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# An Inventory Discount Model under JIT

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**Abstract:** In this study, we focus on a multi-stage supply chain system that operates under a JIT (just-in-time) delivery policy. *Kanbans* play an important role in the information and material flows in a supply chain system. Thus, a *kanban* mechanism is employed to assist in linking different production processes in a supply chain system to implement the scope of JIT philosophy. We develop an optimal model for a multi-stage supply chain system under just-in-time philosophy. In this mode, the significant features of developed model are: 1-We add a retailer to the multi-stage supply chain system. 2-Stage N offers quantity discounts to encourage the retailer to order more and the producer intends to discount the unit production cost if the amount of production is large. 3-We transform our developed model into a more concise version by applying an unequal demand rates for all stages. 4-The most important development of presented model is that the inventory cost of the semi-finished parts shipped to a plant from the preceding stage is considered. Finally the number of kanbans, the batch size, the number of batches and the total quantity over one period are determined optimally.

**Key words:** Supply chain . just-in-time . Kanban . inventory control

## INTRODUCTION

Supply Chain Management (SCM) has been one of the most important and widely discussed topics in manufacturing research over the last ten years. A supply chain is a set of facilities, supplies, customers, products and methods of controlling inventory, purchasing and distribution. The chain links suppliers and customers, beginning with the production of raw material by a supplier and ending with the consumption of a product by the customer. In a supply chain, the flow of goods between a supplier and customer passes through several stages and each stage may consist of many facilities [1].

Such activities are mainly the procurement of materials, the transformation of these materials into intermediate and finished product and the distribution of finished products to the endcustomer. Supply chain management is concerned with the integrated management of the flows of goods and information throughout the supply chain, so as to insure that the right goods be delivered in the right place and quantity at the right time. The SCM literature covers different areas, such as forecasting, procurement, production, distribution, inventory, transportation and customer service, under several perspectives, i.e. strategic, tactical and operational. Supply chain inventory management (SCIM), which is the main concern of this paper, is an integrated approach to the planning and control of inventory throughout the entire network of co-operating organizations, from the source of supply to the end user. SCIM is focused on the ultimate customer demand and aims at improving customer service, increasing product variety and lowering costs [2].

There may be significant material flows (the work-in-process formed by the semi-products) between two consecutive manufacturing facilities (workstations, shops, plants). The plant has different meanings in different context and so does the *kanban*. *Kanban* is a Japanese word for 'card', which practically controls the flow of containers with materials. While the word *kanban* refers to card (or correspondingly the container) in Japanese language, the reference points of the kanban travel, in this research, are assumed as plants. So these plants, or the reference points of kanban travel, in general, could be companies, plants, workshops, workstations, or machines, depending on its context. Also, a kanban can be considered as an AGV (automated guided vehicle), cart, tote, truck, ship, train, etc., depending on the situations.

The just-in-time (JIT) management allows the organization to achieve this goal by increasing the efficiency of the production, reducing the level of wasted materials, time and effort involved in the production process. To implement the JIT philosophy, a kanban technique is introduced as an efficient

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operational mechanism. The improvements in reduction of inventory and wasted labor and enhancement of customer service are usually accomplished through kanban operations. Thus, a kanban mechanism ensures the organizations to run their supply chain systems in JIT policy.

In a kanban-controlled system, production is triggered by the demand at the final stage. At each intermediate stage, it is triggered by its succeeding stage(s)-this process is carried all the way from the final stage to the beginning stage. Thus, the production is controlled (i.e., pulled) by demand of the succeeding stage, the customer of the preceding stage. Thus, this approach leads itself to a very simple control mechanism, best known as kanban (or pull) system.

A transportation system with a kanban mechanism for material transfer in a supply chain must have at least two plants. The material flow and information flow between two adjacent plants form a kanban stage. If a supply chain system consists of only two plants, it is called Single-stage Supply Chain System (SSSCS). If it has more than two kanban stages in series, it is a Multi-stage Supply Chain System (MSSCS).

The goal of this research is to efficiently control the stocks and transportation mechanism in a supply chain system that is operated by a kanban mechanism to achieve the purpose of JIT philosophy, the minimal inventory. It is aimed to increase the efficiency of the production process and reduce the level of wasted materials, time and effort involved in each production stage. The supply chain system considered in this paper consists of three parts: the suppliers, manufacturers and retailers. The general objective of this research is to build an economically viable, efficient logistics system with kanban mechanism. The deliveries of raw materials from the suppliers, the work-in-process (WIP) in production stage and the transshipments of finished goods to retailers are all controlled by the kanbans. A multi-stage supply chain system is to be studied with respect to the kanban mechanism. The number of kanbans and the batch size are to be determined under different system constraints. An economically optimal logistics model for controlling the supply chain system is developed here [3].

## LITERATURE REVIEW

Since a supply chain deals with material flows and information flows across the entire chain, from suppliers of original components to final customers, it comprises at least two major domains: the physical transformation domain (mining, smelting, casting, alloying, machining, assembling; etc.) and the goods distribution domain (conveyance, storage and transportation). The physical transformation domain is formed by several manufacturing enterprises that generate goods through a series of processes provided by different firms. In recent years, in this domain, the ‘‘just-in-time’’ (JIT) principle has been adopted as a supply mechanism in many firms in actual supply chains [4-8]. Originally, the JIT philosophy was developed by the Toyota Motor Corporation through the kanban control for the objective of minimizing inventories. Since the mid-1980's it has become one of the principal methods used as an internal production management system within a single manufacturer. Therefore a great deal of the research in JIT production systems treats lead times for internal (to the firm) supplies as controllable; usually they are assumed to be constant or even zero [9-12]. If we extend the JIT principle as an intra-firm supply mechanism in a supply chain, the lead times become a major factor of concern and the assumption of constant or zero supply lead times becomes no longer tenable. Therefore we contend that lead times should be treated as random variables. So far, to the authors' knowledge, few papers have treated stochastic lead times for JIT supply mechanisms [13] who assume a discrete probability distribution and [5] who consider a uniform distribution. In the goods distribution domain, goods are moved from warehouses to distributors, from distributors to retailers and finally from retailers to customers. The focus of research in this area is inventory management, which is somewhat different from production management.

A kanban technique attracted many researchers since it was first brought to light by [14]. He originally summarized the Toyota approach for determining the appropriate number of kanbans at a workstation. It is applied recently in supply chain systems to efficiently manage the flow of materials [15]. Extended the Toyota approach to fluctuating product-mix problem by using the next period's forecast demand and the last period's observed lead times [15]. Considered the over-planning factor in Toyota's formula for computing the number of kanbans for several production inventory control models [17]. Studied a multi-stage pull system that dealt with production inventory system [18]. Introduced a systematic methodology to manipulate the number of kanbans in a JIT system, where an algorithm to minimize the backlog and WIP was developed for stochastic processing times and variable demand environment [9]. Derived two formulae to calculate the average inventory yielded by fixed-interval withdrawal kanbans and supplier kanbans in a JIT production system and the minimum number of kanbans required for this system was determined by two formulas [11]. Determined the number of kanbans required to transport materials between two workstations for both single-stage and multi-stage kanban systems. In their models, the demand rate was assumed as linear over each of the three phases (inception, maturation and declination) of a product's life cycle. This model eventually determined batch sizes (or the container size), number of kanbans, the dispatching time intervals and the schedule for production [19]. Analyzed a supply chain system by modeling the raw material ordering policy and finished goods delivering policy. An economic batch size for a product with a fixed time-interval was developed [20]. Developed a general-purpose analytical method for performance evaluation of multi-stage kanban controlled production systems. With each stage is associated a given number of kanbans [21, 22]. Determined the delivery policy and the number of kanbans between two workstations. In Nori and Sarker's models, the total cost was expressed as a function of the number of kanbans, shortage cost of materials and holding cost of containers.

**Kanban-controlled supply chain systems:** Although numerous models have been developed to describe supply chain systems, most studies published did not consider many essential characteristics of manufacturing systems such as the supply-retailer's relationship, number of kanbans and kanban operations [16, 22-26]. Considered the kanban operations between two adjacent stages only and they did not link the raw material stage and finished good stage together [11]. Modeled the supply chain systems systemically, but they did not consider the cost of kanban (the setup cost of kanban includes the building cost, transport cost, etc.). Also they thought the number of lots and the number of kanbans needed to ship the lots, were the same [27]. Analyzed a supply chain system, but his technique was not kanban-based [28, 29]. Presented an elegant inventory model for a supply chain system, but his model was meant for a single-stage system and the emphasis was on the development of an order/delivery policy with no relation to the kanban control technique.

## PROBLEM DESCRIPTION

We develop Wang and Sarker [3] model by adding a retailer to the multi-stage supply chain system. In this model, Stage N offer quantity discounts to encourage the retailer to order more and the producer intends to discount the unit production cost if the amount of production is large. Also, we transform our developed model into a more concise version by applying an unequal demand rates for all stages. Also, we assume the inventory cost of the semi-finished parts shipped to plant $i$ ($i=1$ to $N$) from the preceding stage.

The function of the kanban is best explained through the use of an *N-stage* production system as illustrated in Fig. 1. Two adjacent plants, $i$ and $i+1$, in Fig. 1 are isolated for illustrative purpose as shown in Fig. 2.

In a kanban operation, first, a withdrawal kanban attached to a loaded container in a succeeding plant $i+1$ is detached from the container and put into the Kanban Post (WK) where the first part from the container is to be used. Second, the withdrawal kanbans in the post are collected at a fixed or nonfixed interval and brought to the proceeding plant $i$ by the transportation vehicle. The withdrawal kanban indicates such information as the quantity of parts to be filled in a container, the proceeding and succeeding plants involved with the part, the collection interval, etc. The withdrawal kanban is then attached to the container in a store at the preceding plant in place of the production ordering kanban permitting the worker at the preceding plant to produce the required amount of parts; that is, the detached production-ordering kanban triggers the production of the preceding plant. The containers filled with parts together with the withdrawal kanban are brought, in turn, to the succeeding plant by the vehicle. This kanban cycle realizes smooth, timely and wasteless flow of parts between preceding and succeeding plants. In this SC, production is first triggered by the demand at the final stage (the retailer). Production at each stage is triggered by its succeeding stage(s) and the information to the preceding stage is carried by kanbans. This process is carried all the way back to the raw material acquisition stage. In this procedure, production is controlled (i.e., pulled) by demand, as information of demand carried by kanbans flows backwards from the final stage through the intermediate stages to the first stage (the raw material acquisition). A kanban usually includes the information such as part number, description, container, unit load (quantity per container), stock location (from), end process (to) and some optional information (lot size, number of kanbans per lot, machine number of final operation, individual kanban). The role played by kanbans in a supply chain system has a general purpose in the sense that it is not only an information carrier, but also a material carrier (or transporter). A transporter may be a container, a vehicle, or a train. As the system is operated under the JIT philosophy, the stock levels in each stage should run ideally as low as possible. First, the number of kanbans in each stage should be determined; that is, the number of batches at each stage that is to be shipped by kanbans should be determined. Considering the delivering time and loading/unloading time, the number of kanbans needed to transport the batches is determined. Second, the ordering policy for

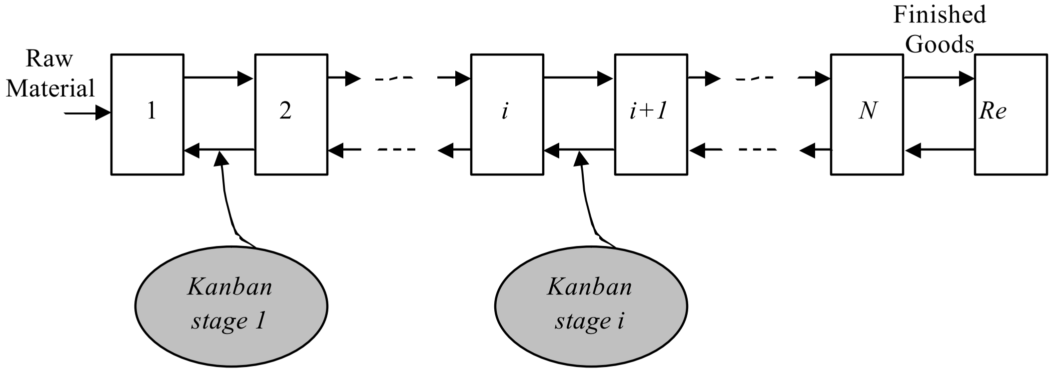


Fig. 1: A multi-stage supply chain system with kanban operations

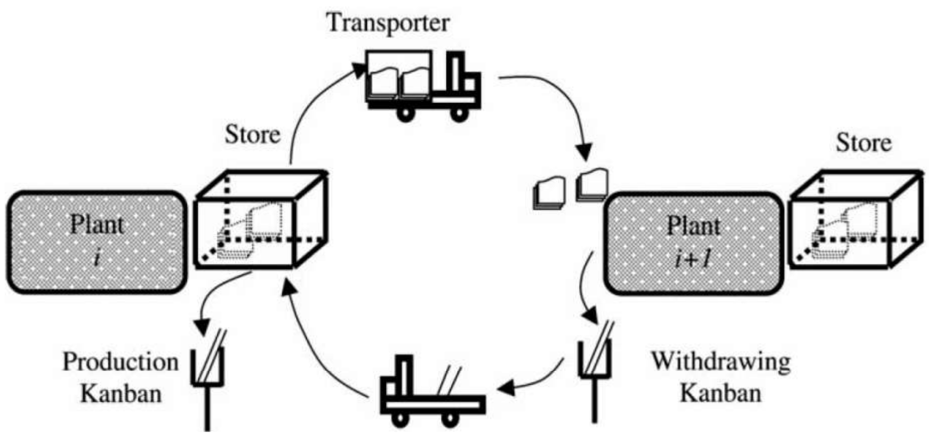


Fig. 2: Operation of kanban production system

the suppliers at the first stage and delivering policy for the retailers at the last stage are to be decided as well. Next, all stages should be linked together to balance the material flow in the supply pipeline. All these apparently discrete problems should be solved collectively based to minimize the SC's total operational cost. Finally, the kanban operations at each stage should be scheduled.

## THE MODEL FORMULATION

The MSSCS model provides the design parameters such as the ordering policy to the suppliers, the delivery policy to the retailers, the batch size and number of batches to deliver WIPs and the total quantity of products in one period in manufacturing stages. From the model, the batch size, the number of batches and the total quantity of products to be produced in each stage over one period are obtained. Next, the materials that need to be transported are set in batches. These batches are transported by kanbans. The operation parameters will be determined in the next section after the design parameters are obtained. The notation used in the model are given below.

In the MSSCS, the total cost for the system includes five cost components: $TC_r$, the cost of raw materials at the first stage; $TC_{wi}$, the inventory cost of *WIP i* at stage $i$; $TC_{w(i)}$, inventory cost of *WIP i-1* at stage $i$; $TC_f$, the cost of finished goods at stage $N$ and $TC'_f$, the cost of finished goods inventory at retailer's stage. They are discussed in details below.

**Cost of raw material inventory:** It is assumed that the demand rate of raw material inventory for the products at the first plant is nothing but $l_r P_1$, is the demand rate of raw material for production in plant 1. $P_1$ is the production rate of plant *1* and $l_r$ is the units of raw material consumed in per semi-finished part in plant *1*. Also $Q_r = l_r\, Q_1$

The orders arrive in lots on time when an order is placed. Shortage is not allowed. So, the input rate (replenishment) is considered as infinite. In this supply chain system, instead of an *EOQ* (economic order quantity) arriving at one time, the company orders raw materials in batches, i.e., the *EOQ* is divided into a number of equal batches, $K_r$. When the production starts, the shipment (one batch) is set at a fixed interval

**Parameters:**

| | |
|---|---|
| i | An index for plant, i = 1,2,...,N + 1; or an index for kanban stage, i = 1,2,...,N, normally kanban stage is between plant i and i+1; |
| $P_i$ | Production rate of plant i (i = 1,2,...,N + 1), units/year; |
| D | The retailer's demand rate, units/year; |
| $l_r$ | Units of raw material consumed in per semi-finished part in stage 1; |
| $l_i$ | Units of semi-finished parts in stage i consumed in per semi-finished part in stage i+1; |
| $H_r$ | Holding cost of raw material inventory, dollar/unit/year; |
| $H_{wi}$ | Holding cost of work-in-process inventory at stage i, dollar/unit/year; |
| $H_{w(i)}$ | Holding cost of semi-finished parts i-1 inventory at stage i, dollar/unit/year; |
| $H_f$ | Holding cost of finished goods inventory at stage N, dollar/unit/year; |
| $H'_f$ | Holding cost of finished goods inventory at retailer's stage, dollar/unit/year; |
| $A_r$ | Setup (ordering) cost at stage 1, dollar/setup (order); |
| $A_{si}$ | Setup (manufacturing) cost at stage i, dollar/batch; |
| $A_{w(i)}$ | Setup (shipping from stage i-1 to stage i) cost at stage i, dollar/ship (setup); |
| $A_f$ | Setup (manufacturing) cost at stage N, dollar/batch; |
| $A'_f$ | Setup (shipping from stage N to retailer) cost at retailer, dollar/ship (setup); |
| I(t) | Inventory level, units; |
| $I_{avg}$ | Average inventory, units; |
| $Pr(Q'_N)$ | Cost of per finished goods for retailer, dollar; |
| $TC_r$ | Inventory cost of raw material, dollars/year; |
| Tcwi | inventory cost of WIP i at stage i, dollars/year; |
| $TC_{w(i)}$ | Inventory cost of WIP i-1 at stage i, dollars/year; |
| $TC_f$ | Cost of finished goods inventory at stage N, dollars/year; |
| $TC'_f$ | Cost of finished goods inventory at retailer's stage, dollars/year; |
| TC | Total cost of a supply chain system, dollars/year; |

**Variables:**

| | |
|---|---|
| $Q_r$ | Total quantity of raw materials ordered over a period T, units/ year; |
| $Q_i$ | Total quantity of WIP i produced over a period T, units/year; |
| $Q_N$ | Total quantity of finished goods produced over a period T, units/year; |
| $Q'_r$ | Order quantity of raw material, units/order; |
| $Q'_i$ | Work-in-process shipping quantity, units/shipment; |
| $Q'_N$ | Finished goods shipping quantity, units/shipment; |
| $K_r$ | Number of order of raw material inventory placed during a period T; |
| $K_i$ | Number of shipments placed during a period T at stage i; |
| $m_i$ | Number of shipments placed during the production time at stage i; |
| $N_T$ | Number of periods placed during a year; |
| T | Cycle time, year; |

during one period. This inventory model is illustrated in Fig. 3.

As the demand rate for the raw material is constant, the batch size will be a fixed quantity. The problem for this inventory model is to determine how much stock should be ordered and how many batches should be placed in a single period, *T*, to meet the demand. This economic batch-size model yields the total raw material cost, $TC_r$, as

$$TC_r = A_r\left(D/Q_N\right)\left(Q_r/Q'_r\right) + H_r I_{avg} \tag{1}$$

Where $K_r = Q_r/Q'_r$ is the number of batches placed per cycle and $N_T = D/Q_N$ is the number of cycles over one year. Since the average inventory $I_{avg} = Q'_r/2$, the above equation is written as

$$TC_r = A_r N_T K_r + H_r \frac{Q'_r}{2} \tag{2}$$

**Cost of work-in-process inventory**

**Inventory cost of work-in-process i at plant i:** The production at plant *i* is carried at a rate of $P_i$ units/year. The parts produced by this plant are defined as *work-in-process i (WIP i). WIP i* inventories built up before they are shipped. As the stock level reaches the lot size, $Q'_i$, the parts are carried by containers from plant *i* to plant *i+1*. The semi-finished parts shipped to plant *i* from the preceding stage are the input of this stage. Since the semi-finished parts are shipped in batches, the number of kanbans, $K_i$, or the batch size, $Q'_i$ (container size) are determined optimally. The level of WIPs at this stage is shown in Fig. 4.

The relations

$$Q_i = \sum_{i=1}^{K_i} Q'_i = K_i Q'_i;\ Q_i = P_i T_{Pi} = m_i \frac{P_i T}{K_i}$$

and

$$N_T = D/Q_N$$

are obtained in plant *i*. The number of items produced at plant *i* in the production time $T_{pi}$ is equal to $Q_i$, that is,

$$Q_i = \int_0^{T_{pi}} P_i dt = P_i T_{pi} = P_i \frac{m_i T}{K_i} \tag{3}$$

The average work-in-process can be calculated by:

$$I_{avg} = \frac{1}{2} Q'_i \left(K_i - m_i + 1\right) \tag{4}$$

Therefore, the inventory cost of *work-in-process i* in stage *i* is given by

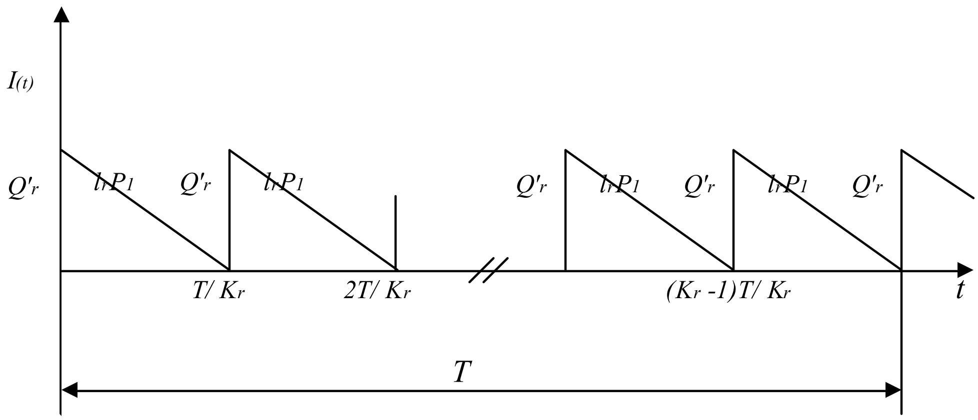


Fig. 3: Raw material inventory at stage 1

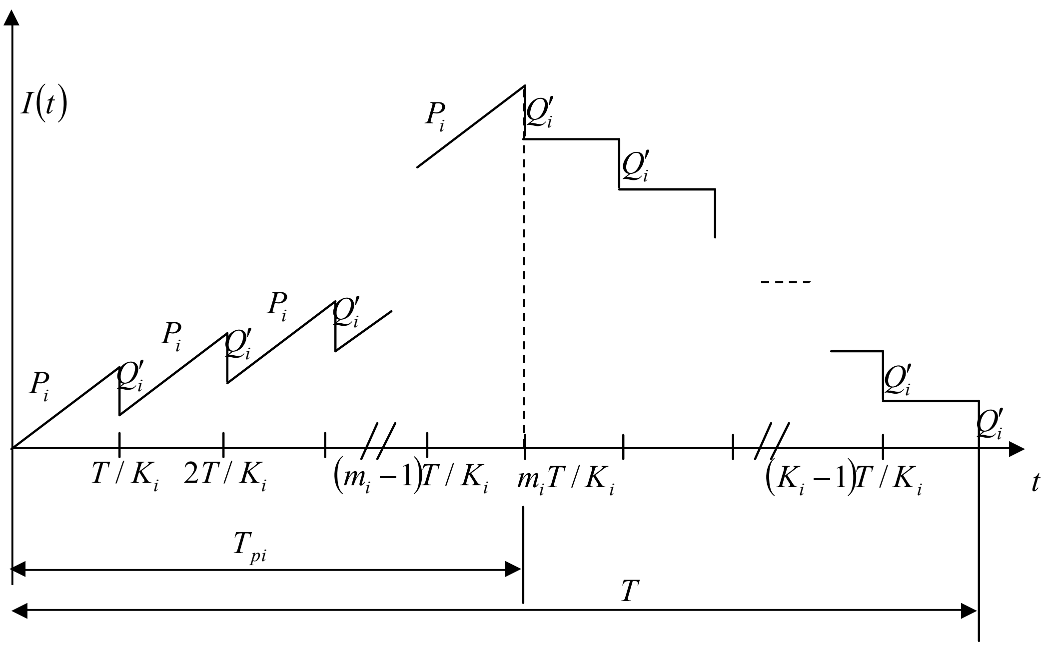


Fig. 4: Inventory of *work-in-process i* at *plant i*

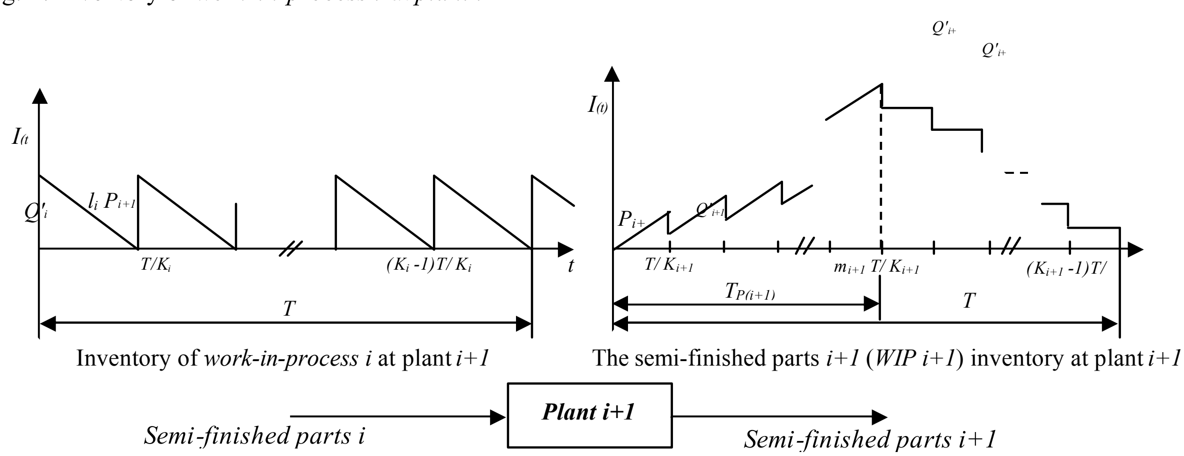


Fig. 5: The work-in-process inventory at plant $i+1$

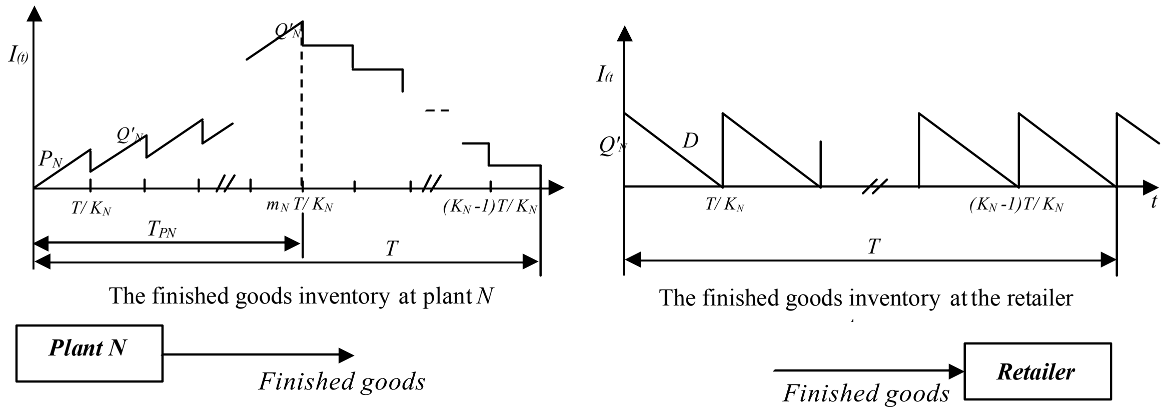


Fig. 6: The finished goods inventory at plant $N$ and at retailer

$$TC_{wi} = A_{si}N_T + H_{wi}\frac{Q'_i}{2}\left(K_i - m_i + 1\right) \tag{5}$$

**Inventory cost of work-in-process i at plant i+1:** You know that the production at plant $i$ is carried at a rate of $P_i$ units/year. The parts produced by this plant are *WIP i* inventories built up before they are shipped. As the stock level reaches the lot size, $Q'_i$, the parts are carried by containers from plant $i$ to plant $i+1$. The semi-finished parts $i$ (*WIP i*) shipped to plant $i+1$ from the stage $i$ are the input of this stage. The semi-finished parts $i$ (*WIP i*) are shipped in batch size, $Q'_i$, to plant $i+1$ and waited for consuming. The level of *WIPs i* at stage $i+1$ is shown in Fig. 5.

It is assumed that the demand rate of *WIP i* inventory for the products at the plant $i+1$ is nothing but $l_iP_{i+1}$, is the demand rate of *WIP i* for production in plant $i+1$. $P_{i+1}$ is the production rate of plant $i+1$ and $x_i$ is the units of *WIP i* consumed in per semi-finished part in plant $i+1$ in addition $Q_i = l_iQ_{i+1}$.

Therefore, the inventory cost of *work-in-process i* in stage $i+1$ is given by

$$TC_{w(i+1)} = A_{w(i+1)}\left(D/Q_N\right)\left(Q_i/Q'_i\right) + H_{w(i+1)}I_{avg} \tag{6}$$

Where $Ki = Q_i/Q'_i$ is the number of batches placed per cycle. Since the average inventory $I_{ave} = Q'_i/2$, the above equation is written as

$$TC_{w(i+1)} = A_{w(i+1)}N_TK_i + H_{w(i+1)}\frac{Q'_i}{2} \tag{7}$$

**Cost of finished goods inventory**

**Cost of finished goods inventory at plant N:** The throughput of the plant N is the finished goods of the N-stage supply chain system. The total stock of finished goods at this stage increases at a rate of $P_N$. The finished goods are shipped to the retailer directly at a fixed interval (Fig. 6). Because of the constant demand rate, the optimal policy of this inventory is determined as a fixed batch size.

The relations

$$Q_N = \sum_{i=1}^{K_N} Q'_N = K_NQ'_N \tag{8}$$

$$Q_N = P_NT_{PN} = P_N\frac{m_NT}{K_N} \tag{9}$$

$$I_{avg} = \frac{Q'_N}{2}(K_N - m_N + 1) \tag{10}$$

are obtained in plant $N$. The number of items produced at plant $N$ in the production time $T_{PN}$ is equal to $Q_N$, that is,

$$Q_N = \int_0^{T_{pN}} P_N\,dt = P_NT_{PN} = P_N\frac{m_NT}{K_N} \tag{11}$$

Therefore, the cost of finished goods inventory in stage $N$ is given by

$$TC_f = A_fN_T + H_f\frac{Q'_N}{2}\left(K_N - m_N + 1\right) \tag{12}$$

**Cost of finished goods inventory at the retailer stage:** We assumed that the production at plant $N$ is carried at a rate of $P_N$ units/year. The parts produced by this plant are *finished goods* inventories built up before they are shipped. As the stock level reaches the lot size, $Q'_N$, the goods are carried by containers from plant $N$ to the retailer. The semi-finished parts $N$-$1$ shipped to plant $N$ from the stage $N$-$1$ are the input of this stage. The finished parts are shipped in batch size, $Q'_N$, to the retailer and waited for consuming. The level of *finished goods* at plant $N$ and at retailer is shown in Fig. 6.

It is assumed that the demand rate of *finished goods* inventory for the retailer is $D$. Therefore, $N_T Q_N = D$.

Therefore, the cost of finished goods inventory at the retailer stage is given by

$$TC'_f = A'_f \left( D / Q_N \right) \left( Q_N / Q'_N \right) + H'_f I_{avg} + D.pr(Q'_N) \qquad (13)$$

In this model, plant $N$ offer quantity discounts to encourage the retailer to order more and the producer intends to discount the unit production cost if the amount of production is large. The discount function, $pr(Q'_N)$, is a Step function. A step quantity discount function is a complicated quantity discount schedule. The step function is shown in Fig. 7, where $q_1$, $q_2$ and $q_3$ represent the incentive quantity levels set by manufacturers and $pr_1$, $pr_2$ and $pr_3$ stand for the basic and discount prices offered, respectively.

The step function defined as:

$$pr(Q'_N) = \begin{cases} pr_1 ; & q_1 \le Q'_N < q_2 \\ pr_2 ; & q_2 \le Q'_N < q_3 \\ pr_3 ; & q_3 \le Q'_N < q_4 \\ \vdots & \vdots \\ pr_{j-1} ; & q_{j-1} \le Q'_N < q_j \\ pr_j ; & q_j \le Q'_N < q_{j+1} \\ pr_{j+1} ; & q_{j+1} \le Q'_N < q_{j+2} \\ \vdots & \vdots \\ pr_{m-1} ; & q_{m-1} \le Q'_N < q_m \\ pr_m ; & q_m \le Q'_N < q_{m+1} \end{cases} \qquad (14)$$

where $pr_1$, $pr_2$,..., $pr_m$ are discount prices which obey the relationships $pr_1 > pr_2 > \ldots > pr_m$ and $q_1, q_2, \ldots, q_m$ which stand for boundaries of the incremental quantities at state $1$ to $m$.

A cost minimization model with a step quantity discount function can be written as

$$\text{Min } D.\,pr(Q'_N) \qquad (15)$$

St:

$$pr(Q'_N) = \sum_{j=1}^{m} pr_j \cdot u_j$$

$$\sum_{j=1}^{m} u_j \cdot q_j \le Q'_N < \sum_{j=1}^{m} u_j \cdot q_{j+1}$$

$$\sum_{j=1}^{m} u_j = 1$$

$$u_j \in \{0,1\}$$

$u_j$ is a 0-1 variable used to indicate which price level should be adopted based on the quantity variable. $u_j = 1$ means $pr_j$ should be selected. Besides, in order to make sure that only one price level is chosen, the constraint

$$\sum_{j=1}^{m} u_j = 1$$

is included.

**Total cost of multi-stage supply chain system:** The total cost of MSSCS, $TC$, can then be written as:

$$TC = TC_r + \sum_{i=1}^{N-1} TC_{wi} + \sum_{i=2}^{N} TC_{w(i)} + TC_f + TC'_f \qquad (16)$$

$$\begin{aligned} \text{Min } TC = & \left( A_r N_T K_r + H_r \frac{Q'_r}{2} \right) \\ & + \sum_{i=1}^{N-1} \left( A_{si} N_T + H_{wi} \frac{Q'_i}{2} (K_i - m_i + 1) \right) \\ & + \sum_{i=2}^{N} \left( A_{w(i)} N_T K_{i-1} + + H_{w(i)} \frac{Q'_{i-1}}{2} \right) \\ & + \left( A_f N_T + H_f \frac{Q'_N}{2} (K_N - m_N + 1) \right) \\ & + \left( A'_f N_T K_N + H'_f \frac{Q'_N}{2} + D.pr(Q'_N) \right) \end{aligned} \qquad (17)$$

St:

$$N_T = D/Q_N$$

$$K_r = Q_r / Q'_r$$

$$Q_r = l_r Q_1$$

$$Q_i = K_i Q'_i \quad i \in \{1,2,\ldots,N\}$$

$$Q_i = l_i Q_{i+1} \quad i \in \{1,2,\ldots,N-1\}$$

$$Q_i = P_i \frac{m_i T}{K_i} \quad i \in \{1,2,\ldots,N\}$$

$$pr(Q'_N) = \sum_{j=1}^{m} pr_j \cdot u_j$$

$$\sum_{j=1}^{m} u_j \cdot q_j \le Q'_N < \sum_{j=1}^{m} u_j \cdot q_{j+1}$$

$$\sum_{j=1}^{m} u_j = 1$$

$$u_j \in \{0,1\}$$

$$pr(Q'_N),\ Q_r,\ Q_i,\ Q_N,\ Q'_r,\ Q'_i,\ Q'_N,\ K_r,\ K_i,\ m_i,\ N_T \ge 0$$

## THE MODEL SOLUTION

In order to solve the model, we simplified the model firstly. For all intermittent stages,

$$Q_i = l_i \cdot Q_{i+1}$$

$$Q_{N-1} = l_{N-} Q_N$$

Therefore

$$Q_i = l_i \cdot l_{i+1} \cdot l_{i+2} \dots \cdot l_{N-1} \cdot Q_N \quad (18)$$

$$Q_i = (\prod_{k=i}^{N-1} l_k) \cdot Q_N$$

Let

$$l_i \cdot l_{i+1} \cdot l_{i+2} \dots \cdot l_{N-1} = \prod_{k=i}^{N-1} l_k = L_i$$

then $Q_i$ is defined as

$$Q_i = L_i \cdot Q_N \quad (19)$$

In the first stage,

$$Q_r = l_r Q_1$$

$$Q_1 = L_1 \cdot Q_N = l_1 \cdot l_2 \cdot \dots \cdot l_{N-1} \cdot Q_N$$

Therefore

$$\begin{cases} L_r = l_r L_1 = l_r \cdot l_1 \cdot l_2 \cdot \dots \cdot l_{N-1} \\ Q_r = l_r \cdot L_1 \cdot Q_N \end{cases} \Rightarrow Q_r = L_r \cdot Q_N \quad (20)$$

$TC_r$ is

$$TC_r = A_r\left(\frac{D}{Q_N}\right)\left(\frac{Q_r}{Q'_r}\right) + \frac{H_r}{2}Q'_r$$

$$TC_r = A_r \cdot \frac{D}{Q_N} \cdot \frac{L_r \cdot Q_N}{Q'_r} + \frac{H_r}{2}Q'_r = \frac{A_r \cdot D \cdot L_r}{Q'_r} + \frac{H_r}{2}Q'_r \quad (21)$$

Let

$$\alpha_r = A_r \cdot L_r \cdot D$$

and

$$\beta_r = \frac{H_r}{2}$$

Then $TC_r$ is defined as:

$$TC_r = \frac{\alpha_r}{Q'_r} + \beta_r Q'_r \quad (22)$$

If this procedure is applied for all cost functions of multi-stage supply chain system, then the total cost of a multi-stage supply chain system will be as follows:

$$\text{Min} \quad TC = \left(\frac{\alpha_r}{Q'_r} + \beta_r Q'_r\right) + \sum_{i=1}^{N}\left(\frac{\alpha_i}{Q_N} + \beta_i\left((L_i - \gamma_i)Q_N + Q'_i\right)\right) \quad (23)$$

$$+ \sum_{i=1}^{N}\left(\frac{\alpha'_i}{Q'_i} + \beta'_i \cdot Q'_i\right) + D \cdot pr(Q'_N)$$

St:

$$pr(Q'_N) = \sum_{j=1}^{m} pr_j \cdot u_j$$

$$\sum_{j=1}^{m} u_j \cdot q_j \le Q'_N < \sum_{j=1}^{m} u_j \cdot q_{j+1}$$

$$\sum_{j=1}^{m} u_j = 1$$

$$u_j \in \{0,1\}$$

$$pr(Q'_N),\ Q_N,\ Q'_r,\ Q'_i,\ Q'_N \ge 0$$

Where

$$\alpha_i = A_{si} \cdot D \quad (24)$$

$$\beta_i = \frac{H_{wi}}{2} \quad (25)$$

$$\gamma_i = \frac{D \cdot L_i^2}{P_i} \quad (26)$$

$$\alpha'_i = A_{w(i+1)} \cdot L_i \cdot D \quad (27)$$

$$\beta'_i = \frac{H_{w(i+1)}}{2} \quad (28)$$

$\ell$ defined as:

$$\ell = \left(\frac{\alpha_r}{Q'_r} + \beta_r Q'_r\right) + \sum_{i=1}^{N}\left(\frac{\alpha_i}{Q_N} + \beta_i\left((L_i - \gamma_i)Q_N + Q'_i\right)\right) + \sum_{i=1}^{N}\left(\frac{\alpha'_i}{Q'_i} + \beta'_i \cdot Q'_i\right) \quad (29)$$

$$+ D \cdot \left(\sum_{j=1}^{m} pr_j \cdot u_j\right) + \lambda_1\left(\sum_{j=1}^{m} u_j - 1\right) + \lambda_2\left(\sum_{j=1}^{m} u_j \cdot q_j - Q'_N\right)$$

If the integer restriction is relaxed, the partial derivatives of $\ell$ will be:

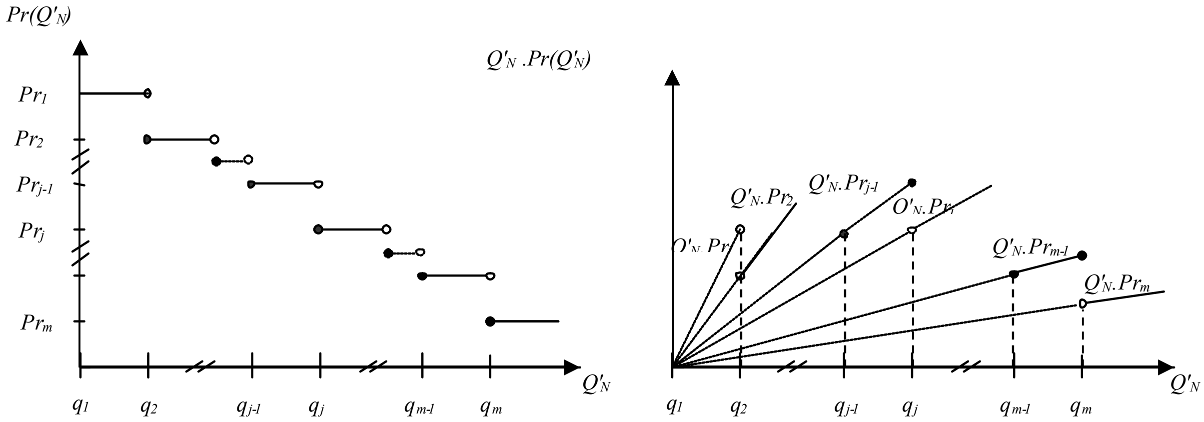


Fig. 7: A step quantity discount function of finished goods for retailer

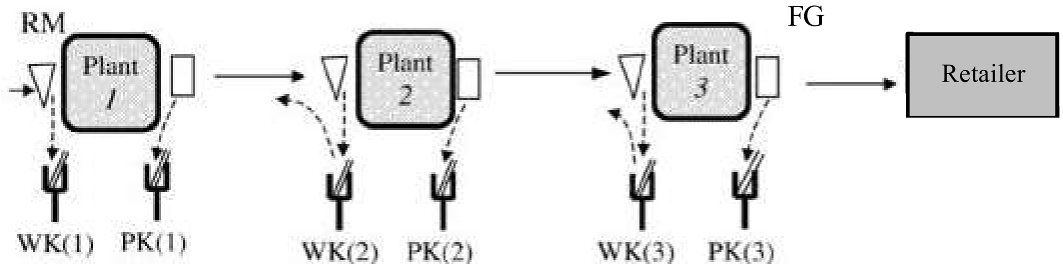


Fig. 8: A three-stage supply chain system with a retailer

$$\frac{\partial \ell}{\partial Q'_r} = -\frac{\alpha_r}{Q_r'^2} + \beta_r = 0 \qquad \Rightarrow \qquad Q_r'^* = \sqrt{\frac{\alpha_r}{\beta_r}} = \sqrt{\frac{2\cdot A_r \cdot L_r \cdot D}{H_r}} \tag{30}$$

$$\frac{\partial \ell}{\partial Q'_i} = \beta_i - \frac{\alpha'_i}{Q_i'^2} + \beta'_i = 0 \overset{i \neq N}{\Rightarrow} Q_i'^* = \sqrt{\frac{\alpha'_i}{\beta_i + \beta'_i}} = \sqrt{\frac{2A_{w(i+1)} \cdot L_i \cdot D}{H_{wi} + H_{w(i+1)}}} \tag{31}$$

$$\frac{\partial \ell}{\partial Q_N} = \sum_{i=1}^{N}\left(-\frac{\alpha_i}{Q_N^2} + \beta_i(L_i - \gamma_i)\right) = 0 \Rightarrow Q_N^* = \sqrt{\frac{\sum_{i=1}^{N}\alpha_i}{\sum_{i=1}^{N}(\beta_i(L_i - \gamma_i))}} \tag{32}$$

Let $Q'_r$, $Q'_i$, $Q_N$ replace $Q'^*_r$, $Q'^*_i$ and $Q^*_N$ respectively. Also let $z$ replace $TC$. Therefore, the general form of multi-stage supply chain system in Eq. (23) is:

$$\begin{aligned} &\text{Min} \quad z = \beta_N \cdot Q'_N + \frac{\alpha'_N}{Q'_N} + \beta'_N \cdot Q'_N + D \cdot (\sum_{j=1}^{m} pr_j \cdot u_j) + C \\ &\text{St:} \sum_{j=1}^{m} u_j = 1 \\ &Q'_N = \sum_{j=1}^{m} q_j \cdot u_j \\ &u_j \in \{0,1\} \end{aligned} \tag{33}$$

Where

$$C = (\frac{\alpha_r}{Q_r'^*} + \beta_r Q_r'^*) + \sum_{i=1}^{N}\left(\frac{\alpha_i}{Q_N^*} + \beta_i(L_i - \gamma_i)Q_N^*\right) + \sum_{i=1}^{N-1}(\frac{\alpha'_i}{Q_i'^*} + (\beta_i + \beta'_i)\cdot Q_i'^*).$$

The $z$ function is simplified as:

$$\begin{aligned} &\text{Min} \quad z = \beta_N \cdot \sum_{j=1}^{m} q_j \cdot u_j + \frac{\alpha'_N}{\sum_{j=1}^{m} q_j \cdot u_j} \\ &\qquad + \beta'_N \cdot \sum_{j=1}^{m} q_j \cdot u_j + D \cdot (\sum_{j=1}^{m} pr_j \cdot u_j) \\ &\text{St:} \sum_{j=1}^{m} u_j = 1 \\ &u_j \in \{0,1\} \end{aligned} \tag{34}$$

$$\underset{j}{\text{Min}}\ \{\cancel{z} = \beta_N \cdot q_j + \frac{\alpha'_N}{q_j} + \beta'_N \cdot q_j + D \cdot pr_j; \forall j\} \qquad (35)$$

This problem can be solved by substituting all values of $q_j$ and $p_{rj}$ in to $\cancel{z}$ .when $\cancel{z}$ is minimized then the $Q'^*_N$ are obtained namely if $\cancel{z}$ is minimized then $Q'^*_N = q_j$.

## THE NUMERICAL EXAMPLE

Example: A three-stage supply chain system with a retailer that is encouraged by quantity discounts, under just-in-time philosophy is shown in Fig. 8 and its parameters are given in Table 1.

Where

$$\alpha_i = A_{si} \cdot D;\ \beta_i = \frac{H_{wi}}{2};$$

$$\gamma_i = \frac{D \cdot L_i^2}{P_i};$$

$$\alpha'_i = A_{w(i+1)} \cdot L_i \cdot D$$

and

$$\beta'_i = \frac{H_{w(i+1)}}{2}.$$

Substituting the values from Table 1 and 2 into Eq. (30, 31), it yields:

$$Q'^*_r = \sqrt{\frac{\alpha_r}{\beta_r}} = \sqrt{\frac{2 \cdot A_r \cdot L_r \cdot D}{H_r}}$$

$$= \sqrt{\frac{2 \cdot 80 \cdot 12 \cdot 5000}{45}} = 461.88 \cong 462$$

$$Q'^*_1 = \sqrt{\frac{\alpha'_1}{\beta_1 + \beta'_1}} = \sqrt{\frac{2A_{w(2)} \cdot L_1 \cdot D}{H_{w1} + H_{w(2)}}}$$

$$= \sqrt{\frac{2 \cdot 100 \cdot 4 \cdot 5000}{20 + 30}} = 282.8 \cong 283$$

$$Q'^*_2 = \sqrt{\frac{\alpha'_2}{\beta_2 + \beta'_2}} = \sqrt{\frac{2A_{w(3)} \cdot L_2 \cdot D}{H_{w2} + H_{w(3)}}}$$

$$= \sqrt{\frac{2 \cdot 120 \cdot 2 \cdot 5000}{22 + 25}} = 225.97 \cong 226$$

$$Q^*_N = \sqrt{\frac{\sum_{i=1}^{N} \alpha_i}{\sum_{i=1}^{N} (\beta_i (L_i - \gamma_i))}} \cong 479 \Rightarrow$$

$$N_T = \frac{D}{Q_N} = \frac{5000}{479} = 10.43$$

$$Q_r = L_r \cdot Q_N = 12 \times 479 = 5748 \Rightarrow K_r = \frac{Q_r}{Q'_r} = \frac{5748}{462} = 12.44$$

$$Q_1 = L_1 \cdot Q_N = 4 \times 479 = 1916 \Rightarrow K_1 = \frac{Q_1}{Q'_1} = \frac{1916}{283} = 6.77$$

$$Q_2 = L_2 \cdot Q_N = 2 \times 479 = 958 \Rightarrow K_2 = \frac{Q_2}{Q'_2} = \frac{958}{226} = 4.23$$

The $z$ function is simplified as

$$\underset{j}{\text{Min}}\ \{\cancel{z} = \beta_N \cdot q_j + \frac{\alpha'_N}{q_j} + \beta'_N \cdot q_j + D \cdot pr_j; \forall j\}$$

This problem can be solved by substituting all values of $q_j$ and $p_{rj}$ in to $\cancel{z}$. If $\cancel{z}$ is minimized then $Q'^*_N = q_j$.

We assume that

$$pr(Q_N) = \begin{cases} pr_1 = 6 & ; \quad 0 \le Q'_N < 100 \\ pr_2 = 5 & ; \quad 100 \le Q'_N < 125 \\ pr_3 = 4.2 & ; \quad 125 \le Q'_N < 150 \\ pr_4 = 4 & ; \quad 150 \le Q'_N < \infty \end{cases}$$

Therefore

Table 1: The system values of a MSSCS

| Demand of finished goods (units/year) | Units of WIP $_i$ consumed in per WIP $_{i+1}$ | Production rate (units/year) | Setup (manufacturing /ordering) cost (dollar/batch) | Setup (shipping from i-1 to i) cost (dollar/ship) | Holding cost of WIP$_i$ at stage I (dollar/unit/year) | Holding cost of WIP$_{i-1}$ at stage i (dollar/unit/year) |
|---|---|---|---|---|---|---|
| D=5000 | $l_r = 3$ | | $A_r = 80$ | | $H_r = 45$ | |
| | $l_1 = 2$ | $P_1 = 25100$ | $A_{s1} = 300$ | $A_{w(2)} = 100$ | $H_{w1} = 30$ | $H_{w(2)} = 20$ |
| | $l_2 = 2$ | $P_2 = 11000$ | $A_{s2} = 200$ | $A_{w(3)} = 120$ | $H_{w2} = 25$ | $H_{w(3)} = 25$ |
| | $l_3 = 1$ | $P_3 = 5600$ | $A_{s3} = A_f = 250$ | $A_{w(4)} = A'_f = 110$ | $H_{w3} = H_f = 35$ | $H_{w(4)} = H'_f = 25$ |

Table 2: The system parameters of a MSSCS

| $L_r = l_r \cdot l_1 \cdot l_2$ | $\alpha_r = A_r \cdot L_r \cdot D$ | | | $\beta_r = 0.5 \times H_r$ | |
|---|---|---|---|---|---|
| $L_1 = l_1 \cdot l_2$ | $\alpha'_1 = A_{w(2)} \cdot L_1 \cdot D$ | $\alpha_1 = A_{s1} \cdot D$ | $\beta'_1 = 0.5 \times H_{w(2)}$ | $\beta_1 = 0.5 \times H_{w1}$ | $\gamma_1 = \frac{D \cdot L_1^2}{P_1}$ |
| $L_2 = l_2$ | $\alpha'_2 = A_{w(3)} \cdot L_2 \cdot D$ | $\alpha_2 = A_{s2} \cdot D$ | $\beta'_2 = 0.5 \times H_{w(3)}$ | $\beta_2 = 0.5 \times H_{w2}$ | $\gamma_2 = \frac{D \cdot L_2^2}{P_2}$ |
| $L_3 = 1$ | $\alpha'_3 = A_{w(4)} \cdot L_3 \cdot D$ | $\alpha_3 = A_{s3} \cdot D$ | $\beta'_3 = 0.5 \times H_{w(4)}$ | $\beta_3 = 0.5 \times H_{w3}$ | $\gamma_3 = \frac{D \cdot L_3^2}{P_3}$ |

$$Q'_N = 75 \Rightarrow \cancel{Z} = 2250 + 7333.3 + 30000 = 39583.3$$
$$Q'_N = 100 \Rightarrow \cancel{Z} = 3000 + 5500 + 25000 = 33500$$
$$Q'_N = 125 \Rightarrow \cancel{Z} = 3750 + 4400 + 21000 = 29150$$
$$Q'_N = 150 \Rightarrow \cancel{Z} = 4500 + 3666.6 + 20000 = 28166.6 \Rightarrow Q_1'^{*} = 150$$
$$Q'_N = 175 \Rightarrow \cancel{Z} = 5250 + 3142.86 + 20000 = 28392.86$$

## CONCLUSION

The significant features of developed model are as follows

- We add a retailer to the multi-stage supply chain system. In this model, Stage N offers quantity discounts to encourage the retailer to order more and the producer intends to discount the unit production cost if the amount of production is large.
- We transform our developed model into a more concise version by applying an unequal demand rates for all stages.
- The most important development of presented model is that the inventory cost of the semi-finished parts shipped to a plant from the preceding stage is considered.

The model developed here can help a manager to respond quickly to consumers-need, determine the right policies to order the raw material, deliver the finished goods and efficiently manage their operations. As a result, an organization can economically benefit from savings by effectively managing a supply chain. The greedy heuristic is very promising in terms of its efficiency and quality of the solution. The problem has potential in implementing the current approach either in an existing system to improve the current operational approach or in a new system where planning needs to be performed to incorporate the results. For future research, the model can further be expanded to converge supply chain system where the system is either supplied with components from several sources in branches or delivers components to other facilities or buyers downstream and the parameters of models can be defined as indeterminist. Also the real constraints can be added to the model.

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