

About Stabilizing Control Bilinaer Systems

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Abstract: Currently, there is a method of theoretical investigation of complex processes that allow a mathematical description, a computer-experiment, i.e. natural science research concerns the methods of computational mathematics. Mathematical study precedes the choice of approximation, ie the question of what factors should be taken into account and which can be ignored. After that, we study the problem by computer simulation. Stability and stabilization problem of the system describing biological process is investigated in the work. Stabilizing control for investigating model which provides stability or solution on determined time interval is obtained.

Key words: Biological system • Model • Control • Equation

INTRODUCTION

We develop a method of regularization and stabilization tasks product shall prevail not based on an explicit parameterization of the original data and the use of the theory of multi-tasking is displayed [1-4]. Also in the works is set strong instability integral equation of the inverse problem of applied biophysics [5]. For the mathematical modeling of biological processes as an independent research area and a characteristic special is the consideration of biological systems as a control object by simulating with the necessary completeness of their behavior through the abstract mathematical model. Obviously, the adequacy of the model of the real system will be determined by the degree of scrutiny of the properties, characteristics and patterns of behavior of real systems, their internal structures in a changing environment. It should be emphasized in the class Biosystems we allocate and maps the cybernetic system which consist of a plurality of elements, combined process information and operating together to achieve a particular objective with high efficiency. The particular form of measuring the effectiveness of the goal depends on determinate conditions and objectives of the system.

Analysis of the biological system and control by them on base of biological models, if interaction between components can be described with help of the variables of linear equation accepting the values from some ensemble and defining action to hierarchy.

In the work system is considered on base bilinear models. Biological system requires overcoming the rigid restrictions connected with requirements of linearity. It's other feature is changeable structure. This characteristic is important in that cases, when restrictions are interposed on controls.

Lemma. Control the type $u^{\circ} = -D^*(t)K(t)x$, $t \in [t_0, t]$ realizes stabilization of the moving the system:

$$\dot{x}(t) = B(t)x(t) + D(t)u$$

$$x(0) = x_0; 0 \leq t \leq T$$

proof of the lemma it is possible to find in [6-8].

Statement of the Problem: Choosing of control $u=u(t)$ is required stabilize motion on final time interval.

Let $\|u\| \leq M, M = \text{const} \geq 0$ then stabilizing control possible to represent in the form:

$$u^0 = -D^*(t) K(t) x, t \in [t_0, T]$$

Process of control by periodic current disease is taken for solving of the problem

This process can be described:

$$\dot{x}_1 = k_{x1}(x_2 - x_1),$$

$$\dot{x}_2 = \beta x_2 - x_2 x_3 - 3,$$

$$\dot{x}_3 = k_{x3}(\sigma x_1 - x_2 x_3 - x_3)$$

with initial condition:

$$x_1(0) = x_1; x_2(0) = x_{20}; x_3(0) = x_{30};$$

where first equation describes concentration of mature plasmazits, second equation concentration of antigens; third equation describes concentration an antibody; β -coefficient of reproduction of carriers of antigen determinant; σ -coefficient of ratio of speed of growth of antibodies by mature plasmazits to concentration of immature plasmazits [9]. We use linearization system of the model, which get, neglecting member $x_2 x_3$.

Then matrix of the system is of the form of:

$$B = \begin{pmatrix} -k_{x1} & k_{x1} & 0 \\ 0 & \beta & 0 \\ k_{x3}\sigma & 0 & -k_{x3} \end{pmatrix}$$

Now with help the matrix we calculate own values the matrix:

$$\lambda_1 = -k_{x1}, \lambda_2 = \beta, \lambda_3 = -k_{x3}$$

As decisions unstable and we add stabilizing control to considered models on final $t \in [t_0, T]$ interval:

$$u^0 = -D^*(t) K(t) x, [10].$$

Fundamental matrix is defined from equation:

$$\dot{\phi}(t) = B(t)\phi(t)$$

where

$$\phi(t) = \begin{pmatrix} e^{-k_{x1}t} & \alpha_1(e^{\beta t} - e^{-k_{x1}t}) & 0 \\ 0 & e^{\beta t} & 0 \\ \alpha_5(e^{-k_{x1}t} - e^{-k_{x3}t}) & \alpha_2 e^{\beta t} - \alpha_3 e^{-k_{x3}t} + \alpha_4 e^{-k_{x3}t} & e^{-k_{x3}t} \end{pmatrix}$$

Now we calculate:

$$Q(t) = \phi^{-1}D$$

Then we have:

$$Q(t) = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ 0 & Q_{22} & 0 \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix}$$

After we calculate elements of the matrix:

$$R(t, T) = \int_{t_0}^T Q(\tau) Q^*(t) d\tau$$

Hereinafter we find:

$$W(t, T) = \phi(t) R(t, T) \phi^*(t)$$

Thereby, solution of the considered system under stabilizing control has the form:

$$x_1 = (\phi_{11}R_{11} + \phi_{12}R_{21})(k_{11} + k_{12} + k_{13}) + (\phi_{11}R_{12} + \phi_{12}R_{22})(k_{21} + k_{22} + k_{23}) + (\phi_{12}R_{13} + \phi_{12}R_{23})(k_{31} + k_{32} + k_{33}),$$

$$x_2 = (\phi_{22}R_{22})(k_{11} + k_{12} + k_{13}) + (\phi_{22}R_{22})(k_{21} + k_{22} + k_{23}) + (\phi_{22}R_{22})(k_{31} + k_{32} + k_{33}),$$

$$x_3 = (\phi_{31}R_{11} + \phi_{32}R_{21} + \phi_{33}R_{31})(k_{11} + k_{12} + k_{13}) + (\phi_{31}R_{12} + \phi_{32}R_{22} + \phi_{33}R_{32})(k_{21} + k_{22} + k_{23}) + (\phi_{31}R_{13} + \phi_{32}R_{23} + \phi_{33}R_{33})(k_{31} + k_{32} + k_{33}).$$

But stabilizing control has the form:

$$u_1 = -\left(\sum_{i=1}^{14} D_i e^{d_i t} - \omega_5 - \omega_4^2\right)x_{10} + \left(\sum_{i=1}^{21} E_i e^{e_i t} + \omega_2 - \omega_5 - \omega_4 \omega_5\right)x_{20} + \left(\sum_{i=1}^{14} F_i e^{f_i t}\right)x_{30},$$

$$u_2 = -\left(\sum_{i=1}^{22} G_i e^{g_i t} - \omega_2 \omega_5 - \omega_1 \omega_3\right)x_{10} + \left(\sum_{i=1}^{25} H_i e^{h_i t} + \omega_1 - \omega_5 - \omega_3^2\right)x_{20} + \left(\sum_{i=1}^{19} L_i e^{l_i t} - \omega_1 - \omega_2 \omega_3\right)x_{30},$$

$$u_3 = -\left(\sum_{i=1}^{13} M_i e^{m_i t}\right)x_{10} + \left(\sum_{i=1}^{20} N_i e^{n_i t} + \omega_1 - \omega_2 \omega_3\right)x_{20} + \left(\sum_{i=1}^1 P_i e^{p_i t} - \omega_1 - \omega_3\right)x_{30}.$$

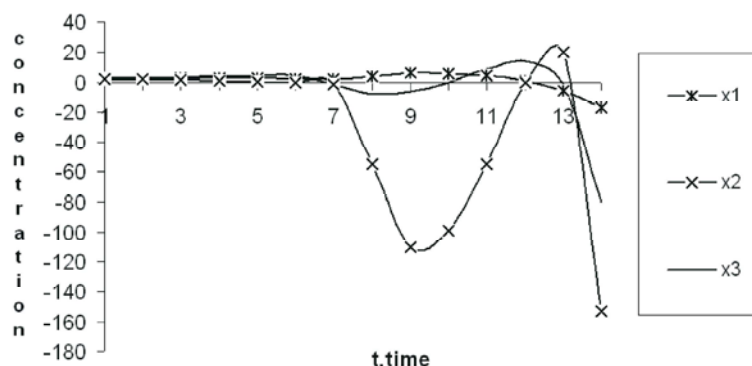


Fig: A graphical representation of the behavior stabilizing control object with

These expressions found define control software. In addition, we investigated the problem using numerical methods.

$$rx1 = 0.48, rx3 = 0.12, \sigma = 14.5, \beta = 0.42.$$

The figure shows an image of the system's behavior with the stabilizing control. numerical calculation was made by the Runge-Kutta method, using the following amounts: a constant which determines the duration of immunity, while natural mortality of antibodies-parameter describing the propagation of carriers determinants. As the graph shows variables can achieve stability. This means that after the introduction of foreign matter into the state of the body between the first hours and up to six hours will be held in a stable mode.

CONCLUSION

The estimation to vicinity approximating models possible to get choosing length of time. Here by image, we can approximate to find the solution of bilinear system through solutions of the linearization systems, providing stabilization of the system on final time interval.

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