

Construction of Primary Fuzzy Cognition Mapping Formathematical Intuition

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Abstract: Exploring into the topic of mathematical intuition needs a self efficacy for balancing intuition and intention. In this paper we discuss the former with the help of symbol grounding problem and the directions for getting the solution to the same. Initially we establish the hypothesis of grounding the notion of intuition with non-symbols which are not members of symbol system as quoted by Harnad and others. Using the framework for fuzzy cognition mapping (FCM), we introduce a new derived mapping of FCMs as Primary Fuzzy Cognition Mapping (PFCM). Finally we construct a PFCM for mathematical intuition with causal relationships with the members in the family of undecidable problems like equivalence between NP and P class of problems G?del's incomplete theorem, Turing Halting problem, Symbol Grounding, etc.

Key words: Mathematical Intuition • Symbol Grounding • G,□del's theorem • Turing Machine • NP Problem
 • Fuzzy Cognition Mapping

INTRODUCTION

Intuition being not based on calculated reasoning; it is a kind of perception without conscious logic. Perception and mathematical intuition have been seen as sources of the non-inferential knowledge.

We notice the evolution of various concepts either by intension or intuition.

- The field of Operations Research was invented out of necessity. During the time of World War II, the search of solution for the set of specific transportation problems led to the framework for Operations Research. Other rich set of problems later found to be inside this framework.
- Boolean Algebra was invented not by any specific application necessary as in (i). Later specific applications of digital systems like computer components were made as by-product of such frameworks.

The above points illustrate the nature of intuition particularly its orientation, either start from intensive thought for solution to generalisation which can be used for family of problems or starting from generalised structures to specific solutions. One can see the

intensions after intuition and the occasions where intuition leads to various intensions for applications.

The emergent behaviour of a team or colony is not same as that of individual member. There seems to be no logical reasoning for this difference.

The sampling of any continuous signal has intervals of infinite discrete points between two samples. Integers are seemed to be based on intension and irrational cuts on real line tend to be gaps for manifestations of intuition.

Gödel's Incompleteness Theorem states [13] that in any consistent formalization of mathematics that is strong enough to define the concept of natural numbers, one can construct a statement that can be neither proved nor disproved within that system.

The halting problem [11] in classical computer science is a decision problem about properties of computer programs on a fixed Turing-complete model of computation, i.e. all programs that can be written in some given programming language that is general enough to be equivalent to a Turing machine. The problem is to determine, given a program and an input to the program, whether the program will eventually halt when run with that input. In this abstract framework, there are no resource limitations on the amount of memory or time required for the program's execution; it can take arbitrarily long and use arbitrarily much storage space, before halting. The question is simply whether the given program

will ever halt on a particular input. In section 2 we introduce the definition and terminology to understand the following results. We further introduce Turing Machine, NP Problems and notion of undecidable problems and we discuss the construction of a fuzzy cognition mapping for the concept of intuition and various relationships with the concepts introduced in the previous sections. Viewing of such maps strengthens the role of intuition in mathematical or computational aspects.

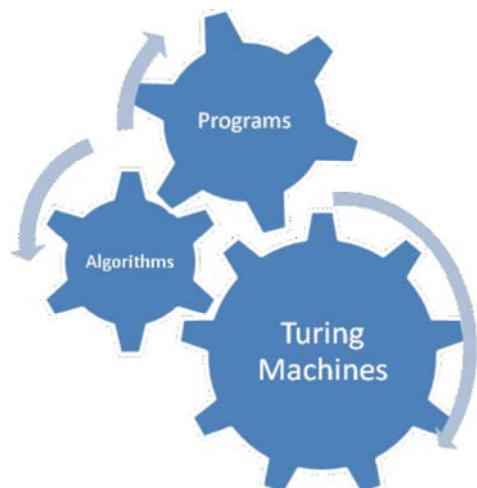
Definitions, Terms and Concepts: In this section we define a relation type with fuzzy weights due the importance of causal relations in cognitive maps. There is a causal relation between two given concepts whenever a relative variation in one of those concepts.[14-15]

Fuzzy Cognitive Maps (FCM) demonstrate cognitive mechanisms in the form of fuzzy directed graphs with the nodes, which basically correspond to “*concepts*” bearing different states of activation depending on the knowledge they represent and the “*edges*” denoting the causal effects that each source node exercises on the receiving node expressed through weights values in the interval $[-1,1]$, which denotes the positive, negative or neutral causal relationship between any two concepts.

Definition: A Primary Fuzzy Cognition Mapping (PFCM) is defined as a special type of FCM where centre node (primary concept) is specially designated and some of the concepts are surrounded with equal weights on the incident links. We call it primary layer. Rest of the concepts are connected to the concepts in the primary layer.

In this paper we select the centre node be the theme “Mathematical Intuition” and the concepts surrounding this is selected computing and cognitive domains.

Turing Machines and Decidability



Finite control of Turing machines cover universal phenomena and shows the existence of unsolvable problems. Decidable problems can be found solution by finite steps of the algorithms either in polynomial time or exponential time. Whether the class of problems that can be solvable by polynomial time (P) is equivalent to the class of problems that can be solvable by Non-deterministic Turing machines in polynomial time (NP) is still undecidable. Turing machines work with pure symbol systems for encoding the input problems and the output is interpreted for its semantics.

Sub-symbolic and Symbolic Systems: AI Research community believes the cognition may in principle be captured by set of computational models. This is the part of mathematical intuition we would like to use here for adding components/concepts to the PFCM.

John Searle [5-9] and others proposed doctrine of computation with strong and weak classification in AI. Weak AI in the literature refers to the reality which holds merely that the computer can be used as powerful tool for understanding the mind. But the “Strong AI” states that the appropriately programmed computer really has (or is) a mind. [1-3]

The three fundamental observations namely, (i) Syntax alone cannot determine semantics (ii) Mind contains semantic contents (iii) Computer programs are entirely made up of syntactical structure, imply a program or mathematical expression of operators never sufficient for having a mind.

Newell and Simon [4] found “Physical Symbol System Hypothesis” as a physical symbol system has the necessary and sufficient means for general intelligent action.

A symbol, then, is an atomic entity, designating some object or concept, which can be manipulated explicitly by a physical symbol system, leading to intelligent behaviour.

Smolensky [10] proposed “*Subsymbolic Hypothesis*” which clearly states that the intuitive processor is a subconceptual connectionist dynamical system that does not admit a complete, formal and precise conceptual level description.

These differences further instantiate the mathematical intuition through mathematical models either like Turing Machines which is a basic representation of ‘digital computer’ in a sequential manner or Neural Network which represents ‘brain neurotic connections’ in a parallel manner. In this paper, we connect these two extremes to establish the existence of elements based on mathematical intuition in between them. As a recent achievement in BCI (brain computer interaction), we justify the usage of

Table 1: Differences in Symbolic and Sub-Symbolic Approaches.

Symbolic	Subsymbolic
Computational level coincides with the representational level.	Computational level lies beneath the representational level.
Objects of computation are also objects of semantic interpretation	Objects of computation are more fine-grained than the objects of semantic interpretation

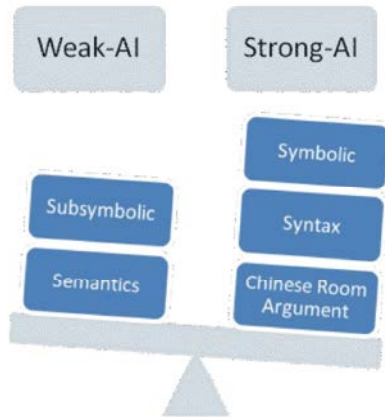


Fig. 1: Weak and Strong AI

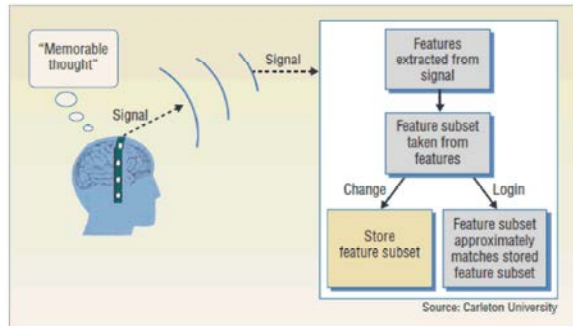


Fig. 2: BCI framework

thought as signal which acts as a password as shown in Figure 2. BCI achieved authorisation of login as shown. This demonstrates thought as material process as part of constructing alphanumeric strings as passwords intentionally. In the long run this type of intimate coupling of external computing resources with brain will establish applications in quite unexpected directions.

Following is the diagram for the PFCM for mathematical intuition theme generated with the help of profound fundamental issues in computer science. There are ample evidences for these issues to be qualified to support the central theme. We call it MI-PFCM.

The example in Figure 3 is not complete in the sense that one can add more concepts in the primary layer as when new theories based on innovative concepts arises either purely intuition or novel combination of earlier concepts.

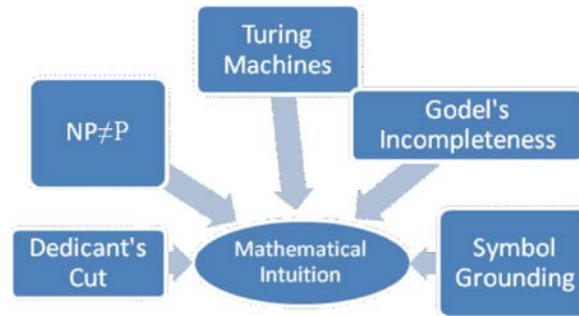


Fig. 3: PFCM-MI

Proposed Framework and Main Results: The PFCM-MI is proposed for collecting the related concepts as nodes. We just concentrate on the primary layer. This doesn't cover the whole picture. The non-primary layers are considered in this section. We shall weave or augment the node concepts in these layers using the chains as follows:

- Halting problem → Universal Turing Machines → Recursively Enumerable sets → Finite state machines → ...
- $NP \neq P$ → Undecidable Problems → Time Complexity of Algorithms → Cooks Theorem → Satisfiability Problem → TSP → Decidable Problems → Finite State Machines → ...
- Symbol Grounding → Symbol system → Chinese Room Argument → Strong AI → Weak AI → ...
- Godel's theorem → Axioms → Set theory → Church Thesis → Theorem proving → ...
- Dedicator's cut → Irrational Numbers → Rational Numbers → Integers → Counting → Godel's Sequences → ...
- Semiotic Networks → Neural Networks → Classification → Connectionism → Semantic Networks → Ontologies

The above chains form the edges or causal relationships for the PFCM-MI and it is not unique as anyone can view and rearrange the concepts in many ways. Here our main interest is to explore the existence of mathematical intuition through these concepts of computing.

We do not cover all the elements in the list of chains 1-6 except few as each diverges into many more domains of interest.

This approach not only gives general framework for total view but also the understanding of interconnections of these chains. The weights associated on the links are fuzzy values from the set {very low, low, medium, high and very high}. The membership values evolves by more inventions, more changes in priorities of importance and influence over the neighbourhood nodes or new connections in different layers. The explicit values are intuitive in nature and they are claimed to exist with the theoretical foundations in the above arguments.

Remarks: One can construct similar PFCMs for mathematical intuition manifested with the concepts in other scientific domains also.

The Primary fuzzy cognitive mapping has the potential for filling the gap between intuition and intension that is exemplified by the concepts linked in the mapping.

The non-primary chains shown in the list in section 3 can be further dealt with similar fuzzy relationships and more concepts in order to study any local theme as central node in such fuzzy maps.

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