

An Efficient Edge Detection Algorithm for Noisy Medical Images

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Abstract: Edge detection is an important technology in the image processing field, particularly in the edges detection and edges extraction. It is the ability to determine the edge of an object, is a primary step in many image enhancement procedures. The edge detection of medical images is existing since years but it is still challenging and scope of research. It has been found that the previous used algorithms were not able to produce optimized or ideal results in different cases. In most applications, medical images contain object boundaries and object shadows and noise. Therefore, they may be difficult to distinguish the exact edge from noise or trivial geometric features. In this paper, we propose a new efficient algorithm for edge detection of noisy medical images based on hyper Entropy. Hyper Entropy is defined in order to suppress noise and adapt to different edge in the image. The performance of proposed algorithm is compared against other methods using images corrupted with various levels of "salt & pepper".

Key words: Edge Detection • Medical Images • Entropy • Noisy Image

INTRODUCTION

Medical images are acquired by a range of techniques across all biological fields, which go far beyond the visible light photographs and microscope images of the early 20th century. Modern medical image may be considered to be geometrically arranged arrays of data samples which quantify such diverse physical phenomena as the diffusion of water molecules through and within tissue, or the time variation of hemoglobin deoxygenating during neuronal metabolism. Medical image processing is currently finding wide application in a large variety laboratory research and clinical practices. Biologists study cells and generate 3-D confocal microscopy data sets; virologists generate 3-D reconstructions of viruses from micrographs; radiologists identify and quantify tumors from MRI and CT scans; and neuroscientists detect regional metabolic brain activity from functional MRI scans and PET. Analysis of these diverse image types requires sophisticated computerized quantification and visualization tools.

Edge detection for Medical image is an important task toward complete image understanding. Since edges are important features of an medical image, great of significant

information contained in image edges. Edge pixels lie between pixels of different classes and exhibit distinct characteristics from other non-edge pixels. The edge detection has been used by target tracking, object recognition, segmentation and etc. A large number of Algorithms for edge detection have been introduced [1-15]. Examples of approaches to edge detection include algorithms such as the Sobel and Prewitt edge detectors which are based on the first order derivative of the pixel intensities [1]. For the second derivative, edges are detected as the location where the second derivative of the image crosses zero. The using of Laplacian-of-Gaussian (LoG) convolution mask is the most common method of the second derivative operator [4, 7, 10] and [13]. However, all of these algorithms tend to be sensitive to noise, which is an intrinsically high frequency phenomenon. Canny [16] proposed an edge detector to solve this problem, which combines a smoothing function with zero crossing based edge detection. Since none of the existing methods can produce the best results for all types of images and levels of noises, finding better method for edge detection is still an active area of research. Medical images are prone to noise and artifacts. Salt & pepper noise is a form of noise. It is typically

manifested as randomly occurring white and black pixels. Salt & pepper noise creeps into images in situations where transients, such as faulty switching fast.

In this paper we introduce a new algorithm to detect edges of noisy medical images based on information theory, which is entropy based thresholding. The proposed approach is decrease the computation time as possible as can and the results were very good compared with the other methods.

The paper is organized as follows: in Section 2 presents some fundamental concepts of the mathematical setting of the threshold selection. The proposed method of edge detection is devoted into Section 3. In section 4 we describe the proposed algorithm. In Section 5, some medical images will be analyzed using proposed algorithm, a comparison with some existing methods will be provided for these images and compare results of our algorithm against other techniques. At last conclusion of this paper will be drawn in Section 6.

Entropy Thresholding: Entropy is defined in terms of the probabilistic behavior of a source of information. Given events e_1, e_2, \dots, e_k occurring with probabilities p_1, p_2, \dots, p_k being k the total number of states, $\sum_{i=1}^k p_i = 1, 0 \leq p_i \leq 1$.

Shannon entropy [17, 18] is defined as:

$$S(p) = - \sum_{i=1}^k p_i \ln p_i \tag{1}$$

If we consider that a system can be decomposed in two statistical independent subsystems A and B , the Shannon entropy has the extensive property (additively)

$$S(A + B) = S(A) + S(B),$$

this formalism has been shown to be restricted to the Boltzmann-Gibbs-Shannon (BGS) statistics.

However, for non-extensive systems, some kind of extension appears to become necessary. Rényi [19-21] was able to extend Shannon entropy to a continuous family of entropy measures. There is extensive literature on the applications of the Rényi entropy in many fields from biology, medicine, genetics, linguistics and economics to electrical engineering, computer science, geophysics, chemistry and physics. The Rényi's entropy measure of order α of an image, $S_\alpha^R(p)$ is defined as:

$$S_\alpha^R(p) = - \frac{1}{1-\alpha} \left(\ln \sum_{i=1}^k p_i^\alpha \right) \tag{2}$$

where $\alpha \neq 1$ is a positive real parameter. This expression meets the BGS entropy in the limit $\alpha \rightarrow 1$. The Rényi entropy is non-extensive in such a way that for a statistical independent system, the entropy of the system is defined by the following pseudo additive entropic rule

$$S_\alpha^R(A + B) = S_\alpha^R(A) + S_\alpha^R(B) + (\alpha - 1) \cdot S_\alpha^R(A) \cdot S_\alpha^R(B) \tag{3}$$

The generalized entropies of Kapur of order α and type β [23, 24] is

$$S_{\alpha,\beta}(p) = - \frac{1}{\alpha - \beta} \ln \left(\frac{\sum_{i=1}^k p_i^\alpha}{\sum_{i=1}^k p_i^\beta} \right), \alpha \neq \beta, \alpha, \beta > 0 \tag{4}$$

In the limiting case, when $\alpha \rightarrow 1$ and $\beta \rightarrow 1$, $S_{\alpha,\beta}(p)$ reduces to $S(p)$ and when $\beta \rightarrow 1$, $S_{\alpha,\beta}(p)$ reduces to $S_\alpha^R(p)$. Also, $S_{\alpha,\beta}(p)$ is a composite function which satisfies pseudo-additive as:

$$S_{\alpha,\beta}(A + B) = S_{\alpha,\beta}(A) + S_{\alpha,\beta}(B) + (1 - \alpha) \cdot (1 - \beta) \cdot S_{\alpha,\beta}(A) \cdot S_{\alpha,\beta}(B) \tag{5}$$

When Entropy applied to image processing techniques, entropy measures the normality (i.e. normal or abnormal) of a particular gray level distribution of an image. When a whole image is considered, the Rényi entropy as defined in (5) will indicate to what extent the intensity distribution is normal. When we extend this concept to image segmentation, i.e. dealing with foreground (Object) and background regions in an image, the entropy is calculated for both regions and the subsequent entropy value provides an indication of the normality of the segmentation. In this case, two equations are need for each region, each of them called priori.

Assume that $f(a, b)$ be the gray value of the pixel located at the point (a, b) . In a digital image $\{f(a, b) | a \in \{1, 2, \dots, M\}, b \in \{1, 2, \dots, N\}\}$ of size $M \times N$, let the histogram be $h(g)$ for $g \in \{0, 1, 2, \dots, 255\}$ with f as the amplitude (brightness) of the image at the real coordinate position. For the sake of convenience, we denote the set of all gray levels as G . Global threshold selection methods usually use the gray level histogram of the image. The optimal threshold t^{opt} is determined by optimizing a suitable criterion function obtained from the gray level distribution of the image and some other features of the image [25].

Assume that t be a threshold value and $R = \{r_0, r_1\}$ be a pair of gray levels with $\{r_0, r_1\} \in G$. Typically r_0 and r_1 are taken to be 0 and 1, respectively. The result of thresholding an image function $f(a,b)$ at gray level t is a binary function $f_t(a,b)$ such that $f_t(a,b) = r_0$ if $f(a,b) \leq t$ otherwise, $f_t(a,b) = r_1$. In general, a thresholding method determines the value t^* of t based on a certain criterion function. If t^* is determined solely from the gray level of each pixel, the thresholding method is point dependent. Let $p_1, p_2, \dots, p_t, p_{t+1}, \dots, p_k$ be the probability distribution for an image with k gray-levels, where p_i is the normalized histogram i.e. $p_i = h_i / (M \times N)$ and h_i is the gray level histogram. From this distribution, we can derive two probability distributions, one for the object (class A) and the other for the background (class B), are shown as follows:

$$P_A : \frac{p_1}{P_A}, \frac{p_2}{P_A}, \dots, \frac{p_t}{P_A},$$

$$P_B : \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, \dots, \frac{p_k}{P_B}, \tag{6}$$

where

$$P_A = \sum_{i=1}^t p_i, \quad P_B = \sum_{i=t+1}^k p_i, \quad t \text{ is the threshold value} \tag{7}$$

The Renyi entropy of order α for each distribution is defined as:

$$S_\alpha^A(t) = \frac{1}{1-\alpha} \left(\ln \sum_{i=1}^t \left(\frac{p_i}{P_A} \right)^\alpha \right)$$

$$S_\alpha^B(t) = \frac{1}{1-\alpha} \left(\ln \sum_{i=t+1}^k \left(\frac{p_i}{P_B} \right)^\alpha \right) \tag{8}$$

The Renyi entropy $S_\alpha^R(t)$ is parametrically dependent upon the threshold value t for the foreground and background. We try to maximize the information measure between the two classes (object and background). When $S_\alpha^R(t)$ is maximized, the luminance level t that maximizes the function is considered to be the optimum threshold value. This can be achieved with a cheap computational effort.

$$t^{\text{opt}} = \text{Argmax} \left[S_\alpha^R(t) + S_\alpha^R(t) + (\alpha - 1) \cdot S_\alpha^R(t) \cdot S_\alpha^R(t) \right] \tag{9}$$

When $\alpha \rightarrow 1$, the threshold value in (3), equals to the same value found by Shannon Entropy. Thus this proposed method includes Shannon's method as a special case. The following expression can be used as a criterion function to obtain the optimal threshold at $\alpha \rightarrow 1$.

$$t_{\text{sh}}^{\text{opt}} = \text{Argmax} \left[S_\alpha^A(t) + S_\alpha^B(t) \right] \tag{10}$$

Now, we can describe the Renyi Threshold algorithm to determine a suitable threshold value t^{opt} and α as follows:

Algorithm 1: Threshold Value Selection (Renyi Threshold)

1. Input: A digital gray-scale noisy image I of size $M \times N$.

2. Let $f(x,y)$ be the original gray value of the pixel at the point $(x,y), (x=1,2,\dots,M, \quad y=1,2,\dots,N)$

3. Calculate the probability distribution

4. For all

I. Apply Equations (6) and (7) to calculate p_A, P_A, p_B and P_B

II. if $0 < \alpha < 1$ then

Apply Equations (8) and (9) to calculate optimum threshold value t^{opt} .

else

Apply Equations (8) and (10) to calculate optimum threshold value.

end-if

5. Output: The suitable threshold value t^{opt} of I , for $\alpha > 0$

In terms of the definition of Kapur entropy of order α and β the entropy of Object pixels and the entropy of background pixels can be defined as follows:

$$S_{\alpha,\beta}^A(p) = \frac{1}{\alpha - \beta} \ln \left(\frac{\sum_{i=1}^t \left(\frac{p_i}{P_A} \right)^\alpha}{\sum_{i=1}^t \left(\frac{p_i}{P_A} \right)^\beta} \right), \alpha \neq \beta, \quad \alpha, \beta > 0 \tag{11}$$

$$S_{\alpha,\beta}^B(p) = \frac{1}{\alpha - \beta} \ln \left(\frac{\sum_{i=t+1}^k \left(\frac{p_i}{P_B} \right)^\alpha}{\sum_{i=t+1}^k \left(\frac{p_i}{P_B} \right)^\beta} \right)$$

The Kapur entropy $S_{\alpha,\beta}(p)$ is parametrically dependent upon the threshold value t for the foreground and background. It is formulated as the sum each entropy, allowing the pseudo -additive property, defined in equation (5). We try to maximize the information measure between the two classes (object and background). When $S_{\alpha,\beta}(p)$ is maximized, the luminance level t that maximizes the function is considered to be the optimum threshold value [26, 27]. This can be achieved with a cheap computational efforts.

$$t^{\text{opt}} = \text{Argmax} \left[\begin{matrix} S_{\alpha,\beta}^A(t) + S_{\alpha,\beta}^B(t) + (1-\alpha) \cdot (1-\beta) \cdot \\ S_{\alpha,\beta}^A(t) \cdot S_{\alpha,\beta}^B(t) \end{matrix} \right] \quad (12)$$

When $\alpha \rightarrow 1$ and $\beta \rightarrow 1$, the threshold value in equation (4), equals to the same value found by Shannon's method. The following expression can be used as a criterion function to obtain the optimal threshold at $\alpha \rightarrow 1$ and $\beta \rightarrow 1$

$$t_{\text{sh}}^{\text{opt}} = \text{Argmax} \left[S_{\alpha}^A(t) + S_{\alpha}^B(t) \right] \quad (13)$$

Edge Detection: A spatial filter mask may be defined as a matrix w of size $m \times n$. So, we will use the usual masks for detecting the edges [1]. The process of spatial filtering consists simply of moving a filter mask w of order $m \times n$ from point to point in an image. At each point (x, y) , the response of the filter at that point is calculated a predefined relationship. Assume that $m = 2a + 1$ and $n = 2b + 1$, where 0 are nonnegative integers. For this purpose, smallest meaningful size of the mask is \ominus , as shown in Figure 1.

Image region under the above mask is shown in Figure 2. In order to edge detection, firstly classification of all pixels that satisfy the criterion of homogeneousness and detection of all pixels on the borders between different homogeneous areas. In the proposed scheme, first create a binary image by choosing a suitable threshold value using Renyi entropy. Window is applied on the binary image. Set all window coefficients equal to 1 except centre, centre equal to \times as shown in Figure 3.

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Then, the entropy of each Central Pixel of image under the window is calculated as $S(C_p) = -P_c \ln(P_c)$

where, P_c is the probability of Central Pixel C_p of binary image under the window. When the probability of central pixel $P_c = 1$ then the entropy of this pixel is zero.

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

Fig. 1: Mask coefficient showing coordinate arrangement

$f(x-1, y-1)$	$f(x-1, y)$	$f(x-1, y+1)$
$f(x, y-1)$	$f(x, y)$	$f(x, y+1)$
$f(x+1, y-1)$	$f(x+1, y)$	$f(x+1, y+1)$

Fig. 2: Image region under the above mask

1	1	1
1	\times	1
1	1	1

Fig. 3: Window coefficients

Table 1: P and S(C_p) of Central Pixel under Window

p	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9
S	0.2441	0.3342	0.3662	0.3604	0.3265	0.2703	0.1955	0.1047

Thus, if the gray level of all pixels under the window homogeneous, then $P_c = 1$ and $S=0$. In this case, the central pixel is not an edge pixel. Other possibilities of entropy of central pixel under window are shown in Table I.

In cases $P_c = 8/9$ and $P_c = 7/9$, the diversity for gray level of pixels under the window is low. So, in these cases, central pixel is not an edge pixel. In remaining cases, $P_c \leq 6/9$, the diversity for gray level of pixels under the window is high. So, for these cases, central pixel is an edge pixel. Thus, the central pixel with entropy greater than and equal to 0.244 is an edge pixel, otherwise not.

The following Algorithm summarizes the proposed technique for calculating the optimal threshold values and the edge detector.

Algorithm 2: Edge Detection

Input: A gray-scale image I of size $M \times N$, that has been calculated from algorithm 1.

2. Create a binary image: For all x, y , if $I(x,y) \leq t^{\text{opt}}$ then $f(x,y)=0$ else $f(x,y)=1$.

3. Create a mask w of order $m \times n$, in our case ($m = 3, n = 3$)

4. Create an $M \times N$ output image g : For all x and y , Set $g(x, y)=f(x, y)$.

5. Checking for edge pixels: Calculate: $a=(m-1)/2$ and $b=(n-1)/2$ For all $y \in \{b-1, b, \dots, N-b\}$ and $x \in \{a-1, a, \dots, M-a\}$, $sum=0$; For all $l \in \{-b, \dots, b\}$ and $j \in \{-a, \dots, a\}$, if $(f(x,y)=f(x+j,y+l))$ then $sum=sum+1$. if $(sum > 6)$ then $g(x,y)=0$ else $g(x,y)=1.6$. Output: The edge detection image g of I .

The Steps of Our Proposed Technique Are as Follows:

- Find global threshold value (t_1) using Renyi entropy. The image is segmented by t_1 into two parts, the object (A) and the background (B).
- By using Kapur entropy, we can select the locals threshold values (t_2) and (t_3) for A and B, respectively.
- Applying Edge Detection Procedure with threshold values t_1, t_2 and t_3 .
- Merge the resultant images of step 3 in final output edge image.

In order to reduce the run time of the proposed algorithm, we make the following steps: Firstly, the run time of arithmetic operations is very much on the $M \times N$ big digital image, I and its two separated regions, A and B. We are use the linear array p (probability distribution) rather than I , for segmentation operation and threshold values computation t_1, t_2 and t_3 . Secondly, rather than we are create many binary matrices f_j and apply the edge detector procedure for each region individually, then merge the resultant images into one. We are creating one binary matrix f_j according to threshold values t_1, t_2 and t_3 together and then apply the edge detector procedure one time. These modifications will reduce the run time of computations

RESULTS

To demonstrate the efficiency of the proposed approach, the algorithm is tested over a number of different grayscale images and compared with traditional operators. The images detected by Canny, LOG, Sobel, Prewitt and the proposed method, respectively. All the concerned experiments were implemented on Intel® Core™ i3 2.10GHz with 4 GB RAM using MATLAB R2007b.

The proposed scheme used the good characters of Renyi-Kapur entropy, to calculate the global and local threshold values. Hence, we ensure that the proposed scheme done better than the traditional methods.

In order to validate the results, we run the Canny, LOG, Sobel and Prewitt methods and the proposed algorithm 10 times for each image with different sizes. As shown in Figure. 4. It has been observed that the proposed edge detector works effectively for different gray scale digital images as compare to the run time of other methods.

Some selected results of edge detections for these test images using the classical methods and proposed scheme are shown in Figures.5, 6 and 7. From the results; it has again been observed that the performance of the

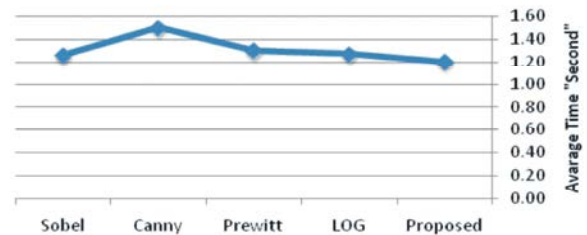


Fig. 4: Chart time for proposed method and classical methods with 512'512 pixel test images.

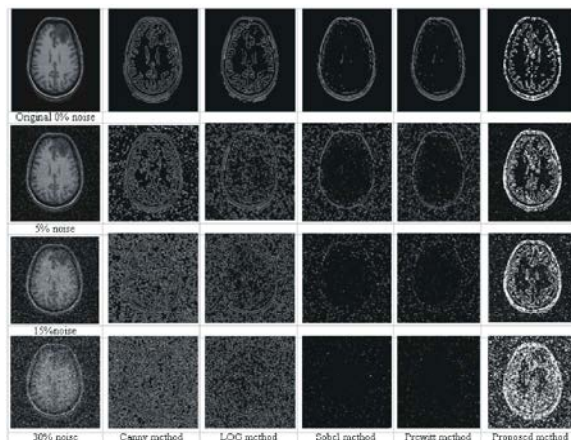


Fig. 5: Performance of Proposed Edge Detector for Brain-MRI image with Various salt and pepper noise

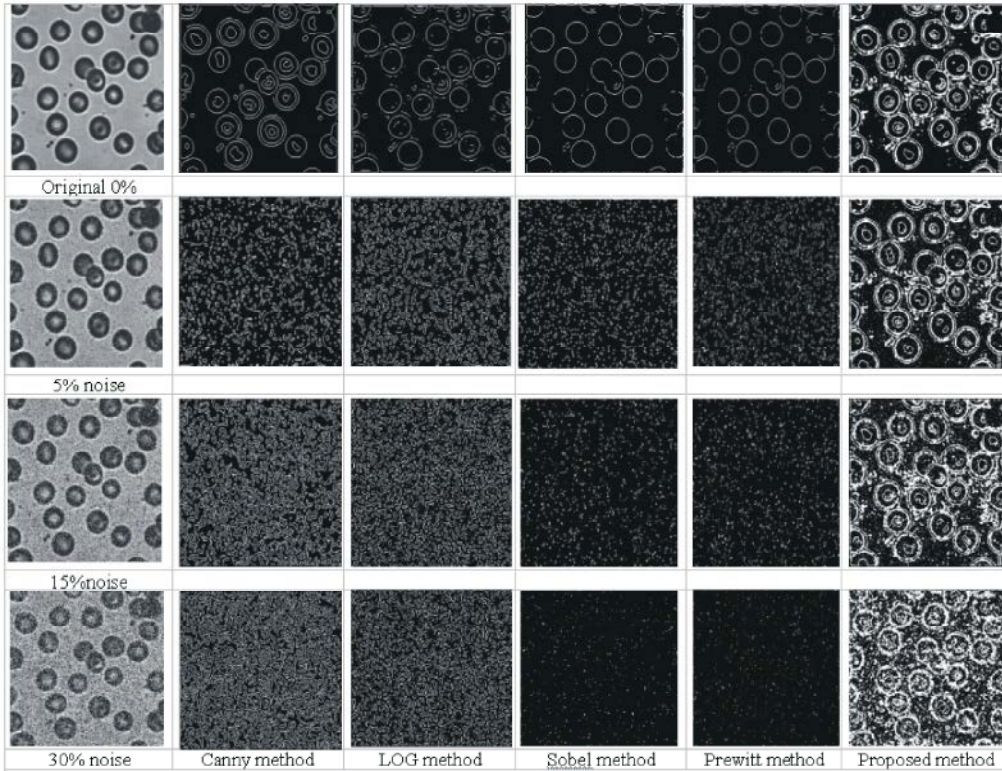


Fig. 6: Performance of Proposed Edge Detector for Blood cells image with Various salt and pepper noise

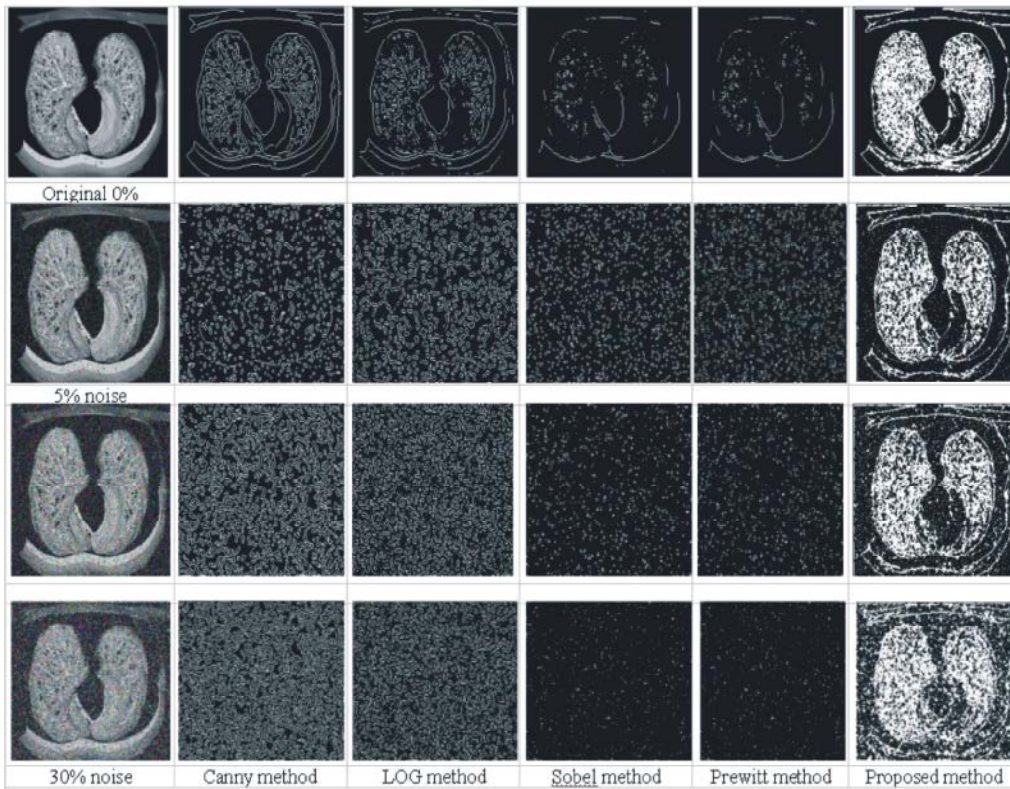


Fig. 7: Performance of Proposed Edge Detector for MRI image with Various salt and pepper noise

proposed method works well as compare to the performance of the previous methods (with default parameters in MATLAB)

CONCLUSION

In this article, a novel algorithm based Renyi-Kapur entropy types is proposed for edge detecting in noise medical images. The proposed algorithm is compared with traditional edge detectors. The experimental results proved that our algorithm is able to detect highest edge pixels in noise medical images. Also it decreases the computation time as possible as can with displayed high noise resilience. Another benefit comes from easy implementation of this algorithm.

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