p_i.
- No repair is allowed.

Let X ={x_1,x_2,…, x_m} denote the state vector of N(V, L) such that x_i = 1 if link l_i is in the operative state and x_i= 0 if link l_i is in the failed state. Hence, the probability of observing a particular state X is given by

$$\Pr[X] = \Pi_{l_i \subseteq L}[q_i + x_i(2p_i - 1)] \qquad (20)$$

The main function of a telecommunication network is to provide connectivity service. Let U $\subseteq$ V be a set of some specified nodes of N(V, L) that is a subset of all the nodes in the network. Network reliability analysis is concerned with the probability that all nodes in U are connected to each other, directly or indirectly. With respect to connectivity, a network can be in either of two states: connected or not connected. Therefore, the structure function is defined as:

$$\Phi(X) = \begin{cases} 1 & \text{if allnodesinUareconnected} \\ 0 & \text{otherwise} \end{cases} \qquad (21)$$

In order to compute the network reliability, we must calculate the expected value of the structure function Φ (X), i.e.,

$$R = E\lfloor\Phi(X)\rfloor = \Sigma_{x\subseteq S}\,\Phi\,(X)\Pr\lfloor X\rfloor \qquad (22)$$

where S is the state space of the all possible network states [16]. If U= V, that is the network to be connected, then Equation (22) refers to all-terminal reliability.

THE PROPOSED LINK RELIABILITY MODEL

Radio distance d_R, is defined as the connectivity range for every node which is compared by calculated distance between a pair of nodes N_i and N_j. If the calculated range between nodes is less than d_R, the nodes are considered to be connected. Obviously the reliability of a fully connected link is 1, but in real ad hoc networks, radio range is not sharp and after a distance radio connection is deteriorated due to weather conditions, quality of transceivers, existence of high rise buildings or hills. Thus the reliability of a link is less than 1 and some how decreases as the distance between the two nodes increases. We present a model for link reliability here.

We assume that for distances less than a fraction of dimension of square region (or diagonal of circular region) of simulation, aD where 0<a<1, there is no reliability degradation. For distances longer than d_R, the link reliability is assumed to be zero since there is no connection. For distances between aD and d_R the link reliability can be modeled using various methods based on the propagation model.

The free space model (FS) is the simplest propagation model. It only assumes that there is a direct path between transmitter t and receiver r. The path must be clear from obstacles. The received power P_r depends on the transmitted power P_t, the gain of the receiver and transmitter antennas (G_t, G_r), the wavelength λ, the distance d between the node pair and a system loss coefficient L. Except the distance d between the nodes, the other factors are system-wide constant parameters. The received power P_r changes with the distance between the sender and the receiver [17-19].

$$P_{r,FS} = \frac{P.G.G_y \lambda^x}{L(4\pi/d)^2} \tag{23}$$

Given the relationship between received power and d, we can define link reliability as a function of distance.

Based on free space propagation model, the reliability of links can be modeled generally as follow:

$$R_L(d) = A/d^2 + B \tag{24}$$

where A and B are constants. Reliability model must satisfy a main condition:

$$R_L(d) \leq 1 \tag{25}$$

Using boundary conditions R_L (aD) = 1 and R_L (d_R) = 0, where D is the dimension of square region (or diagonal of circular region) of simulation, our FS-based model can be introduced as:

$$D_E(n_i, n_j) \leq aD \Rightarrow n_i \text{ is connected to } n_j \text{ and the reliability of link is: } R_L = 1. \tag{26}$$

$aD < D_E(n_i, n_j) < d_R \Rightarrow n_i$ is partially connected to n_j and the reliability of link is:

$$R_L(d) = (a^2 D^2)(1-(d_R^2 / D_E^2(n_i, n_j))/(a^2 D^2 - d_R^2) \tag{27}$$

$$d_R \leq D_E(n_i, n_j) \Rightarrow n_i \text{ and } n_j \text{ are disconnected and Reliability of link is: } R_L = 0 \tag{28}$$

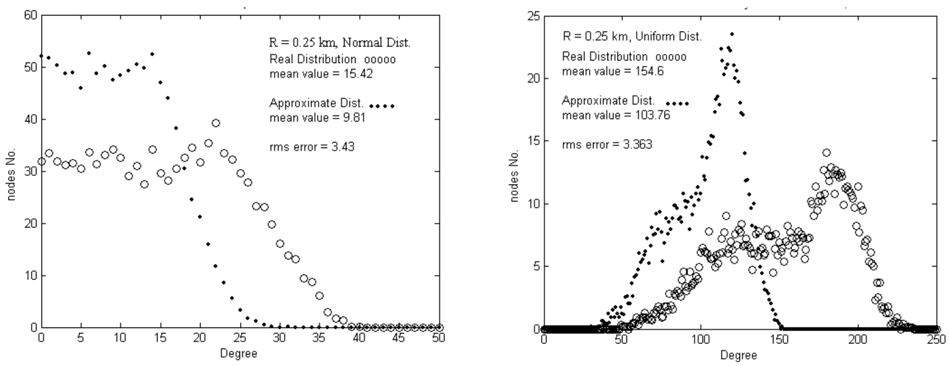
It means that in distances less than a fraction of the dimensions of the region, the reliability of connection is equal to one and for longer distances through radio range, the reliability decreases. For distances equal to or larger than the radio range, no connection exists and the reliability of connection will be equal 0. For distances between the above mentioned limits, the reliability of connection is inversely proportional to the square of the distance between nodes. Default FS radio range is 250 meter [20].

The Two Ray Ground propagation model (TRG) is an improved version of the free space model (FS). We used only FS model as a pilot and more discussions can be found in [21].

EXPERIMENTAL RESULTS IN STATIC AND DYNAMIC CASES

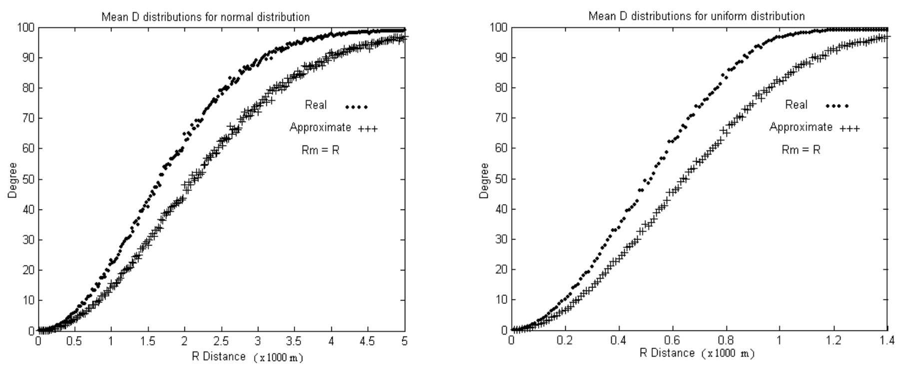
Monte Carlo simulations were carried out in static and dynamic cases. In each case two spatial distributions were used for the nodes in the network-namely normal and uniform. The region occupied by the nodes is considered to be a unit square for uniform spatial distribution and a circle with a radius of 2.5 for normal spatial distribution. Normal and uniform distributions were used as representatives for uniformly and non-uniformly distributed networks. In each distribution, MDDVRR, DDSRR and the percent Hamming (%H) measures for the real and the approximate methods were determined. In static case, the performance of Manhattan method and modified Manhattan method were experimented using Monte Carlo simulations. In section 5.1 the acceptable performance of Manhattan measure for nodes which are distributed normally and uniformly is shown. Then a modification factor is introduced to improve the performance of the Manhattan measure and the results are used in the dynamic case to implement our algorithm which is presented in section 5.2.

Monte carlo simulations in static case: In order to carry out the Monte Carlo simulations in static case two networks were constructed whose nodes had a spatially normal and uniform distribution respectively. In each network, the mean degree of nodes for every R was calculated from which MDDVRR was determined. For degree calculation, we used three different methods: precise method using Euclidean distance, approximate method using Manhattan distance measure and Manhattan measure using modified connectivity range. We used a zero mean unity variance normal distribution in which more than %99 of nodes are located in a circle with a radius of 2.5. We calculated real and approximate measures for a range of R from 0.01 to 1.4 (diagonal of unit square) for uniform and 5 for normal distribution with 0.01 increments. Circular and square regions are representatives for two different regions of nodes in two dimensional space. We used averaging of results to minimize changes. For every R, computations were repeated 10 times and averaged, yielding an rms error of less than %5.



(a) (b)

Fig. 2: DDSRR for (a) normal and (b) uniform spatial distribution of nodes resulted from approximate Manhattan and real method



(a) (b)

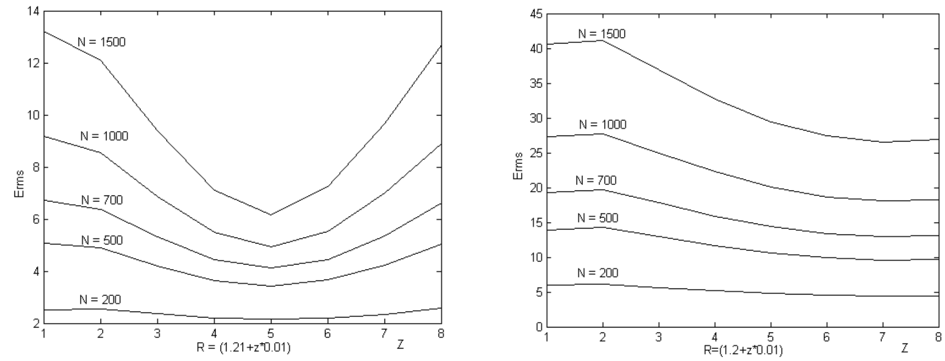
Fig. 3: Mean D distribution for (a) normal and (b) uniform spatial distribution of nodes resulted from approximate Manhattan and real method

Connectivity graph in static case: DDSRR should be calculated for a specific R. For R=0.25 km DDSRR is shown in Fig. 2. The networks include 1000 nodes. Obviously in both distributions real and approximate results have many differences. Using approximate Manhattan measure the maximum frequency of occurrence of a node with a degree specified on the horizontal axis increases and generally the degree of nodes decreases according to real measure.

MDDVRR using real and approximate methods are shown in Fig. 3. Both normally and uniformly distributed networks have the same pattern of MDDVRR, but with different scales. Obviously when R is increased, DDSRR of nodes is increased because the maximum radio range is increased and more nodes can communicate together. In the normally distributed network, MDDVRR reaches its maximum at R~5 km, but in a uniformly distributed network, it reaches its maximum at R~1.4 km. As can be seen from Fig. 3, real mean D distribution is bigger than or equal to the approximate mean D distribution for the same R. It means that using Manhattan approximate method generates some miss and no false positives and this is an advantage as mentioned above. Also Fig. 3 shows that approximate results are less than real results for similar R and both curves are incremental. So we can use a bigger R in approximate calculations to reduce the error. We tried to find the best modification coefficient m to minimize rms error. The average rms error for different values of m was calculated in each distribution. The results were averaged for different values of R. Figure 4 shows the results for different number of nodes. In Fig. 4(a) minimum rms error for different N is for z = 5 and so m = 1.26 and in uniform case, Fig. 4(b), it is satisfied for

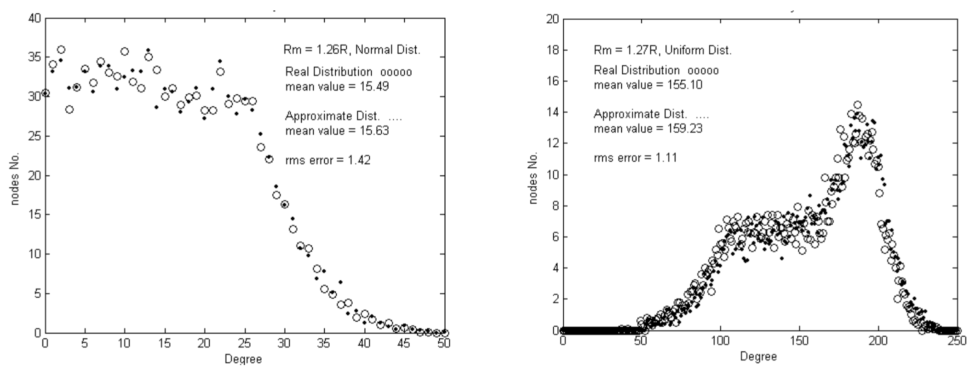
$$z = 7 \text{ and so } m = 1.27.$$

For R_m = 1.27R in uniform distribution the best similarities are seen between real and approximate results because the rms error is minimum. In normal distribution for R_m=1.26R minimum rms error results.



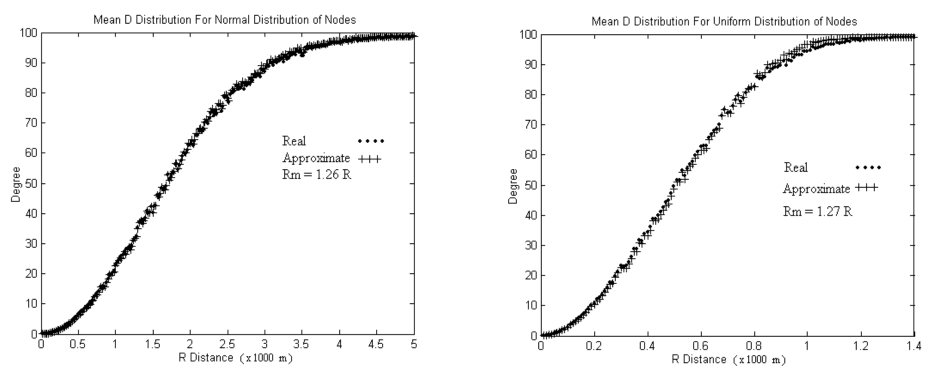
(a) (b)

Fig. 4: RMS error variations versus modification factor for different N for (a) normal and (b) uniform spatial distribution of nodes



(a) (b)

Fig. 5: DDSRR for (a) normal and (b) uniform spatial distribution of nodes resulted from modified approximate Manhattan and real methods



(a) (b)

Fig. 6: MDDVRR for (a) normal and (b) uniform spatial distribution of nodes using modified approximate and real methods

DDSRR curves using modified R in approximate Manhattan measure are shown in Fig. 5 which seem very similar to real one in both spatial distributions of nodes. Therefore modified approximate measure has negligible effect on congestion/interference characteristics of the network.

Figure 6 shows the MDDVRR curves using modified R which decreases rms error appreciably in both distributions. Modified approximate measure has negligible effect on congestion/interference characteristics of network as shown in Fig. 6.

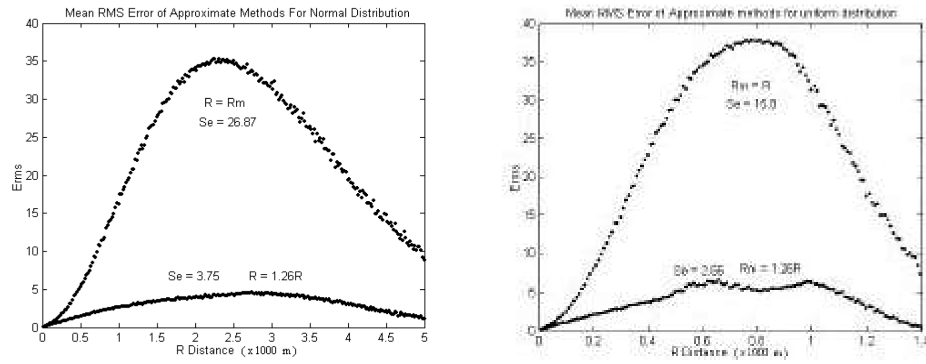


Fig. 7: Mean D distribution rms Error comparison of the best modified R and traditional R for (a) normal and (b) uniform spatial distribution of nodes

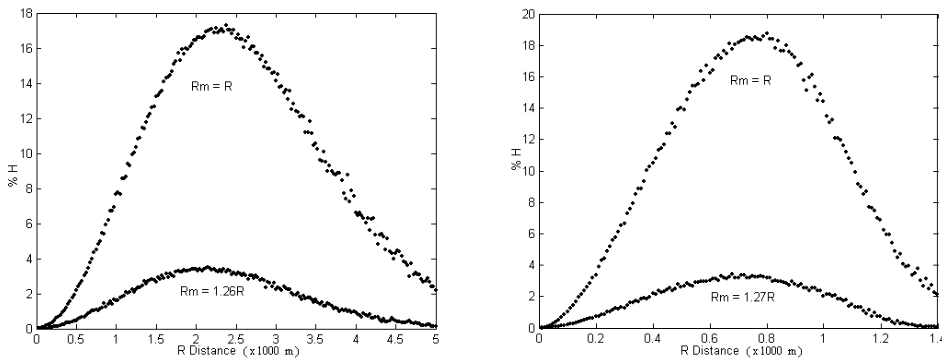


Fig. 8: %H distribution versus R using traditional R and modified R for (a) normal and (b) uniform spatial distribution of nodes

Table 1: Comparison of max. rms error for different methods

	Max. rms error for manhattan measure	Max. rms Error for modified	Manhattan measure fractional reduction of max. rms error
Uniform dist.	38.0	7	5.43
Normal dist.	35.8	5	7.16

Table 2: Comparison of rms error curve limited for different methods

	rms error curve limited area for manhattan measure	rms error curve limited area for modified manhattan measure	Fractional reduction of rms error curve limited area
Uniform dist.	15.8	2.66	5.94
Normal dist.	26.87	3.75	7.17

Similar results from Fig. 2, 3, 5, 6 show that MDDVRR can be used as a interference/congestion measure instead of DDSRR.

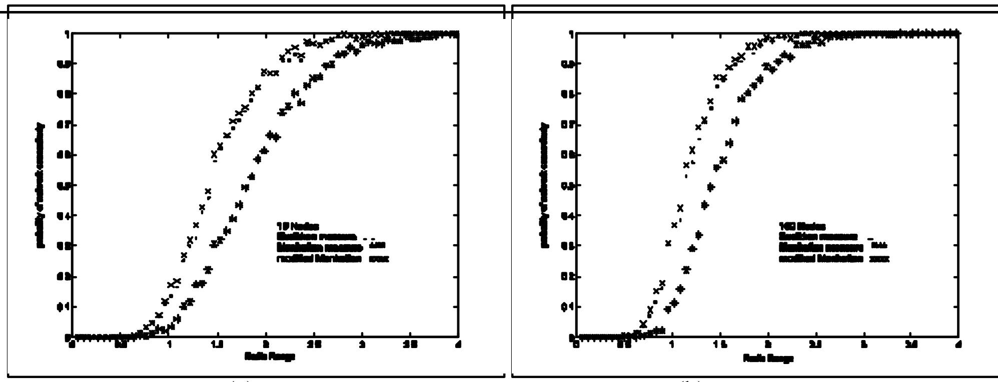
For comparing accuracy of different methods, maximum rms error and rms error curve limited area are used. In Fig. 7 maximum rms error and rms error curve limited area are compared for traditional and modified R.

The rms error for our modified Manhattan method is decreased. This is shown by comparing max. rms error and rms error curve limited area of methods. Max. rms error is decreased by 5.43 and 7.16 times for uniform and normal distribution of nodes respectively as shown in Fig. 7 and summarized in Table 1.

Table 2 shows rms error curve limited area of methods. The rms error curve limited area in using Manhattan measure with modified R, is reduced by 7.17 and 5.94 times for normal and uniform distributions respectively as shown in Table 2.

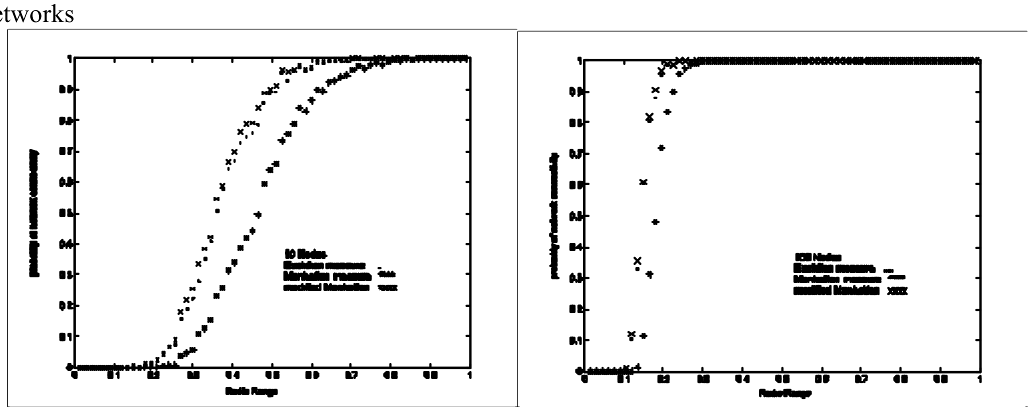
Table 3: Comparison of %H for different methods

	%H for manhattan measure	%H for modified manhattan measure	Fractional reduction of %H
Uniform Dist.	18.5	3.8	4.87
Normal Dist.	17.5	3.8	4.61



(a) (b)

Fig. 9: Connectivity probability using different measures for (a) 10 and (b) 100 nodes in normally distributed networks



(a) (b)

Fig. 10: Connectivity probability using different measures for (a) 10 and (b) 100 nodes in uniformly distributed networks

We determined the connectivity matrices for every R by real and approximate measures and compared them using the graph difference metric, %H. In Fig. 8 curves of %H using traditional and modified R for both distributions are shown

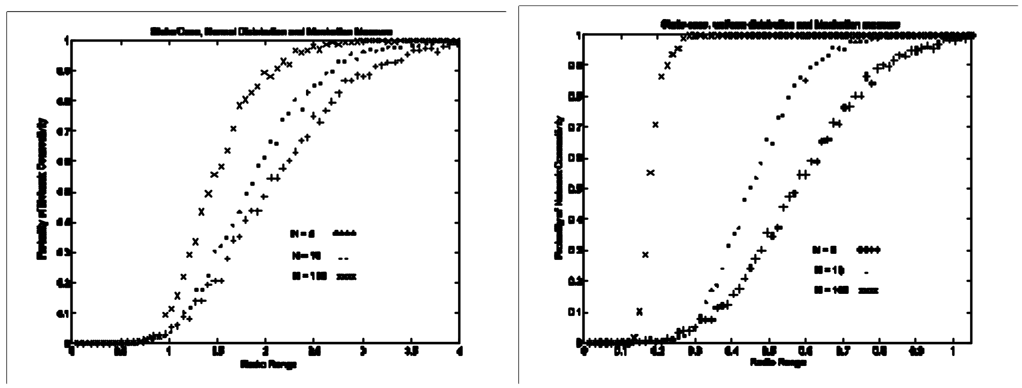
The results are summarized in Table 3. Dissimilarities of approximate methods are shown as %H.

The results obtained show an appreciable improvement in accuracy using our modified Manhattan measure.

Connectivity probability in static network: We used formula (19) in our Monte Carlo simulations to determine the connectivity probability. We constructed 1000 random networks uniformly. Then the connectivity of every network was investigated and the fraction of connected networks was determined. In the static case, link probability was determined for different numbers of nodes (5, 10 and 50). The results show that as the number of nodes increases, the probability of connection increases for the same d_R. The connectivity probability for normally distributed network with different number of nodes using different measures are shown in Fig. 9.

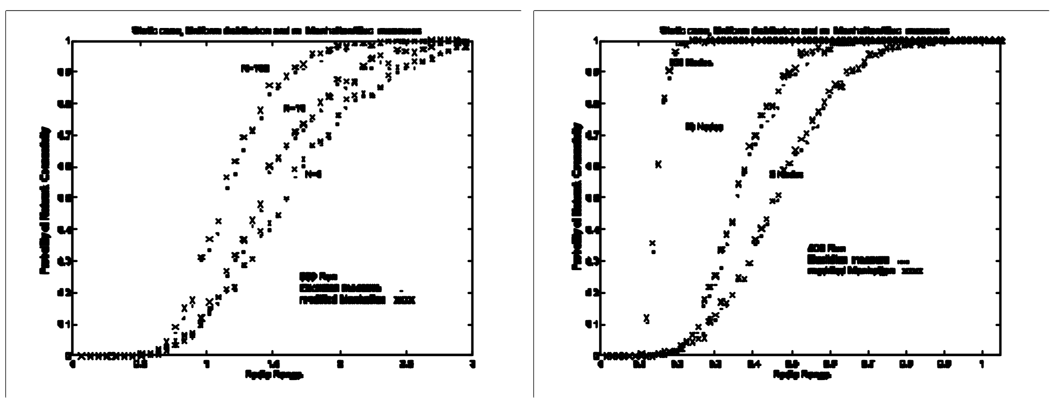
The connectivity probability for uniformly distributed network with different number of nodes using different measures is shown in Fig. 10.

The connectivity probability for uniformly and normally distributed networks with different number of nodes using Manhattan approximate measure are shown in Fig. 11.



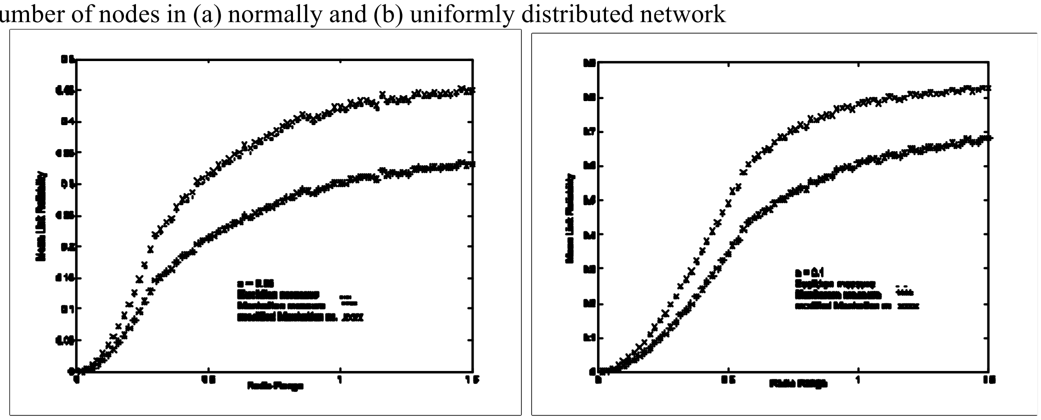
(a) (b)

Fig. 11: Connectivity probability using Manhattan measure for (a) normally and (b) uniformly distributed networks



(a) (b)

Fig. 12: Comparison of connectivity probability using modified Manhattan and Euclidian measures for various number of nodes in (a) normally and (b) uniformly distributed network



(a) (b)

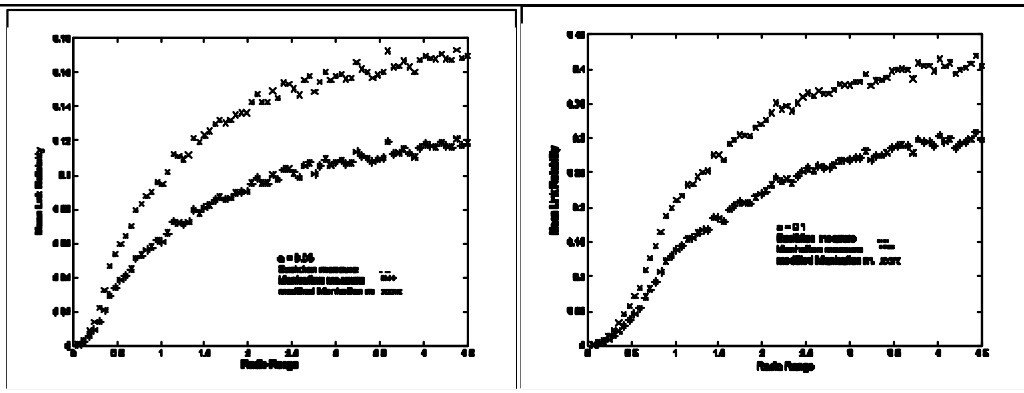
Fig. 13: Mean reliability of links for uniform spatial distribution of nodes using different methods for (a) a = 0.05 and (b) a = 0.1

Also, the connectivity probability was calculated in networks using Euclidian and modified Manhattan measures. Figure 12 shows the comparison of connectivity probability for different node numbers for these methods. The network connectivity probability increases for any given R when there are more nodes as shown in the figure.

The performance of modified Manhattan measure is very close to Euclidian measure and much better than Manhattan measure based on an analysis of the rms errors of the two approximate methods. The rms errors for different measures and different conditions are summarized in Table 4.

Table 4: Rms error for calculating connectivity probability for different parameters and methods

Distribution of Nodes	Number of nodes	rms error for manhattan measure	rms error for modified manhattan measure	Fractional reduction of rms error
Uniform	5	0.1592	0.0125	12.74
	10	0.1398	0.0123	12.57
	50	0.1720	0.0137	12.55
	100	0.1105	0.0123	8.98
Normal	5	0.1577	0.0078	20.22
	10	0.1696	0.0087	19.49
	50	0.1793	0.0087	20.61
	100	0.1154	0.0134	8.61



(a) (b)

Fig. 14: Mean reliability of links for normal spatial distribution of nodes using different methods for (a) a = 0.05 and (b) a = 0.1

Link reliability in static case: For different measures and different distribution of nodes the mean link reliability of the network was evaluated. In our simulations we selected D = 5.9 for uniform distribution and D = 9.1 for normal distribution of nodes.

For static condition the variation of mean probability of links versus d_R using different methods for a = 0.01, 0.03, 0.05 and 0.1 was investigated. To implement our modified Manhattan method, the parameters D and d_R in equations (9) and (10) were multiplied by modification factor. Figure 13 shows the mean link reliability for uniformly distributed network for two different a. Obviously as a decreases the mean reliability of links decreases (a and R on the figures denote a and d_R).

For small values of d_R, according to a and D the link reliability is approximately constant. For larger values of d_R the mean link reliability increases by increasing the d_R.

The mean reliability of links for normally distributed network was determined using different methods. The results for a = 0.03 and 0.05 are shown in Fig. 14. The mean link reliability behaves similar to last case.

In Table 5 the results are summarized for different a and different distribution of nodes. The rms error of calculations using different methods are compared and the reduction of rms error using modified Manhattan method is introduced.

As shown for d_R = 0.7 in uniform distribution of nodes, the mean reliability is equal 0.68 for a = 0.1. It reduces to about 1/2 for a = 0.05 and to about 1/3 for a = 0.03. Also for d_R = 2.5 in normal distribution of nodes, the mean reliability is equal 0.8 for ideal condition (a=1). As a decreases the reliability decreases for the same d_R and for a = 0.1 it is equal 0.36.

Monte carlo simulations in dynamic case: In previous section a modification factor for R was introduced to improve the accuracy of approximate Manhattan measure. We name this method as modified Manhattan in the remainder of the paper. For simulation of dynamic network, boundless simulation area mobility model was used. In order to carry out the Monte Carlo simulations in dynamic case our experiments were set up with nodes with specification (x, y, v, θ) in which v is the velocity and θ is the angle of movement. Velocity v is the distance by which a node moves in θ direction in every step of the calculations. Parameters v and θ are

Table 5: Rms error for calculating mean reliability of links for different parameters and methods

Distribution of nodes	a	R_L	rms error for modified Manhattan measure	rms error for Manhattan measure	Fractional reduction of. rms error
Uniform		R_L for d_R = 0.7			
	0.10	0.68	0.0049	0.1436	29.03
	0.05	0.37	0.0054	0.0978	18.11
	0.03	0.21	0.0038	0.0587	15.45
	0.01	0.004	0.0009	0.0134	14.89
Normal		R_L for d_R = 2.5			
	0.10	0.36	0.0008	0.0934	116.80
	0.05	0.15	0.0005	0.0431	86.20
	0.03	0.07	0.0003	0.0217	72.33
	0.01	0.01	0.0001	0.0039	39.00

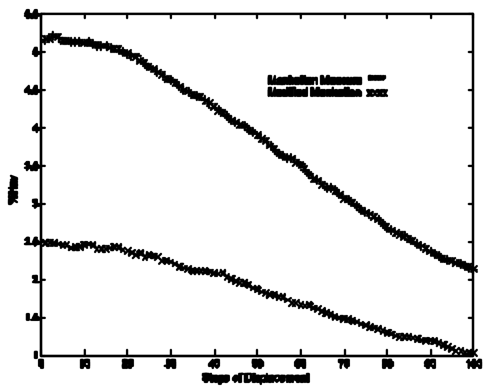


Fig. 15: Comparison of %H distribution for traditional Manhattan method and modified Manhattan method

determined based on uniform distribution because only the number (or percentage) of low speed nodes is important. In other words the time consumption of our algorithm is based on the number of low speed agents and the type of their distribution does not affect the results. In order to generate θ, the random number $\tilde{\theta}$ which is limited between [0,1], is multiplied by 2p. Also in order to limit the velocity of each node, $\tilde{v}$ which is similar to $\tilde{\theta}$ is multiplied by a velocity coefficient, C_v which is less than one. This means that the velocity, v, of every node is the result of multiplication of $\tilde{v}$, C_v and the size of the square region in which the nodes are distributed. In every step, nodes move to a new location based on their velocity and direction and their positions are considered to be fixed during the calculations. To speed up the calculations low speed nodes can be considered to be stationary as considered by Peiravi and Tolooei in [22].

Connectivity graph in dynamic network: Figure 15 shows %H for traditional Manhattan method and modified Manhattan measure in dynamic network. As a realistic example, maximum v (or C_v) was considered 0.01 km per step of calculations and R=0.25 km (as considered by Barrett *et al*. in [10]).

As shown, the accuracy of our modified Manhattan is better than traditional Manhattan method. As the number of steps increases the number of the links decreases, therefore dissimilarity between results of two approximate method decreases.

Computational time for implementing different methods for 100 step movement and computation was measured. Computational time for Euclidian method was 109945 ms while for Manhattan-based approximate methods was 28945 ms. This shows that using our proposed method the speed of calculations improves 3.8 times relative to Euclidian method.

Connectivity probability in dynamic network: Connectivity probability of a mobile ad hoc network using boundless simulation area mobility model was determined for uniformly distributed networks including different number of nodes. Calculations were performed during 20 step of motion. After every step the mobile agents have a pause. The maximum velocity of agents was considered 0. of the dimension of square region in which the nodes were distributed originally, per step.

In Fig. 16 for different number of nodes and different measures connectivity probability of mobile networks are shown.

As shown, as the number of nodes increases, the connectivity probability increases for the same radio range. In Table 6 the rms error of different measures are shown.

As shown using our method the reduction of rms error is very appreciable relative to traditional Manhattan method.

Table 7 shows the computational time of connectivity probability for different methods.

Table 6: Rms error for calculating connectivity probability for different parameters and methods using boundless area mobility model for uniform distribution of nodes

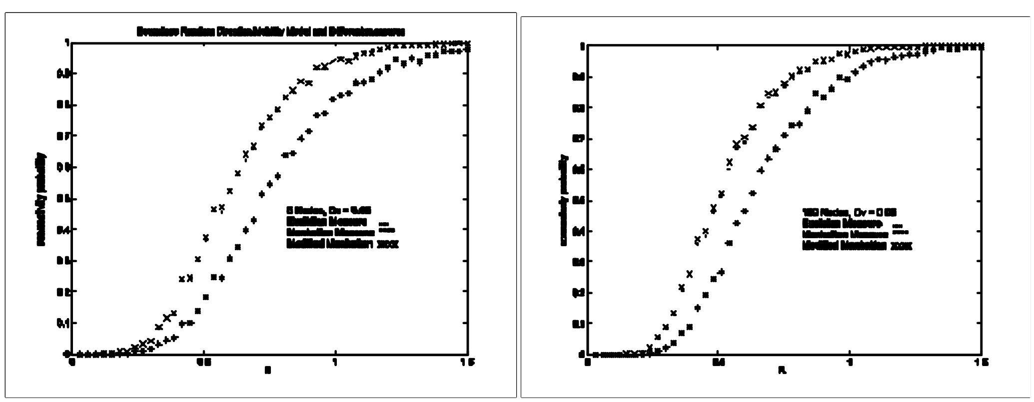
Number of nodes	rms error for Manhattan measure	rms error for modified Manhattan measure	Fractional reduction of rms error
5	0.1260	0.0053	23.77
10	0.1275	0.0049	26.02
50	0.1288	0.0045	28.62
100	0.1281	0.0050	25.62

Table 7: The computational time for connectivity probability for different methods

Number of nodes	Computational time for euclidian measure (ms)	Computational time for modified manhattan measure(ms)	Fractional reduction of computational time
200	16833	9778	1.72
500	231666	155000	1.49

Table 7: Comparison of rms error for different methods

a	R	rms error for Manhattan measure	rms error for modified Manhattan measure	Fractional reduction of rms error
0.05	0.3	0.0374	0.0024	15.58
	0.5	0.0512	0.0032	16.00
	0.7	0.0599	0.0031	19.32
	1.0	0.0672	0.0027	24.89
0.1	0.3	0.0975	0.0042	23.21
	0.5	0.0997	0.0035	28.49
	0.7	0.1055	0.0036	29.31
	1.0	0.1164	0.0033	35.27



(a) (b)

Fig. 16: Comparison of connectivity probability using boundless simulation area mobility model for two different node numbers in (a) 100 and (b) 5 nodes uniformly distributed network

As shown, when the number of the nodes increases 2.5 times, the computational time increases 13.76 times for Euclidian method and 15.85 times for both approximate methods. Therefore, the reduction of computational time will be very effective.

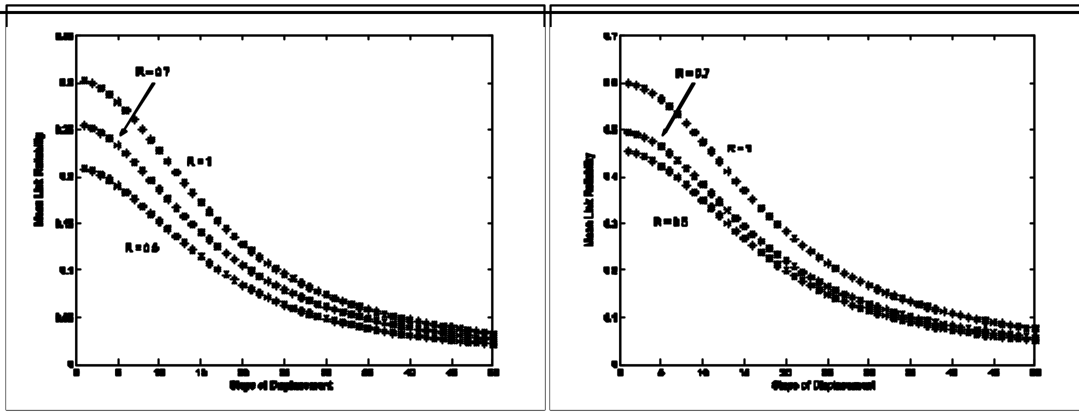
Link reliability in dynamic network: In this case stochastic reliability model is FS-based and mobility model is Boundless Simulation Area mobility model. Fig. 17 shows mean link reliability using Manhattan approximate method for simulation of 50 steps of motion.

Maximum available link reliability depends on a and R. Link reliability increases as d_R or a increase.

Figure 18 shows mean link reliability for uniformly distributed networks including moving agents using

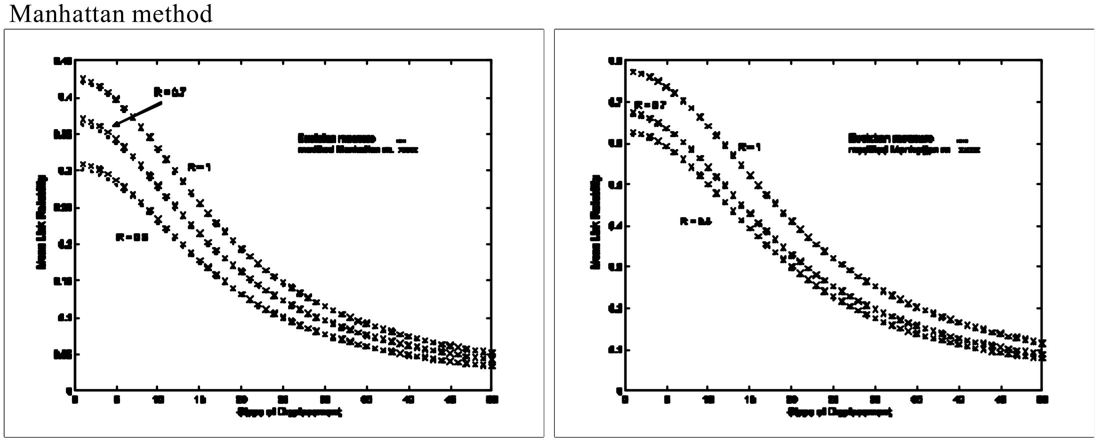
Table 8: The computational time for link reliability for different methods

Number of nodes	Computational time using Euclidian measure(ms)	Computational time using Manhattan measure(ms)	Computational time using modified Manhattan measure(ms)	Fractional reduction of computational Time
1000	166.7	55.6	55.6	3



(a) (b)

Fig. 17: Mean link reliability for uniform distribution and FS model (Cv=0.03), for (a) a = 0.05 and (b) a = 0.1 using Manhattan method



(a) (b)

Fig. 18: Mean link reliability for uniform distribution and FS model (Cv=0.03), for (a) a = 0.05 and (b) a = 0.1 using Euclidian and modified Manhattan methods

Euclidian and our modified Manhattan methods. Both approximate methods require less computational cost relative to the real method. Obviously modified Manhattan measure shows better accuracy relative to traditional Manhattan measure. Using these plots, one can decide different conditions to provide desired reliability.

The rms error of approximate methods and fractional reduction of rms error for different parameters are shown in Table 7. The modified Manhattan method improves the accuracy appreciably relative to the traditional Manhattan method.

From the comparison of computational time for different methods shown in Table 8 it can be see that the modified Manhattan method improves the accuracy appreciably relative to traditional Manhattan method.

RESULTS AND CONCLUSIONS

In this paper a new approximate but fast method for computing connectivity-based methods for static and dynamic networks has been presented. Based on connectivity graph computations, a modification factor is introduced to increase accuracy of approximate Manhattan method. Connectivity graph computations for a normally distributed and a uniformly distributed network each with 1000 nodes have been compared. Three different methods were used for approximating connectivity: real method using Euclidean distance, traditional approximate method using Manhattan measure and our modified Manhattan measure using modified connectivity range, R. The results were compared with the real method to show the effectiveness of our proposed modification factor for Manhattan measure to reduce the approximation error.

Percentage Hamming distance (%H), DDSRR and MDDVRR are used as graph comparison metric and measures. Also computational time and accuracy of methods are used to compare the performance of the methods. In static case, using modified Manhattan method for different distributions, %H decreases more than 4.5 times relative to traditional Manhattan method.

In dynamic case, the dissimilarity decreases to 0.5 relative to traditional Manhattan, while the speed of calculations increases by 3.8 relative to Euclidian method.

Computation of connectivity probability using traditional Manhattan measure and modified Manhattan measure in static and dynamic networks for different conditions was investigated. In static case, using our method %H decreases at least 8.6 times for different conditions relative to traditional Manhattan measure. In dynamic case, using our method, speed of computations increases by about 1.49 for a network including 500 nodes relative to the Euclidian method.

Also computation of propagation-based reliability model, FS model, using traditional Manhattan measure and modified Manhattan measure in static and dynamic networks for different conditions was investigated. In static case, using our method rms error decreases at least 14.89 times for different conditions relative to traditional Manhattan measure. In dynamic case the speed of computations increases by about 3 relative to Euclidian method when using our method.

Our proposed measure requires less calculations with respect to Euclidean measure while it shows better accuracy with respect to traditional Manhattan method. The computational time results reported here have been obtained using Turbo C++ programming on an Intel Pentium D 3.00 GHz PC. Other results obtained using MATLAB programming.

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