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Optimal Crashing of Multi Period-Multi Product Production Planning Problems

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Abstract: To save overhead and penalty costs, in many cases of Multi-Period Multi-Product (MPMP) planning systems, reducing the completion time is highly desirable. Therefore, in this paper we study this problem. We propose an approach in which a MPMP problem is converted into a project management network. Then, considering network concepts and properties, a multi stage novel mathematical model is proposed. Therefore, it is possible to crash each operation in accordance with the relevant cost components in order to decrease total cost by controlling the production planning completion time. Finally, an algorithm is developed. Thus, the proposed approach integrates both project management network and mathematical programming techniques in a production planning problem.

Key word: Multi-period Multi-Production (MPMP) . crashing . network . optimization

INTRODUCTION

In many situations, to save overhead costs (or penalty) or under a pre-defined budget constraint, it can be optimal to reduce the completion time of the project even by expending extra cost to crash the processing time of some activities. In that case, the completion time of a project can be even less than the length of the critical path. It is thus very important form the contractor point of view to decrease project completion time. Actually, a cost benefit analysis may be used in many real practical projects as well theoretical applications.

A Multi-Period Multi-Product (MPMP) problem comprises of matching production levels of individual products to the changes of demand for a number of period into future, subject to capacity constraints. However the machine centers capacity constraints and predecessor relationship may not correctly represent the actual solution in practical cases and can lead to an infeasible solution. To solve these problems, an MPMP problem can be transformed into a project network. Thus, it is possible to determine the sequence of operations and consider the dependencies and precedence logics.

These models generally can be divided into deterministic and uncertain ones. Deterministic models are analyzed by optimization techniques, usually based on linear programming (LP) or other mathematical programming approaches. Uncertain models include probabilistic approaches.

In this paper, by considering all the above mentioned points, a multi stage mathematical programming is studied in which the objective is to minimize the total costs, including overhead and crashing costs. This goal is achieved by crashing some tasks processing times. On the other hand, since the capacity of machine centers is limited and no more than one task can be processed on a single machine, the model determines the sequence of tasks in different machine centers and in different periods as well as the optimal processing time of each task.

In section 2, a survey of the existing literature on MPMP and network crashing is reviewed. The problem statement and the general features of an MPMP network are discussed in section 3. A mathematical programming model, notations, the proposed model and the method of solving the model are developed in section 4. Section 5 presents an experimental investigation and corresponding network and the resulting tables and then the results are analyzed in section 6. Finally, in section 7 conclusions and suggestions for further research are provided.

LITERATURE SURVEY

There are many investigations carried out in the literature incorporating optimization models in MPMP

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problem. As a good instance, initially, Byrne and Bakir [1] developed a hybrid algorithm combining mathematical programming and simulation model of a manufacturing system representing an MPMP problem. They demonstrated that analytical methods working in co-operation with the simulation model results a better solution in comparison with the individual ones. The resulting production plan can be both mathematically optimal and practically feasible. Also in this respect, Kim and Kim [2] proposed an extended linear programming model for a similar hybrid approach. In simulation runs, actual workload of the jobs and utilization of the resources are identified. Information is then passed to the linear programming model for calculating the optimal production plan with the minimal total cost. Byrne and Hossain [3] proposed an extended linear programming model of Byrne and Bakir [1] and Kim and Kim [2]. In their model, in order to introduce the unit load concept of JIT, work load of jobs was sub-divided. While an optimal plan is sought, due to this unit load concept, the model takes account of the requirement of small lot sizes which is one factor of the JIT approach. Incorporation of the unit load concept and the modification of resource requirements and constraints in the proposed LP formulation are expected to help the improvement of the planning model by reducing the level of WIP (Work in Process) and total flow time.

As a related work in considering project and production principles, Noori *et al*. [4] proposed a fuzzy control chart application to MPMP problems. However, they considered uncertainty associated with fuzzy control chart and implemented their approach by using earned value analysis.

Although, there are some works regarding crashing, this concept has not been applied in production planning or especially in MPMP problem directly. However, some of related papers are as follows: Goyal [5] gave a procedure for shortening the duration of a project at low cost. This procedure allows shortening of activities which may have been shortened initially and they happen to be exclusively in the path which has been shortened excessively. Tareghian and Taheri [6] developed a solution procedure to study the tradeoffs of time, cost and quality in the management of a project. This problem assumes the duration and quality of project activities to be discrete, non-increasing functions of a single non-renewable resource. Three inter-related mathematical models are developed such that each model optimizes one of the given entities by assigning desired bounds on the other two. Different forms of quality aggregations and effect of activity mode reductions are also investigated in this paper. Deineko and Woeginger [7] considered the discrete version of the well-known time-cost tradeoff problem for project networks which had been extensively studied in the project management literature. They proved a strong in-approximability result with respect to polynomial time bicriteria approximation algorithms for this problem. Bagherpour *et al*. [8] presented a new approach to adapt linear programming to solve cost time trade off problems. The proposed approach uses two different modeling flowshop scheduling into a leveled project management network. The first model minimizes makespan subject to budget limitation and the second model minimizes total cost to determine optimum makespan over production planning horizon.

There are also some non-deterministic approaches in project crashing problems. Abbasi and Mukattash [9] introduced and developed a method for investigating the application of mathematical programming to the concept of crashing in Program Evaluation and Review Technique (PERT). The main objective was the minimization of the pessimistic time estimate in PERT networks by investing additional amounts of money in the activities on the critical path. Azaron and Tavakoli [10] developed a multi-objective model for the time-cost trade-off problem in a dynamic PERT network using an interactive approach. Feylizadeh *et al*. [11] presented an application of fuzzy goal programming (FGP) in a flow shop scheduling problem where two objectives, namely minimizing completion time and minimizing crashing costs are assumed to be considered simultaneously. Laslo [12] described a stochastic extension of the critical path method time-cost tradeoff model. This extension includes four fundamental formulations of time-cost tradeoff models that represent different assumptions of the effect of the changing performance speed on the frequency distribution parameters of the activity duration, as well as the effect of the random activity duration on the activity cost.

PROBLEM STATEMENT

We study a “Multi Period-Multi Product (MPMP) production planning system”, which consists of some machine centers. Every product has several tasks. Each task has to be processed in a specific machine center and during a given period. Obviously, in a machine center no more than one task can be carried out simultaneously. The tasks of each product must be processed in a given order, which is specific for that product, although different products have different tasks.

Since the machine centers facing limited capacities constraints, it is required to match the production level

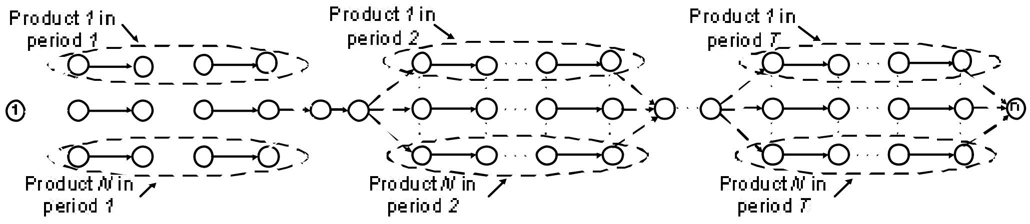

Fig. 1: General feature of an MPMP Network

of products to the variation of demands for a number of future periods, while the constraints such as machine centers capacity limits or the precedence relationships between the tasks of each product are considered.

It is also assumed that the processing length of some tasks can be reduced, although it causes the corresponding processing cost to be increased. On the other hand, shortening the completion time of a project results in saving overhead (and possibly penalty) costs. Therefore, in this research the optimal processing time of each task in each period (as well as the value of the other decision variables) are determined to minimize the total production cost (including producing, holding, shortage, overhead and crushing costs).

To obtain an optimal solution for this problem, the problem is converted into a project network. In this way, dependencies and precedence relationships between the tasks are considered easily by network structure. However, in order to consider the constraints, such as capacity limitations; avoiding simultaneous processing in machine centers, the network is then formulated as a binary, nonlinear programming problem efficiently.

General feature of an MPMP network: General feature of an MPMP network with *T* periods, *N* products and *n* nodes can be shown as follows.

MATHEMATICAL MODEL

As mentioned before, the processing time of a task (or the duration of the network activity which represents that task) can be controlled by the allocation of appropriate amount of resources to that activity. In other words, the duration of an activity can be shortened by consuming more resources. Thus, in this section, the duration of activities are also considered as decision variables.

The proposed model: In this section the proposed model is described. In this model, the objective function (1) minimizes the total cost which includes inventory holding, lost sale, network crashing and overhead costs.

Notations

Indexes and parameters:

t: Period index;
i, j: Product index;
k: Machine centers index;
e: Network events index;
T: The number of periods;
N: The number of products;
K: The number of machine centers;
E: Set of network events;
$(r\text{-}s)$: Activity from node r to node s;
G: The set of activities of the network;
h_{it}: Unit holding cost of inventory for product i in period t;
π_{it}: Unit cost of sales lost for product i in period t;
M_{kt}: Capacity of machine centre k in period t;
D_{it}: Demand for product i in period t;
f_{it}: Unit variable cost of producing product i in period j;
H: Overhead cost per period of time;
c_{rs}: Crash cost per unit time for activity $(r\text{-}s)$;
$\overline{p}_{rs}$: Normal processing time of activity $(r\text{-}s)$;
$\underline{p}_{rs}$: Minimal processing time (crash time) of activity $(r\text{-}s)$;
M: A big number;
U_{kt}: The set of the activities which do not have precedence relation, but are processed in machine center k in period t; $N_{kt} = \{r:(r,s)\in U_{kt}\}$
L_{kt}: The number of possible permutations of set N_{kt};
$A(s) = \{r:(r,s) \text{ is a precedent activity of node } s\}$

Variables

X_{it}: Amount of product i to be produced in period t;
P_{rs}: Processing time to produce a unit of activity $(r\text{-}s)$;
T_e: Planning date for event $e\in E$;
I_{it}: End of period inventory of product i in period t;
J_{it}: End of period deficit of product i in period t;

Obviously, the term $\sum_{t=1}^{T}\sum_{i=1}^{N} c_{ij}\bar{p}_{ij}$ can be deleted because it is constant. Equation (2) controls the inventory balance and it also indicates that backlogging is not allowed. Inequality (3) expresses the capacity constraints of machine centers. Constraint (4) expresses the relationships among the nodes network, that is, it ensures the precedence relations between project activities hold. Constraints (5) and (6) are also used to prevent processing more than one task in a machine center. Following, detail of proposed approach is given which the results maybe found using commercial software such as LINDO®, LINGO ® or GAMZ ®.

$$\begin{aligned} \text{Min}Z = & \sum_{i=1}^{N}\sum_{t=1}^{T} f_{it}.X_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T}\left(h_{it}.I_{it} + \pi_{it}.J_{it}\right) \\ & + \sum_{i=1}^{N}\sum_{t=1}^{T} c_{rs}.\left(\bar{p}_{rs} - P_{rs}\right) + H.(T_{|E|} - T_1) \end{aligned} \qquad (1)$$

$$X_{it} + I_{i\,t-1} + J_{it} - I_{it} = D_{it} \qquad \forall i, \forall t \qquad (2)$$

$$\sum_{i=1}^{N} P_{rs}.X_{it} \le M_{kt} \qquad \forall t, \forall k \qquad (3)$$

$$T_s - T_r \ge P_{rs}.X_{it} \qquad r \in A(s), \forall s \qquad (4)$$

$$T_p \ge T_s - M.Z_l \qquad (5)$$

$$\sum_{w \in U} Z_w = L - 1 \qquad (6)$$

$$\underline{p}_{rs} \le P_{rs} \le \bar{p}_{rs} \qquad (7)$$

$$I_{it}, J_{it}, T_v, P_{rs} \ge 0 \qquad (8)$$

$$Z_w: \text{ Binary Variables} \qquad (9)$$

Solving procedure: Since the model is a nonlinear binary programming, we develop an algorithm customized specially to solve it. In this algorithm, the crushing of activities is performed iteratively.

Step 0: Set

$$P_{rs} = \bar{p}_{rs} \qquad \forall r, s \in G$$

Step 1: Set

$$P_{rs} = \bar{p}_{rs} \qquad \forall r, s \in G$$

equal to the current fixed value. Then solve the model. (The model can be solved by the procedure developed by Byrne and Bakir [1]).

Step 2: Set X_{it} equal to the value obtained in Step 1 and solve the model again. Obtain the optimal value of P_{ij}. Go to Step 1.

Stopping Rule. If the difference between two successive values of objective function is less than an arbitrary value of ε, then stop.

EXPERIMENTAL INVESTIGATIONS

To illustrate our proposed approach, we use the case study presented in Byrne and Bakir [1] consists of three periods, three products MPMP production planning problem, the machine centre have limited capacity. The system comprises of four machine centers, each having one machine and one input buffer. The capacity of each machine center is constant and equals 2400 min/week. The parts are transported between machine centers by two non-accumulating belt conveyors. The cost coefficients demand matrix, process times and process routings are shown in Table 1 through 4 in Byrne and Bakir [1]. It is also supposed that the overhead cost is equal to 10. All system parameters are deterministic and known. The system is considered to be a terminating one, with no guaranteed demand beyond period three. Hence, the holding costs for inventory at the end of period three are high to discourage leftover stocks.

The information for this MPMP problem, such as: activity names, inventory holding costs, backlogging costs, predecessors, normal duration time, crash time, slope of the cost in Table 1. In this table, by normal duration time it is meant length of time of processing of an activity needs to perform normally. However, crashed time is the least possible duration of each activity by spending more resources. The column 'slope of the cost' determines the cost which may be paid to shorten each activity duration time in scope of normal duration time to crash time. Also the network of this problem is shown in Fig. 2 for better understanding the solution of this problem which each resource is written on the corresponding activity.

MODEL DISCUSSION

The mathematical model of section 4.2 was implemented by solving the example presented in "Experimental Investigations" section. Hyper LINGO® 4 was used on a P4 computer with 4800 MHz CPU to obtain the solution. It took 2 seconds. The summary of important results is shown in Table 2. As Fig. 2 shows, the network which represents this example has 45 tasks including 18 dummy ones. The duration of 12 activities which can be crashed within a given range is considered in constraint (7). The total cost and

Table1: Relevant information for MPMP crashing

Activity	Predecessors	Normal Duration time	Crash time	Slope of the cost	Resources	Inventory Holding costs	Backlogging costs
1-2	----	----	----	----	----	----	----
2-5	1-2	5	----	----	MC1	25	400
5-8	2-5	10	----	----	MC4	25	400
8-11	5-8	4	3	100	MC3	25	400
11-14	8-11	----	----	----	----	----	----
1-3	----	----	----	----	----	----	----
3-6	1-3	7	5	120	MC1	30	450
6-9	3-6	7	5	110	MC2	30	450
9-12	6-9	5	----	----	MC3	30	450
12-14	9-12	----	----	----	----	----	----
1-4	----	----	----	----	----	----	----
4-7	1-4	7	----	----	MC1	35	500
7-10	4-7	6	----	----	MC2	35	500
10-13	7-10	10	8	110	MC3	35	500
13-14	10-13	----	----	----	----	----	----
14-15	11-14,12-14,13-14	----	----	----	----		
15-18	14-15	5	----	----	MC1	25	400
18-21	15-18	10	----	----	MC4	25	400
21-24	18-21	4	3	100	MC3	25	400
24-27	21-24	----	----	----	----	----	----
14-16	11-14,12-14,13-14	----	----	----	----	----	----
16-19	14-16	7	5	120	MC1	30	450
19-22	16-19	7	5	110	MC2	30	450
22-25	19-22	5	----	----	MC3	30	450
25-27	22-25	----	----	----	----	----	----
14-17	11-14,12-14,13-14	----	----	----	----	----	----
17-20	14-17	7	----	----	MC1	35	500
20-23	17-20	6	----	----	MC2	35	500
23-26	20-23	10	8	110	MC3	35	500
26-27	23-26	----	----	----	----	----	----
27-28	24-27,25-27,26-27	----	----	----	----	----	----
28-31	27-28	5	----	----	MC1	100	400
31-34	28-31	10	----	----	MC4	100	400
34-37	31-34	4	3	100	MC3	100	400
37-40	34-37	----	----	----	----	----	----
27-29	24-27,25-27,26-27	----	----	----	----	----	----
29-32	27-29	7	5	120	MC1	150	450
32-35	29-32	7	5	110	MC2	150	450
35-38	32-35	5			MC3	150	450
38-40	35-38	----	----	----	----	----	----
27-30	24-27,25-27,26-27	----	----	----	----	----	----
30-33	27-30	7			MC1	200	500
33-36	30-33	6			MC2	200	500
36-39	33-36	10	8	110	MC3	200	500
39-40	36-39	----	----	----	----	----	----

Table 2: Total cost and completion time of the proposed model

	Iteration number							
	1(P0)	2(X1)	3(P1)	4(X2)	5(P2)	6(X3)	7(P3)	8(NLP)
Total cost	301,789.00	293,275.2	280,648.70	280,248.10	274,681.60	273,371.80	273,369.50	272,026.8
Completion time (T40)	12115.97	12115.0	12257.46	12202.87	12392.25	12290.87	12288.62	12235.0

Table 3: Summery results obtained by the proposed approach

	Iteration number							
	1(P0)	2(X1)	3(P1)	4(X2)	5(P2)	6(X3)	7(P3)	8(NLP)
P811	4	3	3	3	3	3	3	3
P36	7	5	5	5	5	5	5	5
P69	7	5	5	5	5	7	7	7
P1013	10	9	9	9	9	9	9	8
P2124	4	3	3	3	3	3	3	3
P1619	7	7	7	5	5	5	5	5
P1922	7	6	6	5	5	7	7	7
P2326	10	9	9	8	8	8	8	8
P3437	4	3	3	3	3	3	3	3
P2932	7	5	5	5	5	5	5	5
P3235	7	5	5	7	7	7	7	7
P3639	10	9	9	10	10	9	9	10
X11	176	150	159	159	178	178	177	172
X12	129	125	128	128	129	129	130	132
X13	129	160	148	148	128	128	128	131
X21	93	111	128	128	100	100	104	101
X22	129	127	122	122	150	150	146	149
X23	129	105	150	150	150	150	150	150
X31	123	124	138	138	144	144	142	148
X32	121	126	129	129	143	143	146	142
X33	121	123	125	125	125	125	125	125

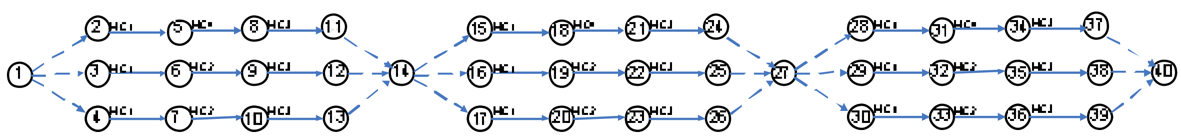

Fig. 2: The example network

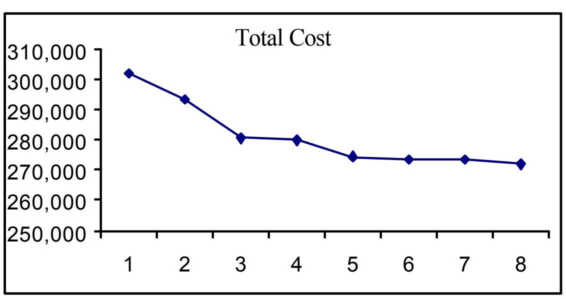

Fig. 3: Total cost of the problem in 8 stages

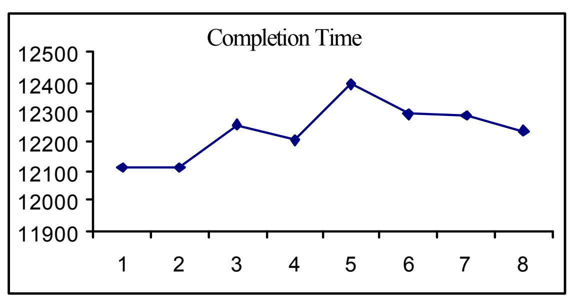

Fig. 4: Completion time of the problem in 8 stages

completion time (with 8 iterations) obtained are shown in Table 2. The summary of results of duration time of activities and the amount of products to be produced in each period are shown in Table 3, as well. According to the results in Table 2, the total cost of the network decreases from 301,789 to 272,026.8. As it is clear, the least cost is related to the Nonlinear Programming (NLP) model which other 7 iterations is converging to its cost. The graphs of Total Cost and Completion Time of the problem (with 8 iterations) are shown in Fig. 3 and Fig. 4.

CONCLUSION AND FURTHER RESEARCHES

In this paper, a mathematical model was developed in order to decrease the total cost of and control the system by controlling the makespan, while crashing of activities is allowed. The model was implemented successfully in an MPMP production planning problem. In order to implement the mathematical model, the MPMP problem was firstly converted into an Activity on Arc (AOA) network. Indeed, manufacturing process is crashed indirectly by using a multi stage mathematical model and project management network approach. Therefore, another advantage of applying the proposed model is to model an MPMP problem while the capacity of machine centers constraints and predecessor relationship are accurately considered. This approach helps manufacturer not only to focus on the total cost and completion time but also consider all the important constraints in the production planning. Further research can focus on applying multi objective programming or/and considering non-deterministic control cases to manage time and cost simultaneously or quality in this system.

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