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Using Newton's Interpolation and Aitken's Method for Solving First Order Differential Equation

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Abstract: In recent years, there has been greater attempt to solving differential equations by analytic methodsand numerical methods. Most of researchers treated numerical approach to solve first order ordinary differential equations. These methods such as RungeKutta method, Taylor series method and Euler’s method, etc. Faith Chelimo Kosgeistudied this problem by combined the newton’s interpolation and Lagrange method. This study will combine of Newton’s interpolation and Aitken's methodas hybrid technique by using these tow types of interpolation to solve first order differential equation.

Key words: Differential equation · Analytic method · Numerical method · Newton’s interpolation method · Aitken's method

INTRODUCTION

In real life situation many problems can be formulated in theform of ordinary differential equation, specially of first order, hence we need to study and solve the differential equations. A numerical method is used to solve numerical problems. The differential equation problem [1-7], consists of at least one differential equation and at least one additional equation such that the system together have one and only one solution called the analytic or exact solution to distinguish it from the approximate numerical solutions that we shall consider. In thispaper, to find the solution of differential equation of first order, Faith C. K [1] studied this problem by using combination of newton’s interpolation and Lagrange method. In this study we will combine of Newton’s interpolation and Aitken's method [2-4, 6, 8]. Finally we verified on a number of examples and numerical results obtained show the efficiency of the methodgiven by present study in comparison with the method of Faith C. K [1]. Let’s consider the following first order differential equation or initial value problem.

$$y' = f(x, y), \qquad y(x_\circ) = y_\circ \tag{1}$$

where f (x, y) is a known function and the values in the initialconditions are also known numbers.

Combined Newton’s Interpolation and Lagrange Method [1]: This study combine both Newton’s interpolation methodand Lagrange method. It used newton’s interpolation method tofind the second two terms then use the three values for y toform a quadratic equation using Lagrange interpolation method as follows;

Newton’s Interpolation Method [1, 6]:

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots + a_n(x - x_0)(x - x_1)\dots a_2(x - x_{n-1}) \tag{2}$$

where

$$a_0 = y_0,\ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)},\ a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \tag{3}$$

etc

Lagrang Interpolation Method [1, 6]:

$$y_n = \frac{(x - x_1) - (x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0) - (x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0) - (x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \tag{4}$$

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Description of the Method: This method will combine both Newton's interpolation methodand Aitken method. It used newton's interpolation method tofind the second two terms then use the three values for y toform a linear or quadratic equations using Aitken interpolation method as follows;

$$f_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + ... + a_n(x-x_0)(x-x_1)...a_2(x-x_{n-1}) \quad (5)$$

where

$$a_0 = y_0, \; a_1 = \frac{f(x_1)-f(x_0)}{(x_1-x_0)}, \; a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} \quad (6)$$

etc

$$y_1 = a_0 + a_1(x-x_0) \quad (7)$$

$$y_2 = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) \quad (8)$$

Note: We can use Newton's Forward Interpolation Formula instead of Newton's divided Interpolation method in (2.1).

Aitken Interpolation Method [6]:

$$P_{o,k}(x) = \frac{1}{x_k - x_o}\begin{vmatrix} y_o & x_o - x \\ y_k & x_k - x \end{vmatrix} \quad (9)$$

$$P_{o,1,2}(x) = \frac{1}{x_2 - x_1}\begin{vmatrix} P_{o,1}(x) & x_1 - x \\ P_{o,2}(x) & x_2 - x \end{vmatrix} \quad (10)$$

$$y_n = P_{o,1,2,...,n}(x) = \frac{1}{x_n - x_{n-1}}\begin{vmatrix} P_{o,1,...,(n-1)}(x) & x_{n-1} - x \\ P_{o,1,...,(n-2),n}(x) & x_n - x \end{vmatrix} \quad (11)$$

Examples: In this section, we will check the effectiveness of thepresent technique (3). First numerical comparison for the following test examples taken in [1].

Example 1:

Solve $\frac{dy}{dx} = 1 - y, \quad y(0) = 0$

By taking the step h = 0.01
First by using Newton's interpolation, we have

Table 1: Solution of $\frac{dy}{dx} = 1 - y$, y(0)=0

x	Combined Newton's Interpolation and Aitken	s
0	0	0
0.01	0.0100	0.009950166251
0.02	0.0199	0.019801326
0.03	0.0297	0.029554466
0.04	0.0394	0.03921056
0.05	0.0490	0.048770575
0.06	0.0585	0.058235466
0.07	0.0679	0.06760618
0.08	0.0772	0.076883653
0.09	0.0864	0.086068814
0.1	0.0955	0.095162581

$$a_0 = 0 = y_0$$
$$a_1 = \frac{f(x_1)-f(x_0)}{(x_1-x_0)} = [\frac{dy}{dx}]_{0,0} = 1$$
$$y_1 = 0 + 1(0.01 - 0) = 0.01$$
$$a_2 = \frac{\frac{f(x_2)-f(x_1)}{(x_2-x_1)} - \frac{f(x_1)-f(x_0)}{(x_1-x_0)}}{(x_2-x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.01} - [\frac{dy}{dx}]_{0,0}}{0.02-0} = -0.05$$

$$y_2 = 0 + a1(0.02-0) - 0.5(0.02-0)(0.02-0.01) = 0.0199$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = x$$
$$P_{0,2}(x) = 0.995x$$
$$P_{0,1,2}(x) = -0.5x^2 + 1.005x$$

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution givenby Faith C. K [1], Table. 1.

Example 2:

Solve $\frac{dy}{dx} = x^2 - y, \quad y(0) = 1$

By taking the step h = 0.01
First by using Newton's interpolation, we have

$$a_0 = 1 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,1} = -1$$
$$y_1 = 1 - 1(0.01 - 0) = 0.99$$
$$a_2 = \frac{\frac{f(x_2) - (x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.99} - [\frac{dy}{dx}]_{0,1}}{0.02 - 0.01} = 0.505$$
$$y_2 = 1 - 1(0.02 - 0) + 0.505(0.02 - 0)(0.02 - 0.01) = 0.980101$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = 1 - x$$
$$P_{0,2}(x) = 1 - 0.99495x$$
$$P_{0,1,2}(x) = 0.505x^2 - 1.00505x + 1$$

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution givenby Faith C. K [1], Table 2.

Example 3:

Solve $\frac{dy}{dx} = y - x, \quad y(0) = 0.5$

By taking the step h = 0.01
First by using Newton's interpolation, we have

$$a_0 = 0.5 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,0.5} = 0.5$$
$$y_1 = 0.5 + 0.5(0.01 - 0) = 0.505$$
$$a_2 = \frac{\frac{f(x_2) - (x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.505} - [\frac{dy}{dx}]_{0,0.5}}{0.02 - 0} = -0.25$$
$$y_2 = 0.5 + 0.5(0.02 - 0) - 0.25(0.02 - 0)(0.02 - 0.01) = 0.50995$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = 0.5 - 0.5x$$
$$P_{0,2}(x) = 0.5 - 0.4975x$$
$$P_{0,1,2}(x) = -0.25x^2 + 0.5025x + 0.5$$

Hence, we can take the approximation solution of linear and quadratic using Aitken Method, if we take quadratic using Aitken Method, we find the same solution givenby Faith C. K [1], Table 3.

Table 2: Solution of $\frac{dy}{dx} = x^2 - y, \quad y(0) = 1$

x	Combined Newton's Interpolation and Aitken	s
0	1	1
0.01	0.99	0.990050166
0.02	0.980101	0.980201326
0.03	0.970303	0.970454466
0.04	0.960606	0.96081056
0.05	0.950101	0.951270575
0.06	0.941515	0.941835466
0.07	0.932121	0.93250618
0.08	0.922828	0.923283653
0.09	0.913636	0.914168814
0.1	0.904545	0.905162582

Table 3: Solution of $\frac{dy}{dx} = y - x, \quad y(0) = 0.5$

x	Combined Newton's Interpolation and Aitken	s
0	0.5	0.5
0.01	0.505	0.504974916
0.02	0.50995	0.50999933
0.03	0.51485	0.514772733
0.04	0.5197	0.519594612
0.05	0.5245	0.524364451
0.06	0.52925	0.529081726
0.07	0.53395	0.533745909
0.08	0.5386	0.538356466
0.09	0.5432	0.542912858
0.1	0.54775	0.547414541

CONCLUSIONS

In this work, we havebeen combined the Newton’s interpolation and Aitken's method to solve first order differential equation, we find the same results given by Faith ChelimoKosgei studied this problem by combined the newton’s interpolation and Lagrange method. This method that we used Aitken interpolationinstead of Lagrange interpolation studied by Faith Chelimo Kosgei, is better than this method for the ease and speed of computational complexity in the Aitken method.

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