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Gaussian Scale Mixture Model for Estimating Volatility as a Function of Economic Factor

[1]*R. Seethalakshmi*, [2]*V. Saavithri and* [3]*C. Vijayabanu*

[1,3] SASTRA University, Thanjavur, Tamil Nadu, India
[2]Nehru Memorial College, Tiruchirappalli, Tamil Nadu, India

Abstract: In this paper the scale mixture of Gaussian distribution is used to model the stock return data in financial market. There are many volatility models and forecasting methods. Some of the models are Historical volatility models, Implied volatility models, Autoregressive Conditional Heteroskedasticity models, models based on Artificial Neural Network. All these models are direct models. In these models the influence of economic factors like price level uncertainty, riskless rate of interest, the equity risk premium and the ratio of expected profit to expected revenue for the economy are not taken into account. Here the volatility parameter 'σ' is treated as a function of an economic factor. The main economic factor considered is the ratio of expected profit to expected revenue. Economic ratio is assumed to follow the exponential distribution. The resultant distribution is fitted to Dow Jones Industrial Average (DJIA) data by estimating the parameters. It is observed that this mixture distribution is a better fit than the GARCH fit.

JEL Classification Code: C1
Key words: Scale mixture of Gaussians · Volatility · Exponential distribution · Laplace distribution · Economic factor · GARCH.

INTRODUCTION

The main characteristic of any financial asset is its return. Return is typically considered to be a random variable. An asset's volatility which describes the spread of outcomes of this variable, plays an important role in numerous financial application. It is a key parameter for pricing financial derivatives. It is input parameter in financial models. Volatility can be forecasted over different time ranges. There are many volatility models and forecasting methods. Some of the models are Historical volatility models, Implied volatility models, Autoregressive Conditional Heteroskedasticity models, models based on Artificial Neural Network. All these models are direct models. In these models the influence of economic factors like price level uncertainty, riskless rate of interest, the equity risk premium and the ratio of expected profit to expected revenue for the economy are not taken into account. Volatility is a function of all these four factors. Remarkable changes in market volatility during the great depression periods are mainly due to change in the ratio of expected profit to expected revenue for economy. Volatility is negatively related to the ratio of expected profit to expected revenues at a time period. This ratio is not obvious but impact volatility because it affects the current value of the market index and standard deviation of future stock prices. The ratio of expected profit to expected revenue captures the effect of two well known determinants of volatility: financial leverage and operating leverage. The greater the amount of dept in the economy, the smaller are expected profit at time one and the greater will be stock market volatility. So an increase in labor costs decreases expected profit at time one and increases stock market volatility. So it is very essential to have a volatility model incorporating the effect of the important economic factor namely, the ratio of expected profit to expected revenue.

Main objective of this study is to predict the volatility as a function of economic factor, the mean economic ratio for a particular time period. The scale mixture of normal distribution is used for modeling the stock return data in financial markets. The volatility σ, parameter of normal distribution, is considered as a random variable Z which is the inverse ratio of expected

Corresponding Author: R. Seethalakshmi, SASTRA University, Thanjavur, Tamil Nadu, India.

profit to expected revenue. Daily data from Dow Jones Industrial Average(DJIA) for the time period 07/01/2002 to 31/12/2003 is considered for this study. The scale mixture distribution is fitted to this data and is compared with GARCH model using AIC index.

This paper is organized as follows: In section 2, probability density function of Normal-Exponential mixture is given. Section 3 gives the model description. In section 4 data is analyzed and section 5 gives the conclusion.

Normal–Exponential Mixture: A,F,Andrews and C.I.Mallows (1974) [1] showed that if the probability density function of some random variable X, $P_X(x)$ is symmetric about zero and the derivatives $P_X(x)$ satisfy

$$\left(-\frac{d}{dx}\right)^k P_X(x) \geq 0 \quad for\ x > 0 \qquad (2.1)$$

then there exists independent variables Y and Z, with Y being a standard normal variable, such that

$$X = \sqrt{Z}\, Y \qquad (2.2)$$

The variable Z is allowed to take on only positive values. A random variable X, which can be expressed as in (2.2) is referred to as a normal variance mixture model or a scale mixture of Gaussians. If the mean of X should be non-zero, (2.2) may be modified by adding a scalar μ corresponding to the actual mean value. Now $P_Z(z)$ is the pdf of Z. The marginal probability density function of X is obtained by averaging over Z, as

$$P_X(x) = \int_0^\infty \frac{1}{\sqrt{2\pi z}} e^{-\frac{(x-\mu)}{2z}}\, p_Z(z)\, dz \qquad (2.3).$$

If Z is an exponential stochastic variable with pdf

$$P_Z(z) = \frac{1}{\lambda} e^{-\frac{z}{\lambda}} \qquad (2.4)$$

and Y is a standard normal variable, then X, generated as $X = \mu + \sqrt{Z}\, Y$, will have pdf $P_X(x) = \int_0^\infty \frac{1}{\sqrt{2\pi z}} e^{-\frac{(x-\mu)^2}{2z}} \frac{1}{\lambda} e^{-\frac{z}{\lambda}} dz$ $= \frac{1}{2}\sqrt{\frac{2}{\lambda}} \exp\left(-\sqrt{\frac{2}{\lambda}}|X-\mu|\right)$ which is the pdf of a Laplace distribution centered at μ. If (X_1, X_2, …..X_n) is a random sample from the scale mixture population then μ is the actual mean value which is known and the maximum likelihood estimator of λ, the variance, is $2\left[\sum_{i=1}^{n}\left|\frac{x_i-\mu}{n}\right|\right]^2$.

Model Description: Many models have been developed for price returns under the assumption that the price return have a finite constant volatility over a given period of time. The first and most common model is Bacheilier - Osborne model [2-4] in 1959. But practically volatility cannot be a finite constant. Hence the idea of infinite volatility appeared. It was introduced by Mandellbrot [5] in 1963. GARCH models [Bollerslev (1986)] assumes non constant volatility. John. J.Binder and Matthias J.Merges [6] (2000), examines the ability of rational economic factors to explain the stock market volatility. Tobias Oliveny and Kennedy Omundi, 2011 [7], studied the effect of macroeconomic factors on stock return volatility in the Nairobi stock exchange, It is more apt to consider volatility as a function of economic factor in a particular time interval.

In this paper, the price returns are calculated using the formula $r_t = \log \frac{p_t}{p_{t-1}}$ where P_t is the closing price at time t and P_{t-1} is the closing price at time t-1. The mean return is evaluated using the actual formula. It is assumed that the return follows normal distribution with parameters μ, the mean and σ, the standard deviation. The volatility σ is assumed to follow exponential distribution with parameter λ. The volatility $\sqrt{\lambda}$ is obtained for the data under consideration. λ is estimated as a parameter of the Laplace distribution which is obtained as a result of mixing of Gaussian and Exponential distribution. It is observed that there is a decrease in volatility as there is an increase in mean ratio of expected profit to expected revenue.

Data Analysis: This paper focuses three companies of DJIA namely American Express Company(AXP), Bank of American Corporation(BAC) and Caterpillar Inc(CAT). The daily data is downloaded fromwww. yahoofinance. com for the period from 07, Jan, 2001 to 31,Dec,2003. Data sets are composed of trading days only i.e. weekends and bank holidays have been removed.

The following Figures [4.1-4.3] give the Laplace density curves for the return X.

For a given value of λ, the parameter of exponential distribution, the following curves (4.4-4.6) give the density curves P(z) for the simulated values of Z.

Table 4.1

	GARCH						Scale Mixture			
	Parameters						parameters			
Company	c	K	GARCH (1)	ARCH (1)	Log likely hood value	AIC	μ	λ	Log likely hood value	AIC
Dow AXP	0.001165	6.22E-06	0.90593	0.080078	1237.5	-2467	4.98E-04	5.72E-04	1693.2	-3382
Dow BAC	0.000389	4.54E-05	0.72102	0.1109	1361.4	-2714.9	4.85E-04	2.86E-04	1866.5	-3729
Dow CAT	0.000883	3.95E-05	0.83689	0.059405	1262.7	-2517.5	8.92E-04	4.50E-04	1753	-3502

From the above table it is seen that the scale mixture distribution is a better fit than the GARCH distribution.

Table 4.2:

Company (DJIA)	Volatility (sqrt(λ))	Mean economic ratio (λ)
AXP	0.04381038	0.0005721
BAC	0.030982136	0.0002861
CAT	0.03887017	0.0004503

From the above table it is observed that the volatility decreases as the mean economic ratio increases.

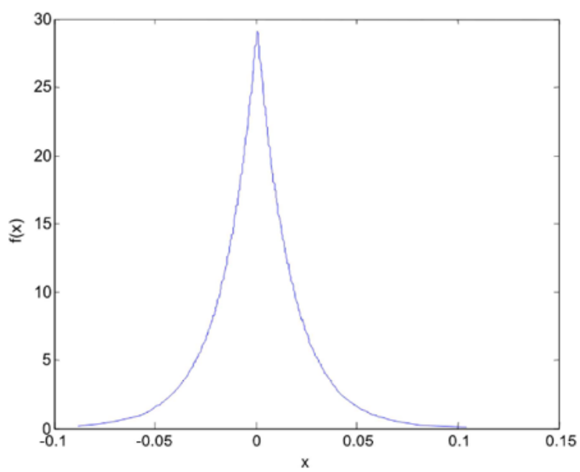

Fig 4.1: Dow Axp Laplace density curve

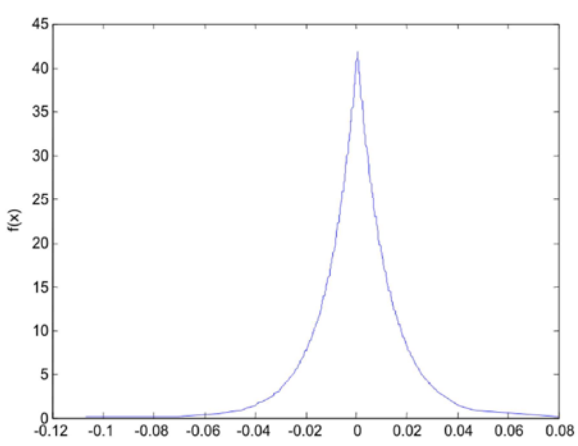

Fig 4.2 - Dow BAC Laplace density curve

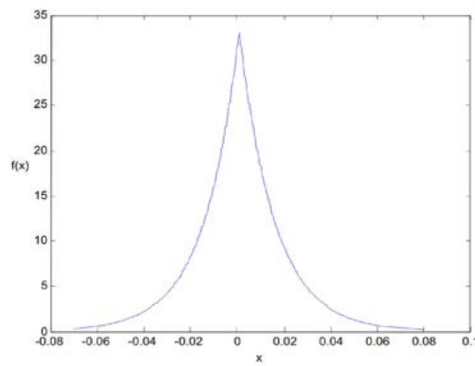

Fig 4.3: Dow CAT Laplace density curve

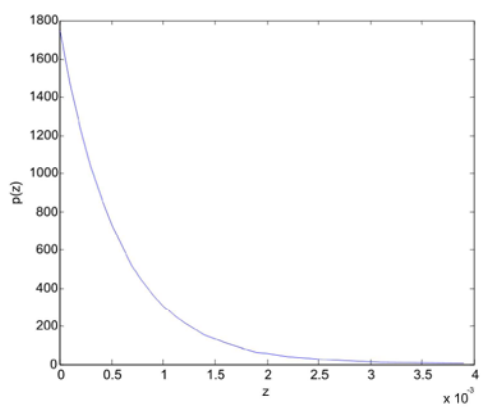

Fig 4.4: Dow AXP exponential density curve

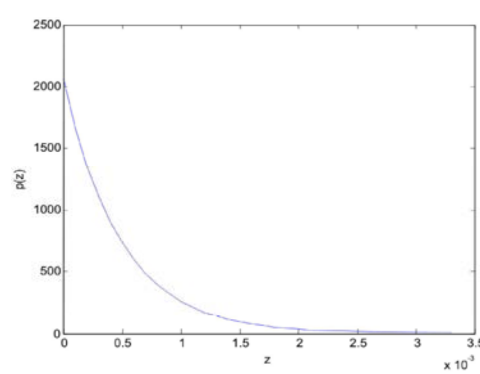

Fig 4.5: Dow BAC exponential density curve

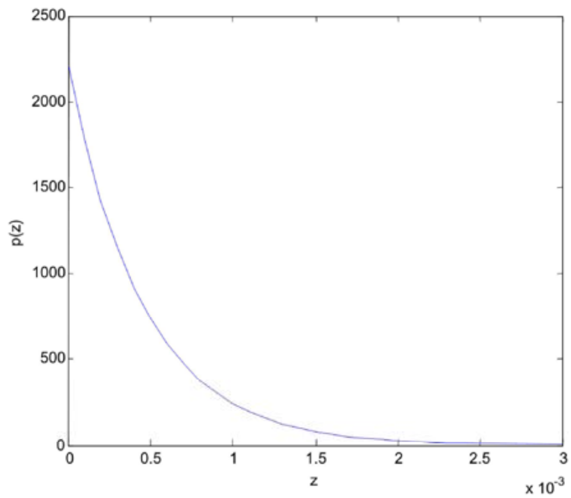

Fig. 4.5: Dow CAT exponential density curve

CONCLUSION

This work is focused on the effect of economic factor in volatility of financial assets. It is shown that the scale mixture distribution is better fit than that of GARCH for financial return series whenever the volatility parameter is assumed as a function of one of the economic factors, the ratio of expected profit to expected revenue. The effectiveness of the model can be improved by incorporating the role of other economic factors also.

REFERENCES

1. Andrews, A.F. and C.L. Mallows, 1974. “Scale mixtures of normal distributions”, Journal of Royal Statistical Society Series B, 36(1): 99-102.
2. Osborne, M.F.M., 1959a. Brownian Motion in the Stock Market, Operations Research, 7: 145-173.
3. Osborne, M.F.M., 1959b. Brownian Motion in the Stock Market, Operations Research, 7, 807-811.- 48.
4. Bachelier, L., 1900. Theory of speculation, A thesis presented to the Faculty of Sciences of the Academy of Paris on March 29, 1900. Originally published in Annales de l'Ecole Normale Supérieure, 27: 21-86.
5. Mandellbrot, B.B., 1963. The variation of certain speculative prices. Journal of Business, 36: 392-417.
6. John, J.Binder and Matthias J. Merges, 2000).“Stock market volatility and economic factors”, Review of quantitative finance and accounting.
7. Tobias Oliveny and Kennedy Omundi, 2011. “The effect of Macro economic factors on stock return volatility in the Nairobi stock exchange, Kenya”, Economics and Finance Review, 1(10): 34.