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The Effect of Quantum Interferences on Emitter Current of Resonant Tunneling Diode and A New Definition for Quantum Capacitance

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Abstract: In this article, the effect of quantum interference between emitter and well electrons in resonant tunneling diode on the amount and direction of emitter current has been discussed. We have shown that the mentioned phenomenon has a considerable effect on diode time behavior. Furthermore, we have compared the results of this new model and the last presented one for the frequency response of diode and we have shown that the new model has a better confirmation by experimental data and a wrong negative sign which has been entered in parameters of last papers will be resolved in new model. Then the exact meaning of quantum capacitance has been discussed and we have shown that the term which is known as quantum capacitance is just a simple storage capacitance and has not any quantum specification to get the "quantum" prefix, where the new model leads to a real quantum capacitance.

Key words: Resonant tunneling diode . quantum capacitance . quantum inductance . storage capacitance

INTRODUCTION

Resonant tunneling diode, RTD, is a fast quantum device with a lot of applications both in analog and digital circuits. So there were many researches and discussions on RTD transient behavior and circuit models and many models have been introduced up to now [1-7]. In this report, we will take some of the recent articles [4, 5] into special consideration, and will explain our statements based on the comparison we will make.

Basically existing models are based on a calculation of the input and output current to/from the well of RTD, and then establishing a differential equation for the well charges. Usually in small-signal condition, the output current (collector current, jc) was assumed as a linear function of well charges, qw:

$$j_C = v_C q_W \tag{1}$$

Where vc is the electron escape rate from well to collector. The input current (emitter current) on the other hand, was usually modeled as the difference of two currents. They were emitter to well current and well to emitter current.

$$j_E = v_0 q_E - v_E q_W \tag{2}$$

Where v_0 is the electron escape rate from emitter to the well, v_E is the electron escape rate from well to the emitter and q_E is the available resonant tunneling charges on the emitter. Then as we said, a differential equation for the well charges was established and solved in order to lead to the device time constant, τ [5] as follows:

$$\frac{dq_W}{dt} = j_E - j_C = v_0 q_E - v_E q_W - v_C q_W \tag{3}$$

$$\Rightarrow q_W = v_0 q_E \tau \left(1 - e^{-t/\tau}\right) \tag{4}$$

$$\tau = \frac{1}{v_E + v_C} \tag{5}$$

Our essential statement in this article is about equation (2). We believe that this kind of emitter current modeling is not correct [6, 7]. In fact, in this model there was an essential assumption according to which the two current parts considered independent and phase correlation between them is ignored. We will see that the assumption is quite wrong and leads to many wrong results. After this introduction, in section 2, we consider transmission through the emitter barrier of RTD when there are electron densities on both sides of it and will introduce new large signal and small signal models for RTD emitter current. We will compare old and new models on the time behavior in section 3, and also check both of them with experimental data. Finally in section 4, we introduce the relation that exists between this discussion and the quantum capacitance concept and suggest an exact quantum capacitance idea.

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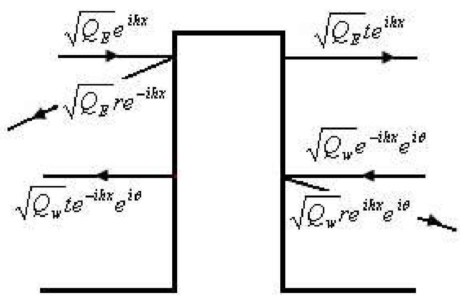

Fig. 1: A tunneling barrier with electron approaching it from its both sides

Large signal and small signal currents through the emitter barrier of RTD: If there is a potential barrier, as shown in Fig. 1, which electron approaches to it from both sides, one way is to consider each side current independent of the other side, and calculate the total current as the difference between those two currents as below:

$$\begin{aligned}&\text{Emitter to well current } j_1 = \frac{\hbar k}{m}|t|^2 Q_E\\&\text{Well to emitter current } j_2 = \frac{\hbar k}{m}|t|^2 Q_W\\&\text{Total emitter current } j_E = j_1 - j_2\end{aligned} \tag{6}$$

Where t is the transmission coefficient through the barrier, k is the electron wave vector, Q_E is the emitter charge density and Q_W is the well charge density. The above formula is similar to (2), so this method is the same method which has been used in the latest articles [5]. But the fact is that, this method is correct only if the input and output electron waves in each side of barrier are completely independent [6, 7]. Whereas in RTD, the electron wave which approaches to the emitter barrier from right side is the same exiting wave from the right, which after passing a small distance (equal to the well width) has been reflected by the collector barrier and is approaching to that side of emitter barrier again. So there is a considerable phase correlation between these two waves. Considering this phase correlation, we can formulate the whole of the problem as shown below:

$$\begin{aligned}&\Psi_I = \sqrt{Q_E}\,e^{ikx} + A\,e^{-ikx} && \text{Wave function at left side of the barrier}\\&\Psi_{II} = Be^{kx} + Ce^{-kx} && \text{Wave function inside barrier}\\&\Psi_{III} = De^{ikx} + \sqrt{Q_W}\,e^{-ikx}e^{i\theta} && \text{Wave function at right side of the barrier}\end{aligned} \tag{7}$$

Inserting the above wave function into Schrödinger equation, assuming wave function and its derivative to be continuous on boundaries as is usual, we will get a set of equations. By solving those equations, the amplitude of reflected wave functions on left and right sides of barrier (A and D respectively), as a function of the input wave function amplitudes on left and right ($\sqrt{Q_E}$ and $\sqrt{Q_W}$ respectively) and the relative phase between them, θ, can be calculated. As the first example we consider the case *QE=QW=1*. The results are drawn in Fig. 2.

From Fig. 2 we get that, always by entering two electrons to the device, two electrons will exit. But when correlation and interference exists, the variations

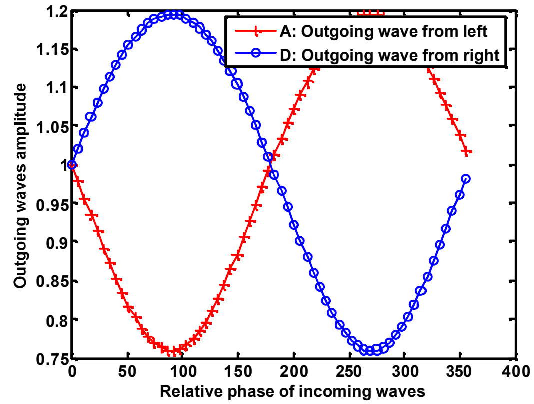

Fig. 2: Outgoing wave amplitudes from left and right, versus relative phase of incoming waves for equal amplitude of incoming waves i.e. Q_W=Q_E=1. Electron energy is equal to 1eV, height of barrier is equal to 1.1eV and barrier width is equal to 2nm

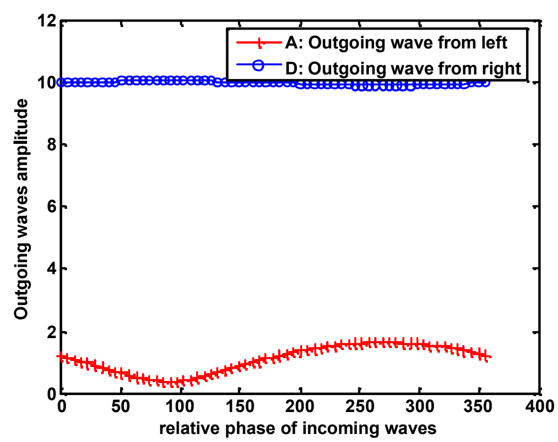

Fig. 3: Outgoing wave amplitudes from left and right, versus relative phase of incoming waves when the amplitude of incoming wave from left (well charge density), is much bigger than the amplitude of incoming wave from right (emitter charge density) i.e. $Q_W >> Q_E$=1). Electron energy is equal to 1eV, height of barrier is equal to 1.1eV and barrier width is equal to 2nm

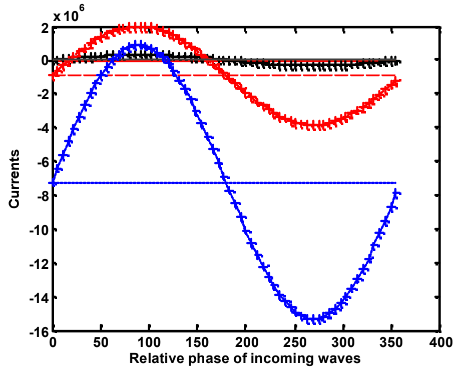

Fig. 4: Emitter current with (+ signed curves) and without (simple curves) correlation effects as a function of relative phase between incoming waves for Q_W=2.5 (Solid line), Q_W =200 (Dashed line) and Q_W =1600 (Dotted line). Electron energy is equal to 1eV, height of barrier is equal to 1.5eV, barrier width is equal to 2nm and Q_E is unit

of relative phase between two entering waves, may cause more electrons to exit from left or right.

As the next example, we consider the case *QW>>QE* (Fig. 3). This is the condition which is happened in RTD.

It is obvious from the figure that, the amplitude's variation of the right side exiting wave is insignificant, but the left side exiting wave variation is significant. In fact when the barrier is opaque enough, the exiting wave variation in each side is a function of entering wave density from the other side, so because the electron density in well is much more than emitter, the variation of exiting wave from emitter side is too much.

Now, we consider the currents. When we consider the correlation, instead of the two independent currents in equation 2, totally there is just a single current term which based on Fig. 2, we can write it as below:

$$J_E = \frac{\hbar k}{m^*}\left(Q_E - |A|^2\right) = \left(|D|^2 - Q_W\right) \qquad (8)$$

In Fig. 4, the above current and the total current without considering the correlation and interferences, equation (6), are drawn on the same plot. From the figure it is obvious that the interference effects can cause the current to become lower or higher and even, as an eye-catching effect, they can change the current direction. This condition is what happens in RTD, exactly for resonant electrons, and in this article we are going to observe its effect on RTD time behavior and by using that, we will introduce the exact quantum capacitance term.

To make a connection between equation 8 and equation 6, we consider that the transmission coefficient τ and reflection coefficient *r* can be found as below from *A* and *D* constants, which have been calculated before:

$$\begin{bmatrix} r \\ t \end{bmatrix} = \begin{bmatrix} \sqrt{Q_E} & \sqrt{Q_W}\, e^{i\theta} \\ \sqrt{Q_W}\, e^{i\theta} & \sqrt{Q_E} \end{bmatrix}^{-1} \begin{bmatrix} A \\ D \end{bmatrix} \qquad (9)$$

And by referring to Fig. 1, current can be calculated as below:

$$J_E = \frac{\hbar k}{m^*}\left(Q_E - \left|\sqrt{Q_E}\, r + \sqrt{Q_W}\, t e^{i\theta}\right|^2\right) \qquad (10)$$

In RTD, it can be simply shown that the relative phase between two waves, θ, is 90 degrees, and *r* and *t* have 45 degree and -45 degree phases respectively. So the current can be shown as below:

$$J_E = \frac{\hbar k}{m^*} Q_E \left(1 - \left(|r| - \sqrt{\frac{Q_W}{Q_E}}|t|\right)^2\right) \qquad (11)$$

This is the emitter Large-Signal current formula in RTD when well and emitter electron correlation is considered. Now, if one wants to ignore the correlations, he may ignore the cross product term in the internal parenthesis of equation 11 to have:

$$J_E = \frac{\hbar k}{m^*} Q_E \left(1 - |r|^2 - \sqrt{\frac{Q_W}{Q_E}}|t|^2\right) = \frac{\hbar k}{m^*}|t|^2 Q_E - \frac{\hbar k}{m^*}|t|^2 Q_W \qquad (12)$$

The above equation is exactly the same as equation 6 which was used in last papers but as we said the interference term which is equal to

$$+\frac{\hbar k}{m^*}\left(2|t\, r|\sqrt{Q_E Q_W}\right)$$

is very important and can change even the sign of the emitter current.

Using equation 11, the emitter Small-Signal current, j_E, can be written as below (v_D and q_W are the small signal voltage applied to RTD and small signal well charge respectively):

$$j_E = \frac{\partial J_E}{\partial V_D} v_D + \frac{\partial J_E}{\partial Q_W} q_W \qquad (13)$$

The first term in equation 13, is equal to $G_D\, v_D$, where G_D is small signal conductance of RTD and the

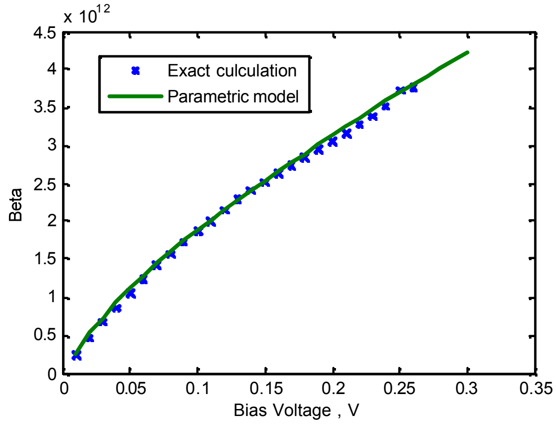

Fig. 5: The coefficient of well charge effect on the small-signal emitter current, ß, as a function of bias in a typical RTD

coefficient of q_W in the second term, after a little calculations, it can be written as below (lets call this coefficient "ß"):

$$\frac{\partial J_E}{\partial Q_W} = \beta = \frac{\hbar K}{m}\left(|t\,r|\sqrt{\frac{Q_E}{Q_W}} - |t|^2 \right) \tag{14}$$

In Fig. 5, we have drawn ß as the function of bias. Using this ß, our formula for the RTD Small-Signal emitter current is written as below:

$$j_E = G_D v_D + \beta q_W \tag{15}$$

The fundamental point is that, ß for the full bias range, even for the NDR region has a positive value, as it is shown in the Fig. 5.

RTD time behavior in previous models and the new one, and a comparison with experiment: If we compare our equation for the emitter current (equation 15) with equation 2, which was used in previous works [5], we get that the form of both equations are the same, except for q_W coefficient which has a positive sign in the new formula and a negative sign in previous one (usually in previous works [5], q_E has been supposed a linear function of input signal, so the first term in both formulas is completely the same). This negative sign is very important and changes the physical picture of RTD completely, as we will see later. Following the path of equations 3 and 4, leads us to this equation for time constant τ in new model:

$$\tau_{new} = \frac{1}{v_C - \beta} \tag{16}$$

Thus the new model rather than the previous one, results a bigger time constant. The value of the quantum inductance which is related to the time constant by $L_q = \tau/G_D$, is also bigger in the new model compared to the old one.

It is evident that the manner of each model in conformity with the experimental data is the final gauge to accept or reject that model. As the experimental test for our new model, we rely on the used data in the reference 5. In that reference, the S measurement results which are shown in Fig. 6 of that reference have been fitted to the circuit model and its results are shown in a table at the end of that reference. Referring to that table, experimental data have been fitted to the circuit model by 2.58ps for τ and 0.79ps for $1/vc$. Now it's very interesting to notice that because τ is bigger than $1/vc$, $1/v_E$ has got a negative sign (equal to -1.14ps). This negative value is completely incompatible to the introduced model in that article. On the other hand, it clearly confirms our presented model. It's because of existing –ß versus v_E in our formula (equation 16). In this way, if the above-mentioned experimental data be used to confirm our model, it leads to 8.7×10^{11} as the value of ß. Now, refer to the Fig. 5, it is obvious that this value is consistent with our results which are fully physics-based and are calculated by solving the Schrödinger equation in the RTD emitter structure (equations 7 to 16). Of course our results are given in more details because we have made ß available with its complete functionality on bias.

The exact meaning of quantum capacitance: The capacitance story in RTD is a very long story. At first, electron charges in RTD well, and its related capacitance and the geometric capacitance were not distinguished correctly [1, 2]. After that, by introducing a term, which was named electron escape rate, the quantum capacitance concept was introduced [4] and when they found out that the correct method of modeling this phenomena in the equivalent circuit model, is using a self, the quantum inductance term was introduced [3, 4]. We believe that the term introduced as "quantum capacitance" and or "quantum inductance" until now, which is basically introduced by equations 1 to 3, is a simple storage capacitance, just like the effect already existed in many classic semiconductor devices, such as the storage of electrons in the base of bipolar transistors. So this element does not have any special feature to relate any quantum prefix to it. On the other hand, anything that has been introduced in this article by equations 1, 14 and 3 is a real quantum capacitance. To clear up this subject, Fig. 6A and 6B about simple storage capacitance and Fig. 7 about

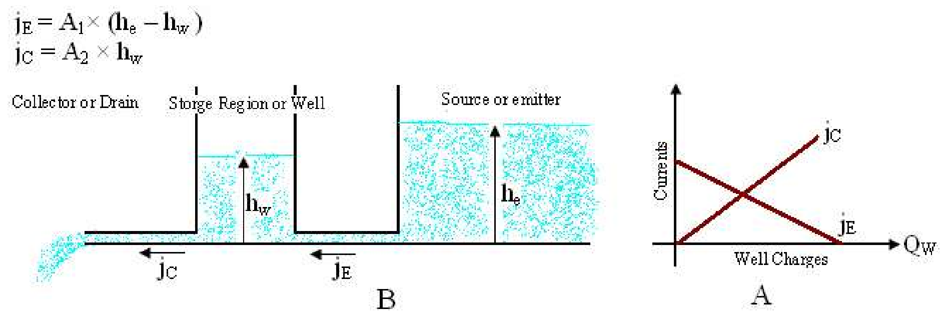

Fig. 6: A-Input current to the well, j_E, and output current from well, j_C, as a function of well charges for a classic storage capacitance
B-A parable of classic storage capacitance. The input flow is a function of difference between the liquid height in storage region and source region and the output flow is a function of liquid height in storage region

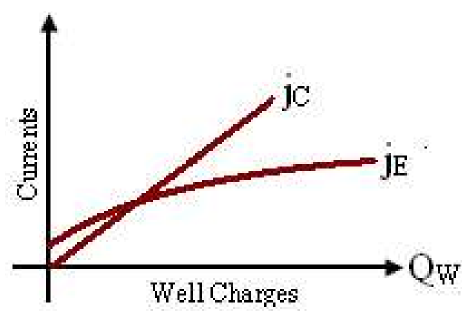

Fig. 7: Input current to the well, j_E, and output current from well, j_C, as a function of well charges for a real quantum storage capacitance

quantum capacitance have been drawn. Figure 6A and 7 show the input and output currents to/from the well as functions of well charges. We can see that the output current, jc, is the same in both of them, but the input current, j_E, has a completely different behavior. In previous model, the input current for $qw=0$ is maximum, and it reduces by increasing qw. In new model, the input current for $qw=0$ is close to zero and it increases by increasing qw (this curve has also a decreasing part that is not the related to our current discussion and so is not drawn in the figure). In fact there is a kind of positive feedback in the newly introduced real quantum capacitance: j_E causes q_W to be increased which it causes increasing j_E too. Of course both old and new systems are finally stable and their equilibrium point is on the intersection point of j_E and jc. Figure 6B shows a parable of classic storage capacitance. This is the same known example which a water bowl is filled from a source by a pipe, and at the same time, there is another pipe which sends out some of its water. It is evident that the rate of input water to the bowl is maximized when the bowl is empty, and little by little the bowl is filled with water and the mentioned rate reduces. Vice versa in the introduced real quantum storage capacitance in this article, as shown in formulas, the input current is minimized at first and increases when the capacitance is filled little by little. It is evident that there is not any allegory for this condition and we can't draw any figure like Fig. 6B for this quantum storage capacitance.

We add up that the quantum capacitance is a kind of storage capacitance, in which against classic storage capacitance, charges can go from low density places (emitter region) to the high density places (well region) and be stored there, and this is the same phenomenon which is happened in RTD.

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