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Estimation of Expected Lifetime and Reliability During Burn in and Field Operation Using Markov Chain Monte Carlo Simulations

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Abstract: Estimation of the mean lifetime and reliability of sophisticated systems is a challenging problem in many engineering applications. This is especially important when the study is concerned with both burn in and field operation periods since the hazard rate is no longer constant and the underlying processes are non-homogenous. In such cases, theoretical development of the solution is very tedious and obtaining results for the expected lifetime and reliability of complex systems is almost impossible. Monte Carlo simulations provide a viable alternative for estimation of the expected lifetime and reliability in such situations. Predictive calculations for the mean time to failures may be carried out using MIL-HDBK-217F for expected operating conditions of the system. However, these estimates are only valid for the field operating conditions assuming that the parts lifetime obeys an exponential probability distribution.

Key words: Monte carlo simulations . burn-in . expected lifetime . reliability . mean time to failure . redundancy

INTRODUCTION

Reliability and expected lifetime are very important issues in modern complex systems, especially in military applications, life-critical applications and high reliability industrial and consumer products. The reliability of a system is affected by the reliability of its components and the way they are interconnected to serve its intended mission under certain operating conditions. Systems have become so complex that the study of their reliability requires extensive modeling and simulation. Moreover, the various portions of the life of any system pose different hazards to the system which makes a thorough study really difficult.

Monte Carlo simulations are very useful in the study of complex systems and many researchers have used them in order to simulate complex, nonstandard multivariate distributions. There are many reliability problems in which sophistication leads us to rely on the results of Monte Carlo simulations. For example, in studying the importance of components, theoretical approaches lead to NP-difficulty. Pan and Tai [1] presented an algorithm to compute variance importance which is a measure of uncertainty importance for system components. A simple equation was derived for this measure and Monte Carlo simulations were used to obtain numerical estimates for a simplified fault tree for a reactor protection system as an example. Their suggested algorithm overcomes non-polynomial difficulty which existed in earlier methods for computing uncertainty importance. Moreover, it was simpler, more accurate and more practical and showed the direct relationship between probabilistic importance and uncertainty importance.

Two notable examples of Markov chain Monte Carlo are the Metropolis-Hastings and Gibbs sampling which are easy methods for the generation of random samples and estimates. The Metropolis-Hastings algorithm which was proposed by Hastings [2] to sample from complicated, high-dimensional probability distributions is more general.

Chib and Greenburg [3] presented a detailed exposition of the Metropolis-Hastings algorithm to simulate multivariate distributions along with derivation and applications.

Gibbs sampling is an especial case of Metropolis-Hastings algorithm on an element by element basis and is explained in full by Casella and George [4]. The Gibbs sampler is a popular Markov chain Monte Carlo routine for generating random variates from distributions otherwise difficult to sample. Therefore, the Gibbs sampling is very useful in specific applications. There are still certain simplifying assumptions needed when either one is used.

Another issue which makes the study of the reliability of complex systems difficult is redundancy

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especially when we deal with non-homogenous processes. Lewis *et al*. [5] presented a Monte Carlo methodology for the reliability simulation of highly redundant systems where two forms of importance sampling, forced transitions and failure biasing were used to effectively simulate large sets of continuous-time Markov equations with the results plotted as continuous functions of time. Lewis *et al*. [5] showed that a modification of the sampling technique allows the simulation of both non-homogeneous Markov processes and non-Markovian processes involving the replacement of worn parts. They examined a few benchmark problems and showed that for problems with large numbers of components, Monte Carlo simulations result in less computing time by as much as a factor of twenty from the Runge-Kutta Markov solver employed in the NASA code HARP.

Existing software for reliability calculations are limited in scope, hard to use and not applicable to burn in period. Therefore, Monte Carlo simulations are used in this study to estimate the expected lifetime and reliability during burn-in and in the field operation. The present research was carried out using Borland C++ programming language on an IBM PC computer.

RELIABILITY AND HAZARD RATE

Reliability is the probability of successful operation during the mission and under pre-specified conditions and can be calculated using various techniques including RBD, Markov state space, analytical, or Monte Carlo Simulations. In this research the Weibull probability distribution function is used to describe the lifetime of a component since the hazard rate is not constant during burn-in and in the filed operation periods of life as can be seen from the bathtub curve of Fig. 1. The hazard rate is a decreasing function of time during the initial period of life, or the burn-in period. Then it flattens out and is nearly constant for the useful period of life of the part. Naturally, this period should be the operation period of the part, if it has gone through its initial burn-in period before. In some industries, this is not the case. They do very little testing and burn-in and let the part experience the initial period of its life in field operation. The high hazard rate during this period implies that there may be many early failures in the field. However, certain warranties are provided for customers so that they may return the failed product and have it repaired at no cost or with minimal expenses. This is naturally not a good option for military, life-critical or high reliability products. The last stage of life as shown in Fig. 1 is the wearout period during which the hazard rate is increasing with time. Some systems such as power plants exist in many

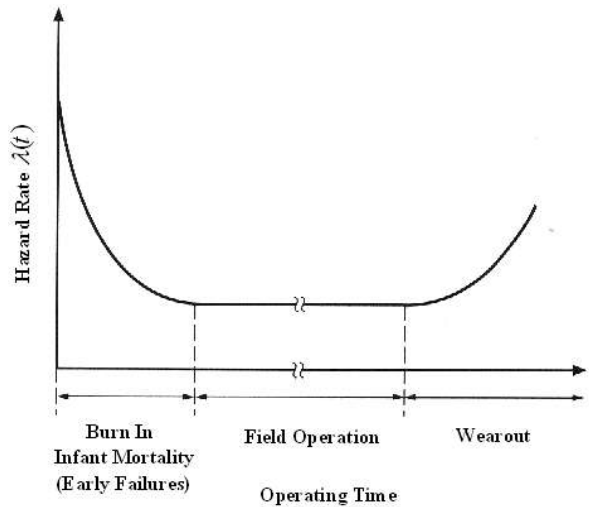

Fig. 1: The bath tub curve for the hazard rate

countries and are still operational in this period of life due to economic reasons. The way to deal with the increasing hazard rate is through planned maintenance which helps postpone the increasing trend in the hazard rate.

The failure rates for the parts which make up the electronic circuits can also be either estimated based on the generic rates of MIL-HDBK-217F [6] or other data sources such as Telcordia [7] model. A comparison of the calculated results of estimates using these two models shows that the Telcordia (Bellcore) calculations are more optimistic than the MIL-HDBK-217 calculations. There are various reasons for the variations between models and this also depends on the particular component and its associated stress factors. The advantage of the Telcordia model is that it provides the capability of considering burn-in data, laboratory test data and field data in the failure rate calculation. Burn-in data is used to determine the first year multiplier, which is an indication of infant mortality. The disadvantage of the Telcordia model is that the operating environments it supports are limited. In addition to three different Ground-based environments, it only supports an Airborne, Commercial environment and a Space, Commercial environment, while MIL-HDBK-217 supports many different types of ground, sea, air and space environments. Moreover, Telcordia uses four standard quality levels which are the same for all components while MIL-HDBK-217 uses quality levels that differ from one part type to another.

The hazard rate for the Weibull probability distribution function may be described as in (1)

$$\lambda(t) = \lambda_1 t^{-\alpha} \qquad (1)$$

where $\lambda_1>0$, λ_1, is the scale parameter and α is the shape parameter. The reliability function is described as in (2)

$$R(t) = e^{\frac{-\lambda_1 t^{1-\alpha}}{1-\alpha}} \tag{2}$$

This function may be used to describe the lifetime and reliability in burn-in, field operation and wearout periods of life. When ($0<\alpha<1$), the hazard rate is a decreasing function of time and may be used in the burn-in period of life. When ($\alpha = 0$), the hazard rate is constant and the reliability function exactly follows the exponential probability distribution function. This may be used to model the field operating life of the component.

THE PROCEDURE FOR MONTE CARLO SIMULATIONS

Monte Carlo simulations are simple to do. The only catch is to get a clear understanding of how to perform them. The usual case is that we have a non-uniform distribution function $f_T(t)$ for the stochastic variable of interest T, say lifetime in reliability studies. We use the cumulative distribution function $F_T(t)$ which is bound to be between zero and one. Thus we first generate uniformly distributed random numbers between zero and one using (3)

$$U = F_T(t) \tag{3}$$

Then we may use the inverse cumulative function to calculate t using (4) as shown in Fig. 2.

$$t = F_T^{-1}(u) \tag{4}$$

This may be done for any component with any assumed probability distribution function for its lifetime. To perform the reliability study for the system, we must follow the following procedure:

- Estimate the cumulative probability distribution of each component in the system.
- Generate a uniformly distributed random number between zero and one for each of the parts in the system and use the inverse of the cumulative distribution function as shown in Fig. 2 to estimate time to failure data for each part in the system.
- Find the time to failure of the system using the failure time data generated for the components of the system. In systems whose reliability block diagram is such that the components are in series, then the time to failure for the system is the same as the time to failure for the weakest component-that is the minimum of the time to failures of the components in the system. On the other hand, if the components in the system are redundant so that they are all in parallel, then the time to failure for the system is the maximum of the time to failures of the components which make up the system. In other systems, the reliability function must be developed to relate the reliability of the parts to the reliability of the system.
- Repeat the procedure outlines in steps 1 to 3 over and over again in order to reduce the error in the estimate by averaging the results. This repetition should be carried out until the changes in the averaged result become less than an accepted amount. If n denotes the number of times for which the simulation is carried out and the results are averaged, then the mean time to failure for the system averaged up until the nth step would be as shown in (5)

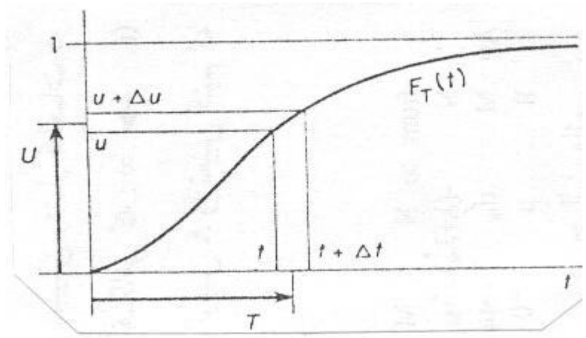

Fig. 2: The random variables T with $F_T(t)$ and uniform U for the generation of data for Monte Carlo simulation

$$MTTF(n) = \frac{\sum_{k=1}^{n} TTF(k)}{n} \tag{5}$$

where TTF(k) denotes the time to failure for the system at the kth step of the simulations.

When n approaches infinity, the MTTF thus estimated would approach a constant value. Thus the system MTTF is given as (6)

$$MTTF = \lim_{n\to\infty} MTTF(n) \tag{6}$$

THE SYSTEM UNDER STUDY

The system under study is a high reliability consumer product. It was thoroughly investigated down

Fig. 3: The reliability block diagram of the system under study showing its various subsystems

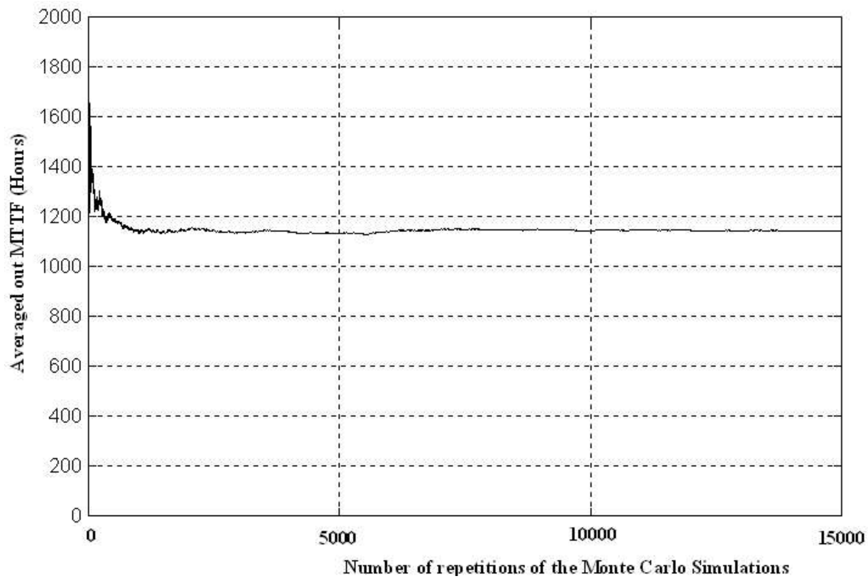

Fig. 4: Averaged out MTTF (hours) results from Monte Carlo simulation runs

to part level in terms of the effect of each part and each subsystem on the operation of the overall system. The system consists of seven subsystems which are comprised of a variety of types of mechanical, electrical and electronic components. The reliability block diagram of the system is shown in Fig. 3 where the series role of the subsystems in the reliability of the system is shown. This is expected of high-reliability consumer products where no extra money is available to implement redundancy. In such products, improved reliability should be built into the system by integration of parts as shown by Peiravi [8], or derating of parts, good manufacturing procedures and proper highly accelerated lifetime testing.as shown by Peiravi and Dehghanmongabadi, N [9].

THE MTTF OF THE SYSTEM FOR ITS FIELD OPERATION PERIOD OF LIFE

The mean time to failure (MTTF) of the system may be found by using the relevant MTTFs of its various subsystems and the reliability block diagram or the reliability model of the system versus its subsystems. To estimate the failure rate for the system during the field operation life, the RBD technique along with failure rate data from MIL-HDBK-217F may be used. For example, the general failure rate for a resistor is as shown in (7)

$$\lambda_p = \lambda_b \pi_T \pi_p \pi_S \pi_Q \pi_E \quad (7)$$

where λ_b is the base failure rate, π_T is the temperature factor, π_p is the power factor, π_S is the power stress factor, π_Q is the quality factor and π_E is the environment factor. There is a similar relationship for other devices with appropriate factors to include stresses and operating environment. Theses calculations were carried out using an excel spreadsheet for all the subsystems and the overall failure rates of the subsystems are shown in Table 1. The failure rates of the parts during burn-in are much higher than their equivalent values in the useful period of life. The probability distribution function of life is also not a simple one. The mean time to failure for the subsystems were estimated using Monte Carlo simulations as outlined above and are tabulated in Table 1.

The mean time to failure of the system during its burn in period (MTTFB) may be estimated by running the simulation once. However, this result would not have a high confidence bound. To get a better estimate, we run the experimental simulation over and over again. Averaging the MTTFB over many runs in Monte Carlo simulations is performed in order to reach an estimate of MTTFB which has a very high confidence bound as shown in Fig. 4. As it can be seen the estimate of MTTFB is very rough in the beginning, but reaches a steady state only after nearly 5000 runs of the Monte

Table 1: Hazard rate and mean time to failure in the burn in period and in the field operation

Ratio of Failure Rate in the Burn in Period to the Failure Rate in the Field Operation	Mean time to fail in the field operation MTTF (Hours)	Mean time to fail in the burn in period-MTTFB (Hours)	Subsystem Hazard Rate in Failures per Million Hours	Subsystem Name	Subsystem
16.908	23635.07	1397.82	42.310	Display Panel	1
33.668	1556.08	46.218	642.640	Drive Panel	2
9.2850	11842.31	1275.40	84.443	Power Supply	3
9.5730	5666.30	591.91	176.482	Control Unit	4
14.797	34482.76	2330.42	28.999	Case	5
33.565	65573.77	1953.62	15.250	Terminals	6
19.337	1009.98	52.53	990.124	Power Distribution	7

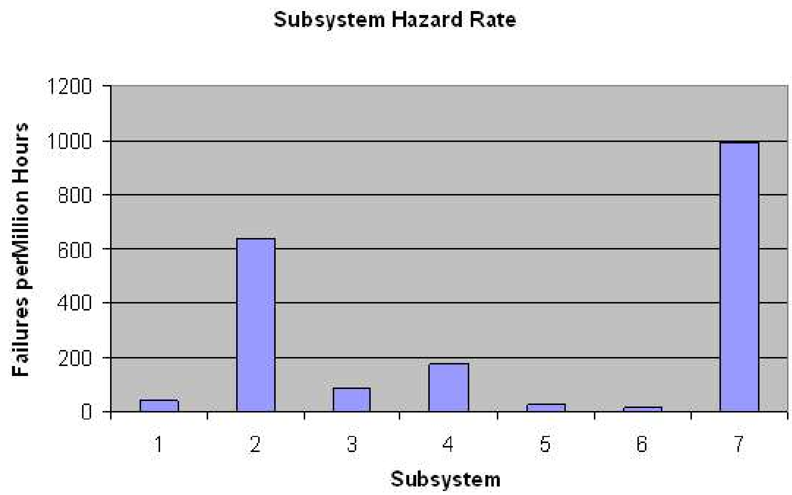

Fig. 5: Subsystem Hazard Rate in Failures per Million Hours

Carlo simulations. This average value given in (5) approaches the true value with a very high confidence bound after 15000 runs of the experimental simulation as shown in Fig. 4.

The mean time to failure in the field operation may be easily obtained from the hazard rate and the results are shown in Table 1.

The last column of Table 1 shows the ratio of the failure rate in the burn in period to the failure rate in the field operation. The results are plotted in the form of a histogram in Fig. 5-7 that show us which subsystems have a higher failure rate and need reliability improvement.

The failure rates of the parts during burn-in are much higher than their equivalent values in the field operation period of life. The probability distribution function of life is also not a simple one during burn in. The mean time to failure for the subsystems were estimated using Monte Carlo simulations as outlined above and the results are shown in Fig. 6.

Using the estimated failure rates for the field operation obtained from the MIL-HDBK-217F and shown in Table 1, the mean time to failure in the field operation is found as shown in Fig. 7.

SYSTEM FAILURE RATE AND SYSTEM RELIABILITY FUNCTION

The failure rates tabulated above and the RBD of the system shown in Fig. 3 may be used to compute the reliability and cumulative unreliability functions for the system as a function of time. Since the subsystems act in a series fashion in the RBD model, the failure rates may be added together to form the system failure rate as in (8)

$$\lambda_{System} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \quad (8)$$

Then the system reliability may be computed as in (9)

$$R_s = e^{-\lambda_{System} t} \quad (9)$$

And the Cumulative Unreliability function may be computed using (10)

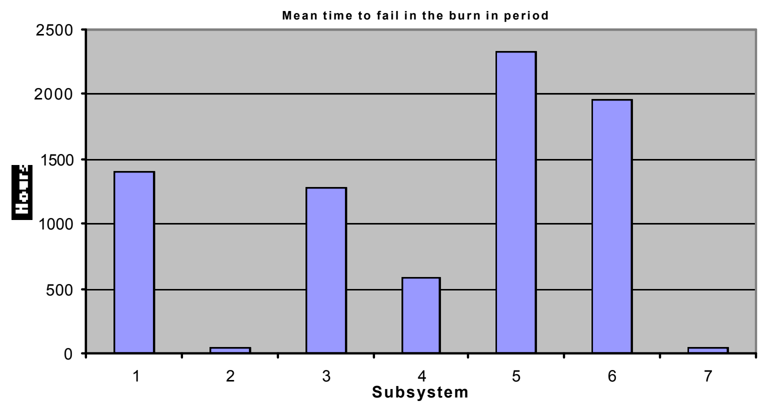

Fig. 6: Mean time to failure of the various subsystems in the burn in period

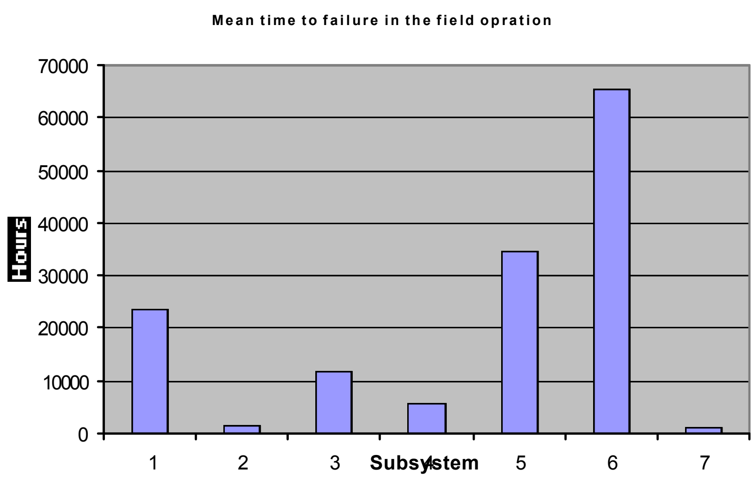

Fig. 7: Mean time to failure of the various subsystems in the field operation

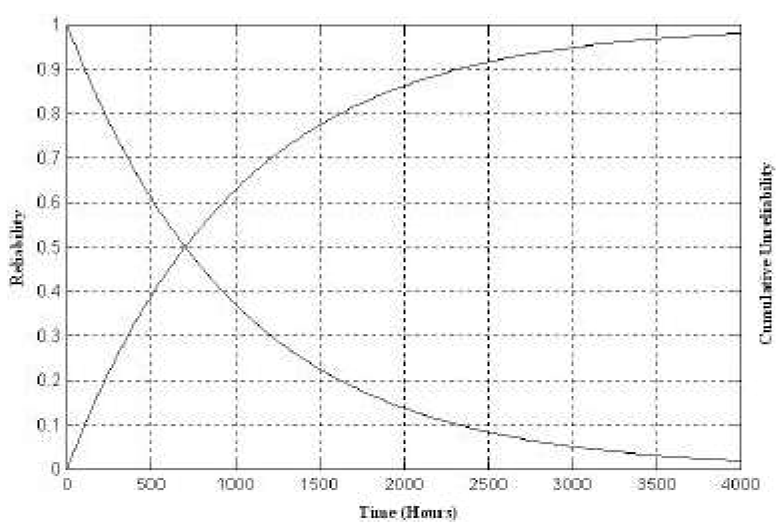

Fig. 8: The reliability and cumulative unreliability function vs. time

$$Q_{System}(t) = 1 - R_{System}(t) \quad (10)$$

The reliability and the cumulative unreliability functions thus computed vs. time are shown in Fig. 8.

DISCUSSION OF RESULTS

There is an important result that we may conclude from the last column of Table 1 in which the ratio of the failure rate of each subsystem in the burn-in period over its failure is in the field operation are tabulated. Note that the highest ratio is nearly 33 and that is for the terminals and drive panel. These two subsystems contain more mechanical parts and wiring than the rest of the system. The next highest ratio is nearly 14 to 19 for the case, the display panel and the power distribution subsystems which contain mechanical parts, fuses, keys, switches and relays. Then the power supply and the control unit which mostly contain power electronic and electronic parts have the lowest ratio of nearly 9. This comparison effectively shows that the nature of the difference between the subsystems also affects their mean lifetime in the burn in period and their expected lifetime in the field. The system MTTF is also obtainable from Fig. 7 and is somewhat less than 1000 hours.

CONCLUSIONS

In this paper, the estimation of the mean time to failure and system reliability for the burn-in and the useful operating period of life were both discussed. It was shown that Monte Carlo simulations can be effectively used in order to estimate the lifetime and reliability in such cases where the non-homogeneous Markovian stochastic nature of the time to failure of the parts makes a theoretical approach almost impossible. Averaging of results after repetitions of the Monte Carlo simulations was used and it was shown that after nearly 15000 iterations, results with a very high confidence bound are obtained. The MTTF of the system was obtained from the reliability plot to be somewhat less than 1000 Hours.

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