

## Empirical Modeling of the Compressive Strength of Concrete Based on Gravel-Sand Ratio and Water-Binder Ratio

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**Abstract:** Empirical modeling of the compressive strength of concrete was carried out based on gravel/ sand ratio and water-binder ratio. The range of process parameters considered were: 21.35 – 43.5 (Mpa), 0.93- 1.15, 0.48 -0.67 for compressive strength of concrete, gravel/ sand ratio, and water-binder ratio respectively. The hydration period was 28 days. The derived empirical model;  $V = 0.78 e^{2.79\beta} - 55.5\epsilon + 48.82$  evaluates the compressive strength of the concrete as the sum of a linear part and an exponential part of gravel/ sand ratio and water-binder ratio respectively. Model -predicted results show that the compressive strength of concrete increases with decrease in the water-binder ratio and increase in the gravel/ sand ratio in accordance with previous research. The validity of the model was rooted on the core model expression  $V - K=N e^{S\beta} - H\epsilon$  where both sides of the expression are correspondingly almost equal. The standard error incurred in predicting the model-based concrete compressive strength relative to the actual results was 0.65%. Computational analysis of actual and predicted results shows that compressive strengths of the concrete per unit gravel/ sand ratio and water-binder ratio were 56.3 & 55.7 (Mpa) and 187.5 & 185.7 Mpa respectively. The maximum deviation of model-predicted results with respect to actual results was < 5%. This translated into over 95% operational confidence levels for the derived model as well as 0.95 dependency coefficient of compressive strength of concrete on gravel/sand ratio & water-binder ratio. The correlation coefficients between compressive strength of concrete and gravel/ sand ratio & water-binder ratio were all > 94%.

**Key words:** Response - Concrete - Compressive strength - Water-cement ratio - Super plasticizer addition

### INTRODUCTION

Gravels are basic building materials resulting from natural disintegration of rocks which are at least 2 mm in diameter. Research [1] has shown that they are usually rounded and often require less amount of cement paste, thereby saving about (4-5) % cement paste.

Researchers [2, 3] have reported that the cost and availability of the aggregates are important when selecting an aggregate and quantity required for a typical civil engineering application.

Report [4] has shown that the technical-know-how on usage of locally available material in the most cost effective manner is basically one of the main challenges facing materials engineer on a project. Economy also affects the type of coarse aggregate being used. Granite is adjudged about twice more expensive than gravel.

This is because granite undergo more processing like blasting of the rock before the final consumer takes it up, whereas gravel is used as mined without any form of processing.

A team of scientists [5] investigated the strength of concrete made from three different types of coarse aggregate namely: crushed granite, washed gravel and unwashed gravel at 20 mm maximum size. Results from the investigation revealed that concrete made from crushed granite give the highest strength value followed by concrete from washed gravel and then unwashed gravel. The scientists concluded that the strength of concrete depends greatly on the internal structure, surface nature and shape of aggregates.

Investigation [6] was carried out on two samples mix of quarry dust. One sample with granite of 20 mm maximum and the other sample on sand and gravel of 28 mm. The results of the investigation revealed that

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the use of quarry dust and granite of 20 mm maximum size improved the concrete strength by 34 % over the strength of concrete with sand and gravel of maximum size 28 mm. Observation [7] has shown that aggregates are like the skeleton of concrete. Suggestions from the research indicate that the aggregates must not be coated to give enhanced compressive strength of concrete. Failure in concrete originates within the aggregate-matrix interface when the concrete mass is stressed. This is because the interface is the weakest medium of the composite system. Furthermore, irregular shaped aggregate with rough surface texture should be used in place of smooth and rounded ones to obtain greater compressive strength.

A study [8] conducted on Sand fines (clay/silt) and water-cement ratio has shown both parameters as causative factors of weakness in concrete strength. In this research, fines content in sand of 2%, 4%, 6%, 8%, 10% and 12% as well as water-cement ratio of 0.55, 0.6 and 0.7 were used. The concretes were prepared using the basic mix 1:2:4. Workability test on fresh concrete as well as compression and split tensile strengths were conducted in accordance with BS 1881. The results of the study revealed that workability of concrete decreased as fines content increases. At the same level of fines content, workability increases when the water-cement ratio increased. An increase in the compressive strength of concrete was recorded with increase in fines content up to 4%. There was a decrease in the compressive strength with increase in all water-cement ratios. The research also showed the same trend for the tensile splitting strength results. Based on the foregoing, the scientists concluded that, fines content of 4% in sand and water-cement ratio of 0.55 are most appropriate for production of structures-oriented concrete.

Results of investigation [9-16] on the use of silica fume in concrete mix revealed an enhancement to the concrete properties. The research also shows that binding materials play very important role in the quality, durability and strength of the cement mortar.

A similar investigation [17] regarding applicability of silica fume in concrete mix aimed at achieving a high strength concrete revealed clearly a higher increase in compressive strength and significant reduction in slump even though the average input concentration of super plasticizer was 0.75%. Further work [18] on applicability of silica fume in concrete mix was really aimed at partially replacing cement with silica fume and then investigating both physical and durability

properties such as cyclic freezing and thawing, sulphate attack and alkaline silica reactivity. The results of the investigation showed maximum compressive strength after a hydration period of 28 days on replacing 15% of cement with silica fume. In this research, a variable dosage of super plasticizer (0.35%) was added as water - binder ration.

The paper presents empirical modeling of the compressive strength of concrete based on grave-sand ratio and water-binder ratio.

### MATERIALS AND METHODS

The concrete cube size measuring 150x150x150mm in dimension was used. The batching of the concrete cubes was by weight. The concrete was produced using a range of process parameters: gravel/sand and water-binder ratio; 0.93 - 1.15 and 0.48- 0.67. The hydration period is 28 days. The cement used is Portland limestone Cement (PLC). The aggregates used conformed to BS877. The concrete cubes were lubricated with oil before the mixed concrete was placed inside it in order to reduce friction between the concrete and the cubes. When the concrete was properly mixed, the concrete cubes were filled one-third of their height and compacted 150 times. The cubes were later filled to two-third of their height and finally filled completely. In each of the layer, the concrete cubes were compacted 150 times respectively. The concrete cubes were cast and cured for 28 days respectively. At the end of the hydration period, the concrete cubes were crushed to determine, their compressive strength [19].

### RESULTS AND DISCUSSION

Table 1: Variation of compressive strength of concrete V with gravel/sand ratio  $\beta$  and water-binder ratio  $\epsilon$  respectively [19]

$(\beta)$	$(\epsilon)$	(V)
0.93	0.67	21.35
1.00	0.65	25.10
1.07	0.63	28.85
1.13	0.61	32.60
1.15	0.48	43.50

Computational analysis of the actual results shown in Table 1, gave rise to Table 2 which indicate that;

$$V = K = Ne^{Sp} - I_j \epsilon \tag{1}$$

Introducing the value of K, N, S,  $I_j$  and S into equation (1) reduces it to;

$$V - 48.82 = 0.78e^{2.79\beta} - 55.5\epsilon \tag{2}$$

Re-arranging equation (2) gives;

$$V = 0.78 e^{2.79\beta} - 55.5\epsilon + 48.82 \tag{3}$$

The derived model is equation (3).

where,

K = 48.82, N = 0.78, S = 2.79 and J<sub>γ</sub> = 55.5 are all equalizing constants. (determined using C-NIKBRAN [20])

(ε) = Water-binder ratio

(β) = Gravel/ sand ratio

(V) = Compressive strength of concrete (Mpa)

**Boundary and Initial Conditions:** A cube sized concrete block 150 x 150 x 150mm produced from a mixture of sand, aggregates and cement was considered and subjected to compressive test using appropriate crushing loads. The concrete is assumed to be unaffected by dissolved gases in the atmosphere.

The considered range of the compressive strength of concrete, gravel/ sand ratio and water-binder ratio are 21.35 – 43.5(Mpa) and 0.93 -1.15 and 0.48-0.67 respectively. The hydration period is 28 days.

Table 2: Variation of V - K with N e<sup>Sβ</sup> - J<sub>γ</sub>ε

V - K	N e <sup>Sβ</sup> - J <sub>γ</sub> ε
-27.47	-26.74
-23.72	-23.98
-19.97	-19.53
-16.22	-15.60
-5.32	-7.34

**Model Validity:** Equation (3) is the derived model. The validity of the model is rooted on the core model equation (1) where both sides of the equation are correspondingly almost equal. Table 2 also agrees with equation (1) considering values of V-K and N e<sup>Sβ</sup> - J<sub>γ</sub>ε evaluated from the actual results in Table 1. Furthermore, the derived model was validated by comparing the compressive strength of concrete predicted by the model and that obtained from the experiment. This was done using various analytical techniques which includes computational, statistical, graphical and deviational analyses.

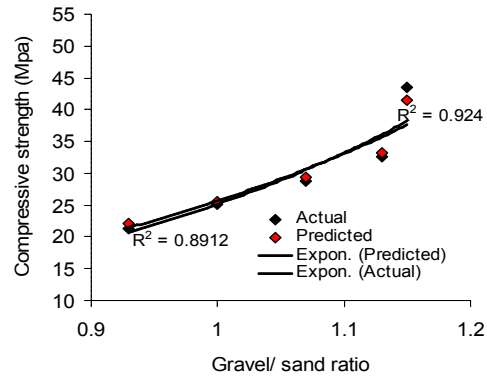


Fig. 1: Coefficient of determination between compressive strength of concrete and gravel/ sand ratio as obtained from actual and model-predicted results

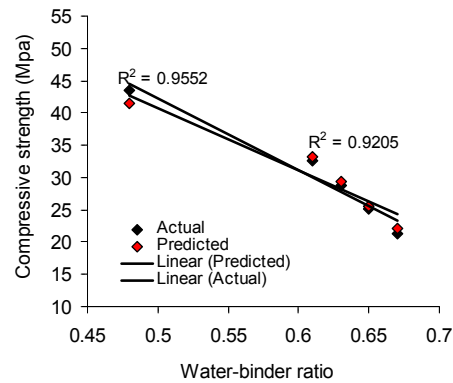


Fig.2: Coefficient of determination between compressive strength of concrete and water-binder ratio as obtained from actual and model-predicted results

**Computational Analysis:** Compressive strength of concrete per unit gravel-sand and water-binder ratios

The compressive strength of concrete per unit gravel/ sand ratio V<sub>β</sub> (Mpa) was calculated from the equation;

$$V_{\beta} = V / \beta \tag{4}$$

Re-written as

$$V_{\beta} = \Delta V / \Delta \beta \tag{5}$$

Equation (5) is detailed as

$$V_{\beta} = V_2 - V_1 / \beta_2 - \beta_1 \tag{6}$$

Where,

$V_{\beta}$  = Change in the compressive strengths  $V_2$ ,  $V_1$  at gravel-sand ratios  $\beta_2$ ,  $\beta_1$ .

Considering the points (0.93, 21.35) & (1.13, 32.6) and (0.93, 22.08) & (1.13, 33.22) as shown in Fig. 3, designating them as  $(\beta_1, V_1)$  &  $(\beta_2, V_2)$  for actual and model-predicted results, and then substituting them into equation (6), gives the slopes: 56.3 and 55.7 Mpa respectively as compressive strength of concrete per unit gravel- sand ratio.

Similarly, on considering the points (0.67, 21.35) & (0.61, 32.6) and (0.67, 22.08) & (0.61, 33.22) as shown in Fig. 4, designating them as  $(\epsilon_1, V_1)$  &  $(\epsilon_2, V_2)$  and also substituting them into equation (6), gives the slopes  $V_{\epsilon}$ : -187.5 and -185.7 Mpa to be the compressive strength of concrete per unit gravel- sand ratio as obtained from the actual and model-predicted results respectively.

The negative signs preceding the values indicate that the slopes of the curves (Fig. 4) relating compressive strength of concrete and water-binder ratio are all negative. Therefore, the real values of the compressive strength of concrete per unit water-binder ratio are 187.5 and 185.7 Mpa for the actual and model-predicted results respectively.

Previous work [19] indicates that the compressive strength of concrete increase with decrease in the water-binder ratio and increase in the gravel/ sand ratio. In accordance with the previous research work, results from the empirical model agree with the trend of relationship between the compressive strength of concrete and gravel/ sand & water-binder ratio.

**Statistical Analysis**

**Correlation:** The correlation coefficient between compressive strength of concrete and gravel/ sand ratio & water-binder ratio were evaluated from the coefficients of determination in Figs. 1 and 2 (designated as results of the actual and derived model) using equation (7). These results are 0.9440 and 0.9612 & 0.9773 and 0.9594 respectively.

$$R = \sqrt{R^2} \tag{7}$$

**Standard Error (STEYX):** The standard error incurred in predicting the model-based compressive strength relative to values of the actual results is 0.65%. The standard error was evaluated using Microsoft Excel version 2003.

**Graphical Analysis:** The validity of the derived model was further verified by plotting values of the actual,

besides the model-predicted results using Microsoft Excel (version 2003) to evaluate the trend of both results. Figs. 3 and 4 indicate very close alignment of curves and shapes which depicted significantly similar trend of data point's distribution for the actual and derived model-predicted compressive strength. This shows proximate agreement between both results.

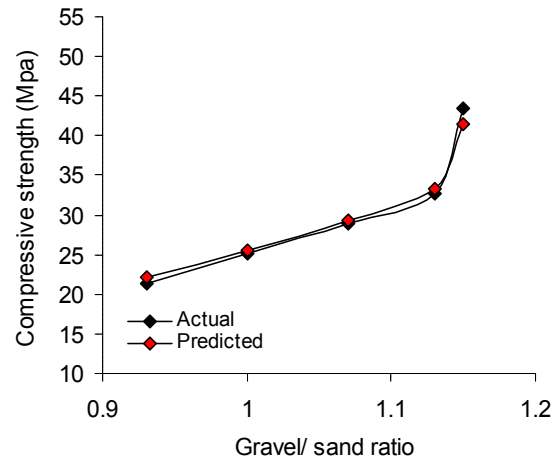


Fig. 3: Variation of concrete compressive strengths of concrete with gravel/sand ratio as obtained from actual and model-predicted results

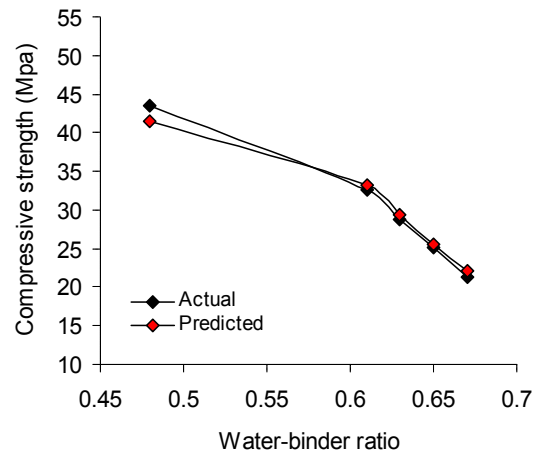


Fig. 4: Variation of compressive strengths of concrete with water-binder ratio as obtained from actual and model-predicted results

**Deviational Analysis:** A deviational analysis of the results of concrete compressive strength obtained from the actual and model-prediction shows little deviation on the part of model-predicted results (relative to the actual). This was attributed to the fact that the effects of the surface properties of the cement which played vital roles during the hydration were not considered during the model formulation. This necessitated the

introduction of correction factor, to bring the model-predicted concrete compressive strength to those of the corresponding experimental values.

The deviation  $D_v$ , of model-predicted compressive strength from the corresponding actual result was given by;

$$D_v = \left( \frac{V_p - V_e}{V_e} \right) \times 100 \quad (8)$$

where

$V_e$  and  $V_p$  are compressive strengths evaluated from experiment and derived model respectively

Fig. 5 shows that maximum deviation of model-predicted compressive strength from the actual results was less than 5%. This translates into over 95% model operational confidence. The figure shows that the least and highest deviations of model-predicted results (from actual results) are 1.34 and - 4.64 %.

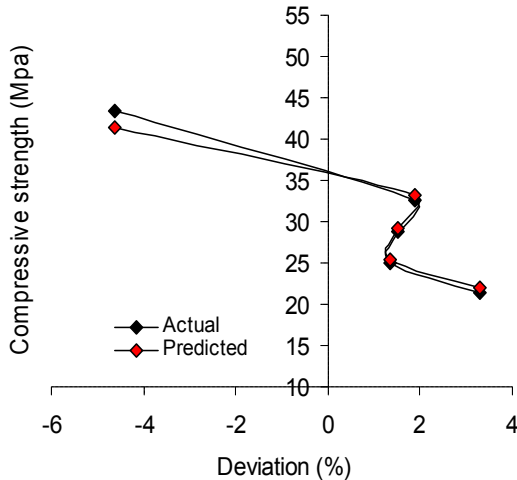


Fig. 5: Deviation of model-predicted results from actual values relative to compressive strength

These deviations correspond to model-predicted compressive strengths of concrete: 25.44 and 41.48 (Mpa); gravel/sand ratios: 0.65 and 0.48 and water-binder ratios: 1 and 1.15 respectively.

Correction factor,  $C_f$  to the model-predicted results was given by;

$$C_f = - \left( \frac{V_p - V_e}{V_e} \right) \times 100 \quad (9)$$

Comparative analysis of Figs. 5 and 6 show that the evaluated correction factors are negative of the deviation as shown in equations (8) and (9).

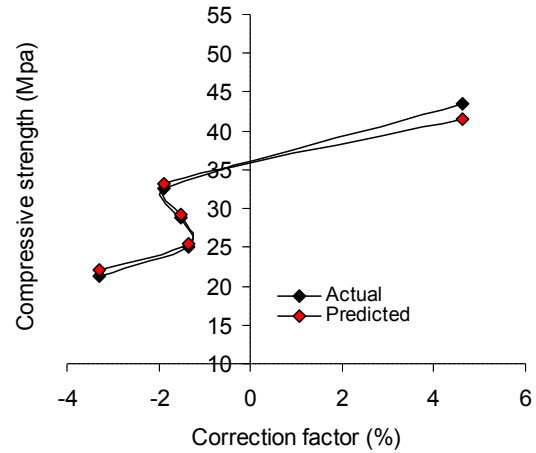


Fig. 6: Correction factor to model-predicted results relative to compressive strength

The correction factor took care of the negligence of operational contributions of the effects of surface properties of the cement which actually affected the concrete hydration process. Introduction of the corresponding values of  $C_f$  from equation (9) into the model gives exactly the corresponding actual compressive strength. Fig 6 indicates that the maximum correction factor to the model-predicted results was less than 5%. Fig 6 shows that the least and highest correction factors to the model-predicted results are - 1.34 and 4.64 %. These correction factors also correspond to model-predicted compressive strengths of concrete: 25.44 and 41.48 (Mpa); gravel/ sand ratios: 0.65 and 0.48 and water-binder ratios: 1 and 1.15 respectively.

It is pertinent to state that the negative and positive signs preceding numerals in reporting deviation and correction factors merely indicate deficit and surplus respectively. The actual deviation or correction factor is just the numeral.

## CONCLUSION

Empirical modeling of the compressive strength of concrete was carried out based on gravel- sand ratio and water-binder ratio. The derived empirical model;  $V = 0.78 e^{2.79\beta} - 55.5\epsilon + 48.82$  evaluates the compressive strength of the concrete as the sum two parts; a linear part and an exponential part of gravel-sand ratio and water-binder ratio respectively. Model-predicted results show that the compressive strength of concrete increases with decrease in the water-binder ratio and increase in the gravel/ sand ratio in accordance with previous research. The validity of the model was rooted

on the core model expression  $V - K=N e^{Sb} - H\epsilon$  where both sides of the expression are correspondingly almost equal. The standard error incurred in predicting the model-based concrete compressive strength relative to the actual results was 0.65%. Compressive strengths of the concrete per unit gravel/sand ratio and water-binder ratio were 56.3 & 55.7 (Mpa) and 187.5 & 185.7 Mpa as obtained from actual and model-predicted results respectively. The maximum deviation of model-predicted results with respect to actual results was < 5%. This translated into over 95% operational confidence levels for the derived model as well as 0.95 dependency coefficient of compressive strength of concrete on gravel-sand ratio & water-binder ratio. The correlation coefficients between compressive strength of concrete and gravel-sand ratio & water-binder ratio were all >94%.

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