

## Using Wavelet Analysis in Crack Detection at the Arch Concrete Dam under Frequency Analysis with FEM

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**Abstract:** The dam is a part of civil works for the irrigation and flood control purposes. Specially, arch concrete dams have important characteristics because of high safety, economical design, designing complexity and its applications. Most of structure failures take place because of ingredients fracture. This event occurred by cracks with their development is considered as a serious threat for the behavior of the structure. The exhibition methods of the cracks are classified in groups of Structural Health Monitoring (SHM) methods. The effect of cracks on the behavior of a structure is local stiffness differences that have great effects on the dynamic treatment of the structure. This matter is very significant in the difference between natural frequency and mode shapes, so analysis it will be lead to detect the crack. The new and useful method that has located in the of signal Analysis discussion has encounter with researches reception that is named as Wavelet Transform (WT). This transform is one of the useful mathematic transform methods with have a high ability in recognition of inconsistencies by index of Wavelet Transform graph in the shape of one or two close points that have noises in relation which other points. So, in this paper, first the theory of wavelet analysis is presented including continuous and discrete wavelet transform followed by its application to SHM. Then, using frequency analysis response of dam with ABAQUS software, crack detection possibility has been researched in dam structure under Wavelet analyzing in MATLAB software toolbox.

**Key words:** Arch dam . frequency analysis . crack . wavelet analysis . ABAQUS

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### INTRODUCTION

Water with good quality and in sufficient quantity is a basic requirement for humanity. Reservoirs and concrete dams that create those reservoirs provide a means to balance the fluctuation of natural water flow. Multipurpose reservoirs can serve for drinking water, irrigation in agriculture, production of clean renewable energy, recreation and flood protection. So, concrete arch dams and detection of probable damage on its body play a vital role in the infrastructure of many states for the provision of water resource and saving money.

It is necessary to use scientific methods for reducing cost and time and also to meet industry requirements. Finite Element Method (FEM) is employed to solve different engineering problems in permanent, temporary, linear and nonlinear states [1]. Among finite element method software, the ABAQUS software is used because of high accuracy and ability of dynamic analysis such as earthquake and water wave loading on structures. This software is accurate research software and is employed in industry and university settings [2]. In order to Detection of damage,

investigation of its incidence effects is necessary. According to theoretical principles of structure, there is a relationship between dynamic and static response and stiffness as a result [3]. Any sudden change in stiffness leads to dynamic and static response variation. This condition will help to an estimate damage and investigation of structural response before and after failure (Structural health monitoring) [4].

Wavelet method is a new and effective method for the realization of structural damage. There is a close relationship among wavelet method and frequency and time issues. The ability of this method is high in recognition of damage location. Wrong and lieu (1998) proposed articles about fracture discovery by wavelet method. They explained fracture recognition in beam by simple support (5). Currently, Lie and Chen (2001) published another article and they proposed a method based on Wavelet Finite Element Method (WFEM) in order to recognition of damage specifications. This method addresses to realization of damaged location like beam Fracture [6].

For many years, damage identification methods have been studied by a number of researchers while the importance of Nondestructive Evaluation (NDE) of the

civil infrastructure has been significantly increasing. Damage detection is the first level of the more general problem of damage identification. A further analysis and subsequent levels of damage identification of a structure include: classification and severity of damage, determination of the location of damage, prediction of possible breakdown or failure, and estimation of the remaining service life [7, 8]. The modal analysis method one of many NDE methods, is based on the fact that the change of structural properties causes a variation in the different modal parameters; namely natural frequencies, damping ratios and mode shapes. Many analytical and experimental studies have been conducted to establish analytical correlations between damage severity and modal parameters. About 60 years ago Kirmser reported the relationship between natural frequencies and the introduction of a crack in an iron beam [9]. The use of mode shapes to study the dynamic behavior of structures requires that a number of accelerometers be installed on the structure. However, a wavelet analysis can make it possible to evaluate the soundness of structures by using only one accelerometer strategically located on the structure [10, 11]. Moreover, in order to detect damage using modal based methods, a complete dynamic analysis is often required and is usually performed by a finite element analysis method to locate and quantify the damage. This procedure has several difficulties: First, it is not always possible or convenient to measure the vibration response of a structure before damage. Second, it is often not feasible to conduct a detailed dynamic analysis of a complete structure. Third, it is sometimes difficult to obtain accurate material properties for a dynamic analysis. Furthermore, it is not easy to extract local information caused by small damage from modal parameters that characterize the global behavior of a structure [12]. In order to eliminate these difficulties wavelet-based damage detection has been considered by several researchers over the last decade. Historically, wavelets were first applied in geophysics to analyze data from seismic surveys, which are used in oil and mineral exploration, to get 'pictures' of layering in surface rock. In fact, geophysicists only rediscovered wavelets; mathematicians had developed them to solve abstract problems some twenty years earlier but had not anticipated their applications in signal processing [13]. While Fourier analysis consists of the breaking up of a signal into sine waves of various frequencies and phases, wavelet analysis is a breaking up of a signal into shifted and scaled versions of a mother wavelet or basis function. These results in variable sizes of a window function and make it possible to detect the discontinuities and breakdown points of data that other analyzing methods usually

miss. The first researcher known to have applied wavelet to vibration analysis is Newland [14-16]. He applied a wavelet analysis for the study of vibration of buildings caused by underground trains and road traffic by which he found the similarities between the response signals in each floor.

### WAVELET THEORY

**Basis function:** The Fast Fourier Transform (FFT) is a perfect tool for finding the frequency components in a signal. A disadvantage of the FFT is that frequency components can only be extracted from the complete duration of a signal. The frequency components are obtained from an average over the whole length of the signal. Therefore it is not a suitable tool for a non-stationary signal such as the impulse response of cracked beams, vibration generated by faults in a gearbox, and structural response to wind storms, just to name a few. These types of problems associated with FFT can be resolved by using wavelet analysis. Consequently, wavelet analysis has recently been considered for damage detection and Structural Health Monitoring (SHM). It provides a powerful tool to characterize local features of a signal. Unlike the Fourier transform, where the function used as the basis of decomposition is always a sinusoidal wave, other basis functions can be selected for wavelet shape according to the features of the signal. The basis function in wavelet analysis is defined by two parameters: scale and translation. This property leads to a multi-resolution representation for non-stationary signals. As mentioned before, a basis function (or mother wavelet) is used in wavelet analysis. For a wavelet of order N, the basis function can be represented as

$$\psi(n) = \sum_{j=0}^{N-1} (-1)^j c_j (2n + j - N + 1) \quad (1)$$

Where  $c_j$  is coefficient. The basis function should satisfy the following two conditions [17, 19]: The basis function integrates to zero, i.e.

$$\int_{-\infty}^{\infty} \psi(t) dt = 0 \quad (2)$$

It is square integrable or, equivalently, has finite energy, i.e.

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt < \infty \quad (3)$$

Eq. (2) suggests that the basis function be oscillatory or have a wavy shape. Eq. (3) implies that

most of the energy in the basis function is confined to a finite duration. The important properties of basis functions are ‘orthogonality’ and ‘biorthogonality’. These properties make it possible to calculate the coefficient very efficiently. There is no redundancy in the sense that there is only one possible wavelet decomposition for the signal being analyzed. However, not all basis functions have these properties. A frequently mentioned term in the definition of a basis function is ‘compact support’, which means that the values of the basis function are non-zero for finite intervals. This property enables one to efficiently represent signals that have localized features.

**CONTINUOUS WAVELET TRANSFORM (CWT)**

The CWT is defined as:

$$W(a,b) = \frac{1}{\sqrt{a}} \int f(t) \cdot \psi^* \left( \frac{t-b}{a} \right) dt \tag{4}$$

Where a and b are scale and translation parameters, respectively and  $\psi^*$  is the complex conjugate of  $\psi$ . The basis function  $\psi$  is represented as

$$\psi_{jk}(t) = 2^{j/2} \psi(2^j t - k) \tag{5}$$

If the scaling parameter a, is  $0 < a \leq 1$ , it results in very narrow windows and is appropriate for high frequency components in the signal f(t). If the value of a, is  $a \geq 1$  it results in the very wide windows and is suitable for the low frequency components in the signal. According to the uncertainty principle (also known as Heisenberg inequality), the resolution in time and frequency has the following relationship:

$$\Delta t \Delta f \geq \frac{1}{4\pi} \tag{6}$$

And  $\Delta f$  is proportional to the center frequency f, which leads to:

$$\frac{\Delta f}{f} = C \tag{7}$$

Where C is a constant. Therefore, the time resolution becomes arbitrarily good at high frequencies, while the frequency resolution becomes arbitrarily good at low frequencies [19]. This property helps to overcome the limitation of Short Time Fourier Transforms (STFT) in which the time-frequency resolution is fixed. In order for an inverse wavelet transform to exist, the mother wavelet should satisfy the admissibility condition defined as:

$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty \tag{8}$$

Where  $\Psi$  is the Fourier transform of  $\psi$  [20]. Eq. (4) can be represented as

$$W(a,b) = \langle f(t), \psi_{a,b}^*(t) \rangle \tag{9}$$

Therefore, CWT is a collection of inner products of a signal f(t) and the translated and dilated wavelets  $\psi_{a,b}(t)$ . The value of the scale a is proportional to the reciprocal of the frequency which results from:

$$F\left[\psi\left(\frac{t}{a}\right)\right] = |a| \psi(a\omega) \tag{10}$$

Where F[] denotes the Fourier transform.

**DISCRETE WAVELET TRANSFORM (DWT)**

The main idea of DWT is the same as that of CWT. While the CWT requires much calculation effort to find the coefficients at every single value of the scale parameter, DWT adopts dyadic scales and translations (i.e. scales and translations based on powers of two) in order to reduce the amount of computation, which results in better efficiency of calculation. Filters of different cutoff frequencies are used for the analysis of the signal at different scales. The signal is passed through a series of high-pass filters to analyze the high frequencies, and through a series of low-pass filters to analyze the low frequencies. In DWT the signals can be represented by approximations and details. The detail at level j is defined as

$$D_j = \sum_{k \in Z} a_{j,k} \psi_{j,k}(t) \tag{11}$$

Where a Z is the set of positive integers. The approximation at level J is defined as:

$$A_j = \sum_{j > J} D_j \tag{12}$$

Finally, the signal f(t) can be represented by [20]

$$f(t) = A_j + \sum_{j \leq J} D_j \tag{13}$$

As opposed to the CWT where only a wavelet function is used, in DWT a scaling function is used, in addition to the wavelet function. These are related to low-pass and high-pass filters, respectively. The

scaling function  $\phi(t)$  must satisfy the following three conditions [18]:

- It integrates to one:

$$\int_{-\infty}^{\infty} \phi(t) dt = 1 \quad (14)$$

- It has unit energy:

$$\int_{-\infty}^{\infty} \phi(t) dt = 1 \quad (15)$$

- The set consisting of  $\phi(t)$  and its integer translate is orthogonal:

$$\langle \phi(t), \phi(t - n) \rangle = \delta(n) \quad (16)$$

The scaling function can also be represented as:

$$\phi(n) = \sum_{j=0}^{N-1} c_j \phi(2n - j) \quad (17)$$

$$\phi_{j,k}(t) = 2^{j/2} \phi(2^j t - k) \quad (18)$$

Which are similar to Eq. (1) and (5), respectively. Not all wavelet functions have scaling functions. Only orthogonal wavelets have their scaling functions. This DWT can be very useful for on-line health monitoring of structures, since it can efficiently detect the time of a frequency change caused by stiffness degradation. Further details about wavelet theory can be found in the literature [17, 19, 20].

### STRUCTURAL HEALTH MONITORING (SHM)

In order to extend the life of facilities more attention should be given to infrastructures and buildings. Even though structures are normally designed to last 50-100 yr, overloads, excessive usage, exposure to extreme weather or environmental conditions and other unexpected factors can cause more rapid deterioration of structures. In general, the development of successful health monitoring methods depends on two key factors: sensing technology and the associated signal analysis and interpretation algorithm. Over the past 10 yr, wavelet theory has been one of the emerging and fast-evolving mathematical and signal processing tools for vibration analysis [21]. This new signal processing tool made it possible to decompose and reconstruct the measured raw data efficiently. Al-khalidy et al. published numerous papers about damage detection using wavelet analysis [22, 23]. Their

main objective was to develop an on-line system to monitor the damage rate of a structure to assure its safety during severe environmental loading such as earthquake events. Estimating the damage rate of structures caused by low cycle fatigue loads is important to assure the safety of structures. However, it is difficult to detect the fatigue signals and to estimate the damage rate of structures from the detected signals. They applied the orthonormal discrete wavelet transform to the detection of fatigue signals from the observed signals contaminated by noise. In the orthonormal basis, the wavelet expansion of a function  $x(t)$  and the coefficients of wavelet expansion are defined as:

$$\pi(t) = \sum_j \sum_k \alpha_{j,k} \psi_{j,k}(t) \quad (19)$$

$$\alpha_{j,k} = \int_{-\infty}^{\infty} x(t) \psi_{j,k}^*(t) dt \quad (20)$$

Where  $\alpha_{j,k}$  are the coefficients of the wavelet expansion of  $x(t)$ . The structure in the study is represented by a simple mass-spring-dashpot model and the governing differential equation of motion of the system is given by:

$$m \frac{d^2}{dt^2} x(t) + c \frac{d}{dt} x(t) + kx(t) = f(t) = y(t) + \sum_{i=1}^n S \delta(t - \tau_i) \quad (21)$$

Where  $m$ ,  $c$  and  $k$  are system mass, viscous damping coefficient, and stiffness, respectively;  $x(t)$  is the displacement response and  $f(t)$  the external excitation is assumed to be a sum of the seismic excitation,  $y(t)$  and a sequence of impulses occurring at random times. The impulses model the fatigue damages occurring at random times  $\tau_i (i=1,2,\dots,n)$  with a magnitude  $S$ . The wavelet transform clearly picked up the exact time of impulse occurrence in the input. The researchers found that several factors played a role in the successful detection of the impulses in the input force, such as sampling rate, wavelet regularity, and signal-to-noise ratio. They also found that increasing the sampling rate makes it easier to detect the impulses. The higher the sampling rate is, the more the signal will overcome the noise. However, the magnitude of the wavelet coefficients  $\alpha_{j,k}$  decreases by 23 orders of magnitude when increasing the sampling rate from 8 to 100 Hz. They suggested that if the noise level is low, high sampling rates should be adopted and vice versa.

Robertson *et al.* [24] presented a wavelet-based method for the extraction of impulse response functions (Markov parameters) from measured input and output data. The input data and the Markov parameters are

approximated by the locally orthogonal 8 term Dubieties wavelet functions, which leads to a simple representation of the convolution integral in the wavelet domain. They also performed structural system identification using the Markov parameters extracted by the wavelet analysis [25]. The modes, mode shapes and damping parameters of the state space based model were found.

Kitada [26] proposed a method of identifying nonlinear structural dynamic systems using wavelets. This method made it possible to determine stiffness and damping coefficients of a structure with severe material nonlinearity without any assumption about nonlinear characteristics of the structure. Other research studies aimed at determining modal parameters using wavelets can be found in the literature [27-29].

Hou *et al.* [30] provided numerical simulation data from a simple structural model with breakage springs. The governing equation of motion of the system is given by

$$m \frac{d^2}{dt^2} x(t) + c \frac{d}{dt} x(t) + k(t)x(t) = f(t) \quad (22)$$

The system stiffness  $k(t)$  is expressed by:

$$k(t) = \sum_{i=1}^n k_i(t) \quad (23)$$

Where  $k_i(t)$  represents the stiffness of the  $i$ th spring in the system at time  $t$ . If breakage of a spring occurs due to an excessive response,  $k_i(t)$  is defined by:

$$k_i(t) = \begin{cases} k_{i0}, & \text{if } |x(t)| \leq x_i^* \forall t \leq t \\ 0, & \text{otherwise} \end{cases} \quad (24)$$

Where  $k_{i0}$  and  $x_i^*$  are the initial stiffness and the thresh-old value of the  $i$ th spring, respectively. However, if a spring is broken because of fatigue,  $k_i(t)$  is determined by:

$$k_i(t) = \begin{cases} k_{i0}, & \text{if } N(t) < N_i^* \forall t \leq t \\ 0, & \text{otherwise} \end{cases} \quad (25)$$

Where  $N(t)$  is the total number of cycles of the response in the time interval  $[0, t]$  and  $N_i$  is the allowable number of cycles for the  $i$ th spring. In order to calculate Eq. (22) fourth-order Runge-Kutta integration is used. The discrete wavelet transformation of the response curve showed clear spikes, which they attributed to the occurrence of structural damage. The authors also investigated the noise intensity and damage

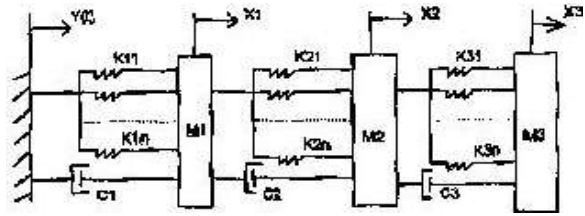


Fig. 1: A sketch of three DOF spring-mass-dashpot model (from [25] with permission from CRC Press)

severity. They provided a delectability map that represents a qualitative relationship between the noise intensity and damage level. In conclusions, the authors state that structural damage or the change in system stiffness may be detected by spikes in the details of the wavelet decompositions of the response data. In another research performed by Hou and Noori [31] a MDOF model with three degrees of freedom is used, as shown in Fig. 1, instead of a SDOF model.

The total inter-mass stiffness  $k_i(t)$  is expressed by:

$$k_i(t) = \sum_{j=1}^n k_{ij}(t) \quad (26)$$

Where  $k_j(t)$  represents the stiffness of the  $j$ th spring of the  $i$ th mass in the system at time  $t$ .

The objective (compared to their previous study) is to identify the location of damage in the structure. The details in discrete wavelet transform of each external excitation show the moment when a spring is broken. Therefore, the location of damage can be identified.

Hou and Hera [32] proposed pseudo-wavelets to identify system parameters, and the associated pseudo-wavelet transform was developed. One of the pseudo wavelets that is based on the Fourier amplitude response function for a linear SDOF system subjected to an impulse input is defined as

$$W(\omega; \omega_0^*, \zeta_0^*) = \beta \frac{\omega_0^{*2}}{\sqrt{(\omega^2 - \omega_0^{*2})^2 + (2\zeta_0^* \omega \omega_0^{*2})^2}} \quad (27)$$

Where  $\omega_0^*$  is the shifting parameter,  $\zeta_0^*$  is the scaling factor, and  $\beta$  is a normalizing factor. The pseudo-wavelet transform (PWT) can then be defined as:

$$C_f(\omega_0^*, \zeta_0^*) = \int_0^\infty F(\omega) W(\omega; \omega_0^*, \zeta_0^*) d\omega \quad (28)$$

Where  $C_f$  is the pseudo-wavelet transform of  $F(\omega)$ . For a MDOF system, the truncated PWT was used and was found to be much more accurate compared to the

non-truncated pseudo-wavelet transform approach. The truncated PWT involves constructing a truncated response spectrum by setting the transform to zero for any frequency outside the frequency range determined in the neighborhood of a local maximum.

Another approach to locate damage is to study the curvature mode shapes and wavelet maps. The residual of a mode shape is the difference between the damaged and undamaged mode shapes. The curvature of the residual of a mode shape is the second derivative of the residual of this mode shape. This is referred to as the curvature mode shape. Wavelet transforms can then be applied to the curvature mode shapes. However, when applied to stationary signals (such as mode shapes of a vibrating structure), curvature mode shapes and wavelet maps fail to determine the exact location of the damage. Amaravadi *et al.* [33] proposed a new technique that combines these two methods for enhancing the sensitivity and accuracy in damage location. First, the curvature mode shape is calculated from the residual of individual mode shapes. Then a wavelet map is constructed for each curvature mode shape. The proposed technique was experimentally verified using a lattice structure made of vertically stacked aluminum beams, and a cantilever beam. Damage was simulated as side notches in one of the beams. The results show that the proposed method accurately predicts the location of the damage. Since the wavelet number indicates the location of damage, the accuracy of this method depends on the number of wavelet coefficients contained in the signal.

In WT analysis, the frequency resolution becomes quite poor in the high frequency region. To overcome this drawback Sun and Chang [11] applied the wavelet packet transform (WPT) instead of WT to the dynamic signals measured from a structure. WPT is another extension of WT that provides complete level-by-level decompositions. The wavelet packet component signal  $f_j^i(t)$  can be expressed by a linear combination of wavelet packet functions  $\psi_{j,k}^i(t)$  as follow:

$$f_j^i(t) = \sum_{k=-\infty}^{\infty} c_{j,k}^i \psi_{j,k}^i(t) \quad (29)$$

and the coefficients  $c_{j,k}^i$  are obtained from

$$c_{j,k}^i = \int f(t) \psi_{j,k}^i(t) dt \quad (30)$$

provided that the wavelet packet functions are orthogonal. For damage detection, location and severity, a three-span bridge model is used. The response of the bridge model to an impact force is measured and then decomposed by WPT. The next step

is to calculate the component energy at each level and this energy is used as inputs into neural network models for damage assessment. The researchers found that a neural network model is capable of identifying the presence of damage that corresponds to as small as 4% of the rigidity reduction in any element. For damage location and severity, another neural network model could locate and quantify moderate (10-20% EI reduction) and severe (20-30% EI reduction) damages. Liu *et al.* [34], Wu and Du [35], and Hwang *et al.* [36] also used the wavelet packets or wavelet basis neural network for damage detection.

Gurley *et al.* [37] developed a wavelet-based coherence and bi coherence technique in order to detect intermittent first-and higher-order correlation between a pair of signals. The classical approach for reduction of variance is to perform ensemble averaging by using localized time integration. In this study, the introduction of a variable integration window was predicated on the multi resolution character of wavelets and high-lighted that the lack of ensemble averaging results in much of the observed spurious coherence. These correlation schemes can be applied in problems involving wave-structure interactions or seismic response of structures where intermittent correlation between linear and nonlinear processes is of interest.

Accordingly, by obtaining geometrical dimensions of karoon1 case studied dam from related designing map and also its mechanical and physical characteristics in damaged and safe cases, the damage was modeled by ABAQUS software. Support condition and physical properties are the same for both states.

### MODELING OF THE DAM BY ABAQUS SOFTWARE

A case study has been done to make finite element model. It can be a real model or simulated one. Besides of difficulty of creating and analyzing a real model rather than a simulated one, its results will be accurate and real. Meanwhile, by investigation of dam model and prediction of structure behavior, probable weak points under applied load can be realized. So, the information of Karoon1 (Fig. 1 and 2), double-curvature arch dam, was prepared and its real model was created exactly from designing map. In this dam, internal and external radiuses, internal and external angles of arches are vary depend on their level. The axis which passes through arches' center is not asymmetrical. So, this dam is considered as one of the most complex dams. Table 1 summarized the specifications of mentioned dam body.

The finite element model of both safe (Fig. 3) and damaged (Fig. 4 and 5) states of dam was created by

Table 1: Geometrical specification of karoon1 dam

6 m	Thickness at Crest	Double curvature arch dam	Dam type
177.5 m	Normal water level	372 m	Length of crest
0.2	Poason concrete coefficient	200 m	Height from foundation
2400 kg/m <sup>3</sup>	Concrete density	33.5 m	Thickness at base

Table 2: The frequencies of the first four modes

Fourth mode frequency	Third mode frequency	Secound mode frequency	First mode frequency	Sample
3.3423	2.9860	2.1455	1.8680	Safe
3.3181	2.9735	2.1354	1.8614	Cracked

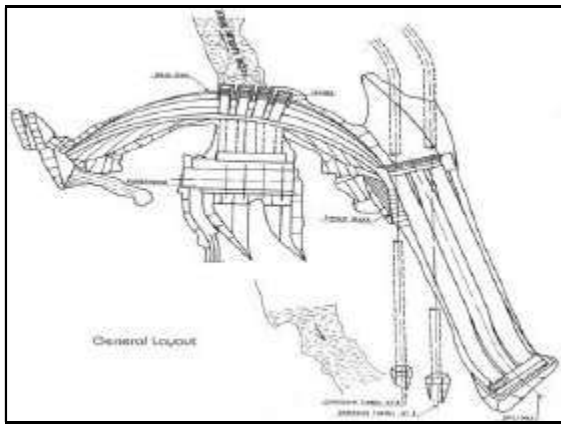


Fig. 1: Plan of Karoon1 dam

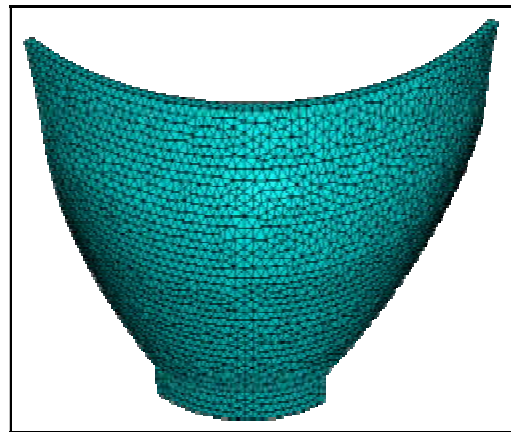


Fig. 3: Finite element mesh of safe dam body



Fig. 2: Upstream view of karoon1 dam

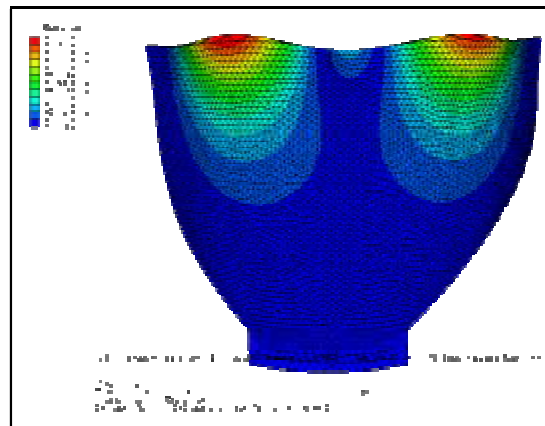


Fig. 4: The model of cracked dam under frequency analysis

ABAQUSE software and then analyzed in the frequency bond of (0-100 Hz). The dam has a quadratic surface. Therefore the 3-dimension solid element (C3D20RH) was used in modeling of dam's geometry to fit the finite element model with its real geometry. A crack with dimensions of 10m (in height) by 4m (in depth) by 0.2m (in width) has been supposed at the crest level in midpoint of up stream of dam (Fig. 5).

### FREQUENCY ANALYSIS OF DAM

In this study, the responses of two Frequency analyses were studied. Results showed that the frequency has been reduced in models that have crack. Table 2 presents the values of the first four frequencies of both cracked and safe models.

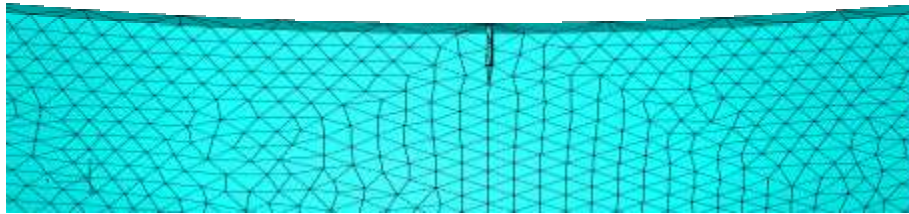


Fig. 5: The details of meshing of cracked dam in ABAQUS software

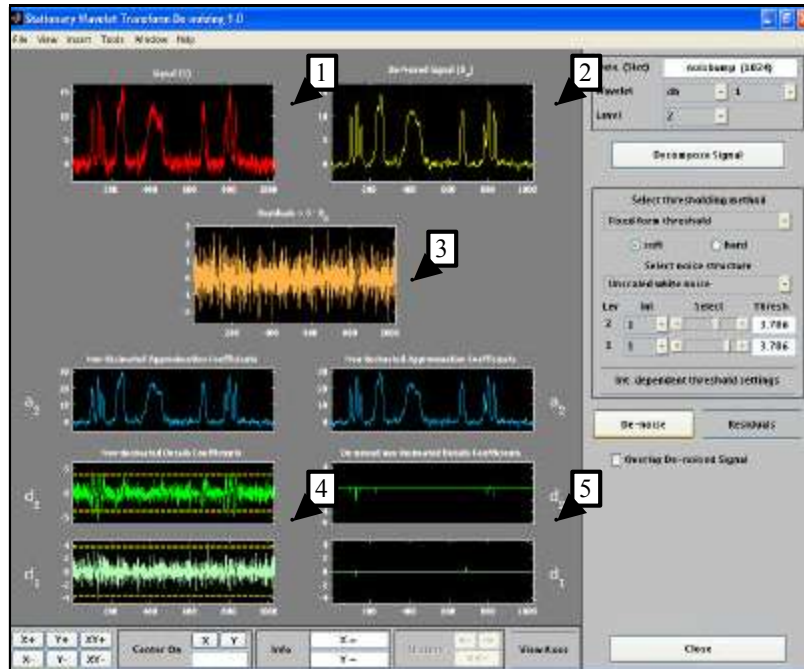


Fig. 6: SWT window in MATLAB software

### GRAFICAL WINDOW OF STATIONARY WAVELET TRANSFORM

A set of frequencies, which are being created during the short period of time, could repeat in another period of time. These types of signals are called stationary signals. Figure 6 shows SWT window.

According to Fig. 4, followings are identical in graphical window:

- Input signal 2-Signal without noise
- Signal with noise 4. Decomposed: signal with noise
- Decomposed: signal with out noise

The indices of analyzed signals ( $d_1$ ,  $d_2$ ) relate to selected level number.

### ANALYSIS OF SAFE-DAM WITH WAVELET

After analysis of dam by ABAQUS software, the responses were used for wavelet analysis by MATLAB

software in special toolbar of wavelet. In safe model, in the first frequency mode, the displacement responses of 96 points at crest level with equal distances have been took in two directions; involve width direction of dam ( $U1$ ) and reservoir length direction ( $U2$ ). This response that had 1.8650 Hz frequency has been analyzed in different scales. Because of paper space limitation, five results by Sym2 wave (scale 2) have been shown in Fig. 7 and 8.

### ANALYSIS OF CRACK-DAM WITH WAVELET

Similarly, the response of cracked dam was considered in first frequency mode in ninety-six points of dam at crest level in equal distances. Displacements are in width direction of dam ( $U1$ ), reservoir direction ( $U2$ ) and perpendicular direction to dam width axis ( $U3$ ). Figure 9 shows cracked dam analyzed model. Fig. 10-12 indicate results.

According to the figures, the graphs involve considerable rise at or around crack location. But there



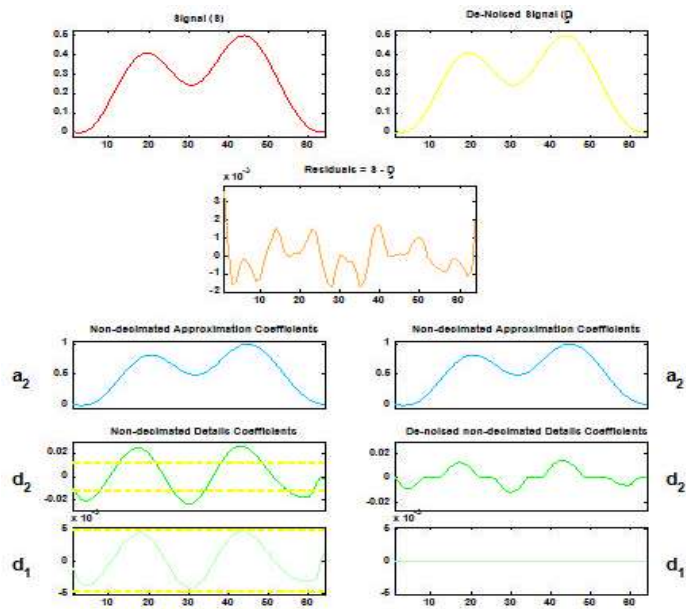


Fig. 7: The graph of SWT for safe sample response under Sym2 wavelet analyzer (Mode1-U1)

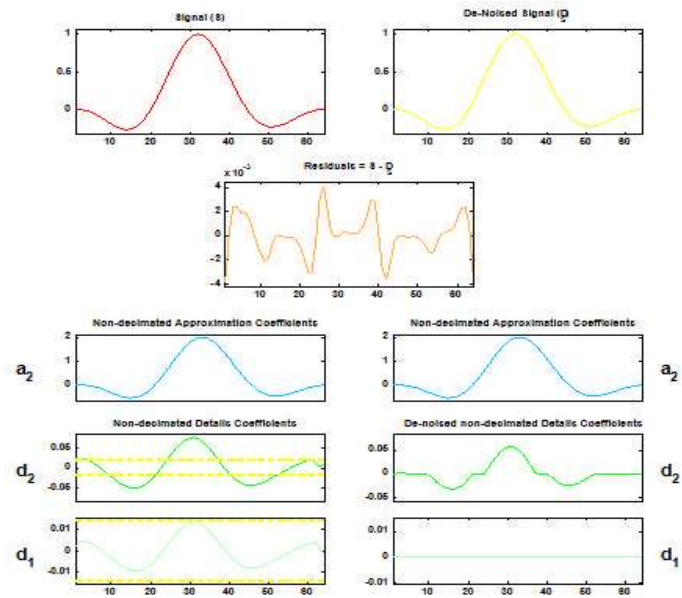


Fig. 8: The graph of SWT for safe sample response under Sym wavelet analyzer (Mode1-U2)

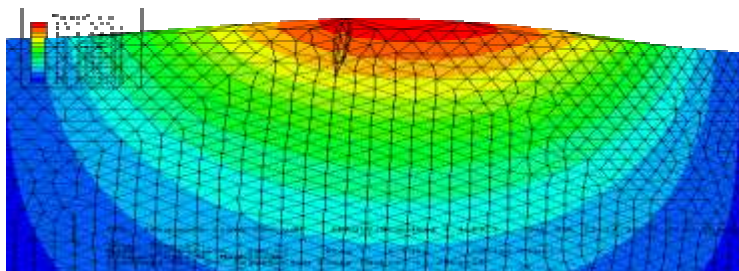


Fig. 9: Cracked dam analyze mode1 by ABAQUS software

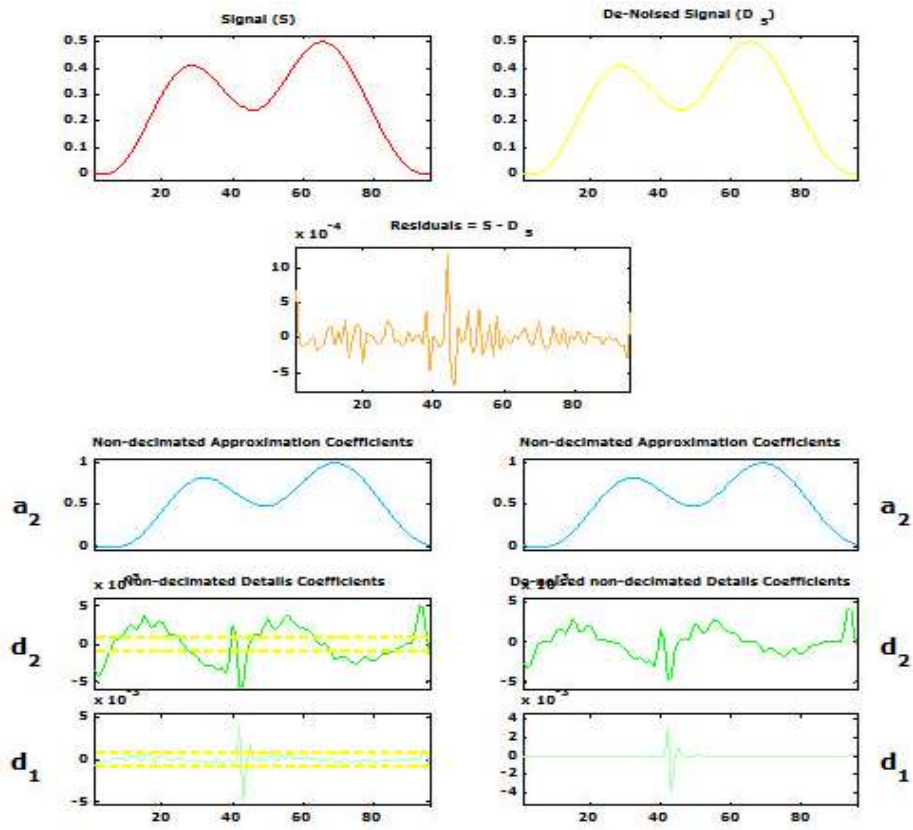


Fig. 10: The graph of SWT for response of cracked dam under Sym2 wavelet analyzer (Mode1-U1)

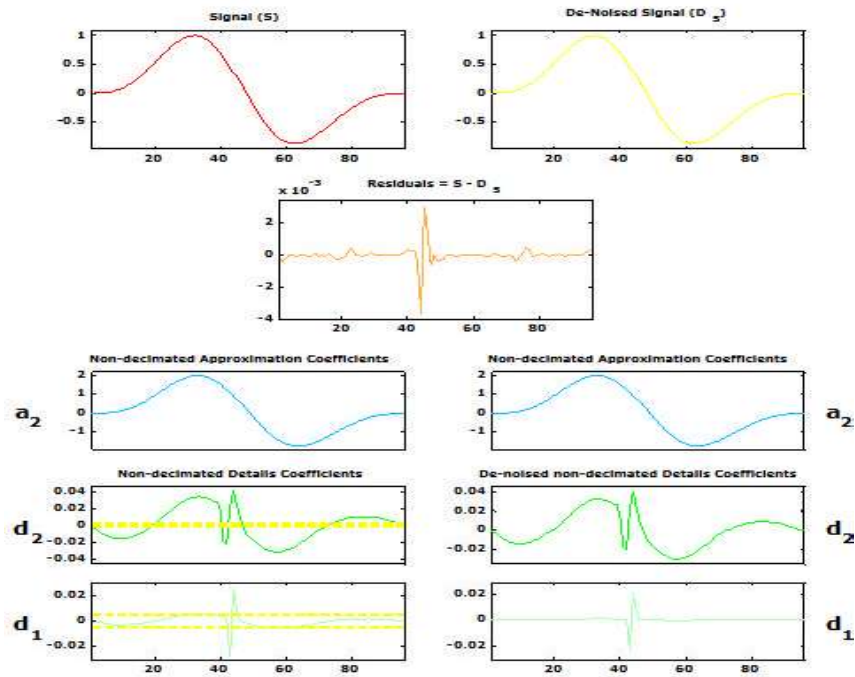


Fig. 11: The graph of SWT for response of cracked dam under sym2 wavelet analyzer (Mode1-U2)

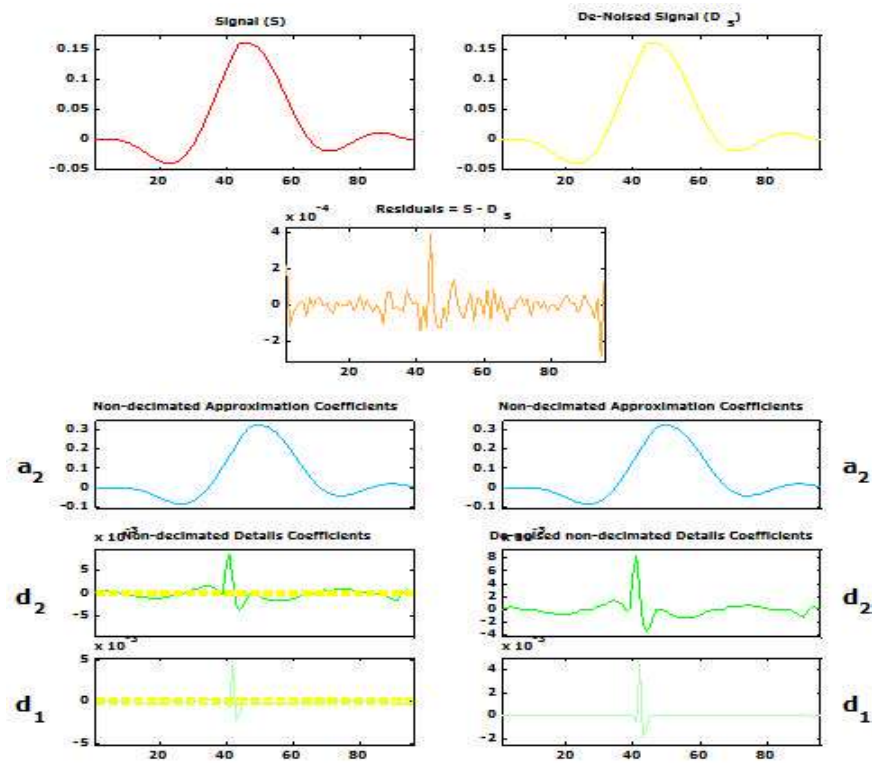


Fig. 12: The graph of SWT for response of cracked dam under sym2 wavelet analyzer (Mode1-U3)

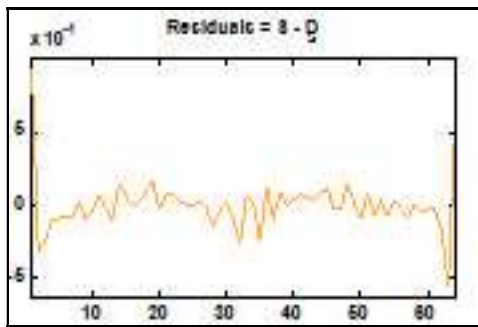


Fig. 13: SWT for response of safe dam under Ciof wavelet analyzer (Mode1-U2)

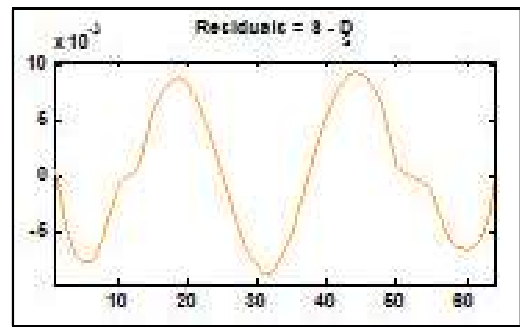


Fig. 15: SWT for response of safe dam under Bior wavelet analyzer (Mode1-U2)

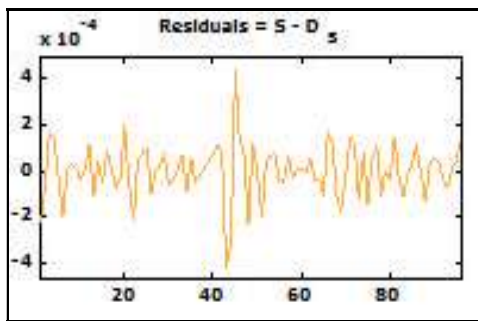


Fig. 14: SWT for response of cracked dam under Ciof wavelet analyzer (Mode1-U2)

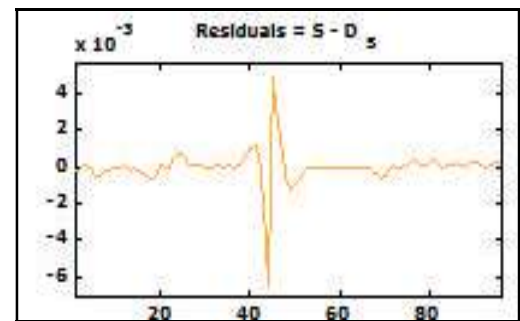


Fig. 16: SWT for response of cracked dam under Bior wavelet analyzer (Mode1-U2)

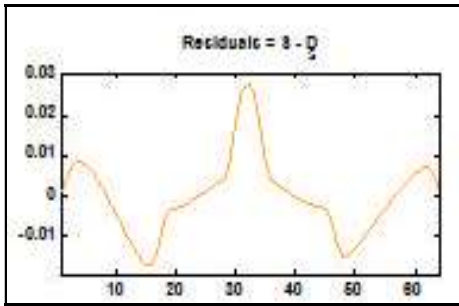


Fig. 17: SWT for response of safe dam under Db wavelet analyzer (Mode1-U2)

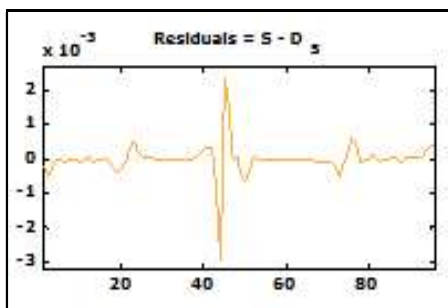


Fig. 18: SWT for response of cracked dam under Db wavelet analyzer (Mode1-U2)

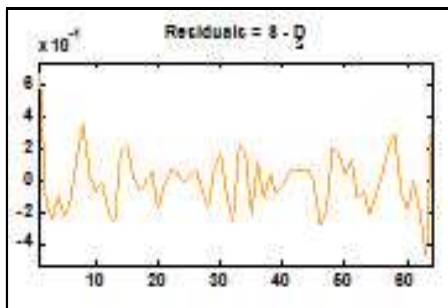


Fig. 19: SWT for response of safe dam under Dmey wavelet analyzer (mode1-U2)

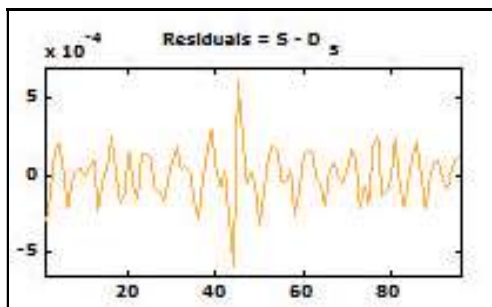


Fig. 20: SWT for response of cracked dam under Dmey wavelet analyzer (Mode1-U2)

was no noise or any harmony in graphs of safe dam (Fig. 7 and 8). In other words, the location of crack is in central of crest level in location of 48<sup>th</sup> point (center of 96 selected points). Wavelet-transform has been used to recognition of damage place for characteristic of SWT which are different from other methods. It has high clearance ability to distinguish of different frequencies. SWT can show the location of frequency changes that these locations are points that they have been damaged.

The results of analysis with other wavelets have been shown in Fig. 13-20 only in noise state (because of paper limitation) in safe and damaged dam in reservoir direction (*U2*). As figures indicate, recognition of crack location is possible.

### CONCLUSION

- Wavelet-transform has high ability in analysis of static and dynamic response signals. This property is clear in recognition of discrete and uncoordinated situations such as sudden fluctuation in stiffness.
- The effects of damage on wavelet coefficient graph are more compressed in low scales and the location of noise is clear. Meanwhile, increase in scale which is corresponding with decrease in frequency leads to wide spreading noise and low clearness.
- Selection of proper scale is important in wavelet analysis. Higher scales are recommended for recognition of small damages and low scales are recommended for recognition of damages near support.
- By investigation of wavelets type in low scales, Dmey, Sym, Bior, Ciof, Db and Haar wavelets are introduced as a useful wavelets in crack detection. As experienced in this research, Coif, Db and Sym wavelets have suitable efficiency in recognition of crack location rather than other wavelets.

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