World Applied Sciences Journal 3 (3): 522-530, 2008
ISSN 1818-4952

Investigation of Dynamic Properties of Cantilever Castellated Beams in Comparison with Plain-webbed Beams using White Noise Excitation

Mohammad Ali Lotfollahi-Yaghin and Hamid Ahmadi

Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran

Abstract: In order to investigate the dynamic properties of castellated beams, the excitation intensity of dynamic mode shapes for different loading patterns must be considered. Obviously, under a specific loading pattern, all of dynamic mode shapes are not excited. In other words, the response of the structure is dependent on the exerted loading. In this article, the powerful FEM software, ANSYS is used for investigation of dynamic properties of castellated beams. At first, a modal analysis is performed on both plain-webbed and castellated cantilever beams with two different types of lateral bracing and their dynamic properties and mode shapes are studied. This analysis shows that the presence of large web openings may have a severe penalty on the load carrying capacities of castellated beams under the dynamic gravitational loads. Then a white noise dynamic load with an appropriate frequency range is exerted on both plain-webbed and castellated beams and their various dynamic responses are studied. It can be seen that the loading pattern is a very important factor for investigation of dynamic properties of cantilever castellated beams.

Key words: Castellated beam . cantilever beam . flexural mode shape . finite elements method . response spectrum . white noise . PSD

INTRODUCTION

Modern techniques of fabricating steel members allow for welded I-beams to be easily fabricated and it is often economical to produce such beams with equal flanges and slender unstiffened webs using standard hot-rolled beams [1]. Castellated beams are such structural members, which are made by flame cutting a rolled beam along its centerline and then rejoining the two halves by welding so that the overall beam depth is increased by 50% for improved structural performance against bending [2]. Therefore, application of these structural members may lead to substantial economies of material. Basically, the reasons for fabricating castellated beams are as follows [1]:

Section height will be increased that results in the enhancement of moment of inertia, section modulus, stiffness and flexural resistance of the section. Also the weight of the profile will be decreased which, in turn, the weight of the whole structure will be reduced. The utilization of the existing profiles is optimized and also plate girders are not needed. Finally, the passage of services through the web openings will be easily provided.

The presence of large web openings may have a severe penalty on the load carrying capacities of castellated beams, depending on the shapes, the sizes and the location of the openings.

The widespread use of castellated beams as structural members in multistory buildings, commercial and industrial buildings and portal frames, has prompted several investigations into their structural behavior.

According to the authors' knowledge, despite the considerable volume of research on the structural behavior of castellated beams and dynamic behavior of plain-webbed beams, dynamic behavior of castellated steel beams is not considerably investigated.

In this article, dynamic properties of cantilever castellated beams with two different types of lateral restraints are investigated and are compared with the dynamic properties of plain-webbed beams with the same height and length of 27 cm and 310 cm, respectively. At first, a modal analysis is performed on both plain-webbed and castellated beams and their dynamic properties and mode shapes are studied. In modal analysis of structures, it is necessary to assume the number of considered modes. The number of modes depends on the importance of investigation and the required accuracy. Unfortunately, the weight of the effect of higher modes is not known precisely. So the selection of number of modes depends on the experience of the researcher.

After the modal analysis, a uniform distributed white noise dynamic load with appropriate frequency range is exerted on both plain-webbed and castellated

Corresponding Author: Dr. Mohammad Ali Lotfollahi-Yaghin, Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran

beams with two different types of lateral bracing and their various dynamic responses are studied. In the case of white noise loading, it is not necessary to select the number of modes at first. After running the analysis, all of the active modes which are proportionate to the loading conditions are appeared with their appropriate weight of effect. Under the white noise dynamic loading, various responses of the beams such as displacement, flexural moment and acceleration are monitored for different loading frequencies along the beams and PSD (power spectral density) diagrams of these responses are investigated.

LITERATURE REVIEW

The widespread use of castellated beams has prompted several investigations into their structural behavior.

Web distortion and flexural-torsional buckling of the web and flange were studied by Hancock *et al*. [3]. Trahair studied the flexural-torsional buckling of castellated beams [4, 5]. Web Buckling in Thin Webbed Castellated Beams was investigated by Redwood and Zaarour [6]. Lateral-torsional buckling of castellated beams was studied by Nethercot and Kerdal [7]. Failure Mode for Castellated Beams was investigated by Kerdal and Nethercot [8]. The moment-gradient factor in lateral-torsional buckling on inelastic castellated beams was considered by Mohebkhah [9]. A finite-element model for the inelastic nonlinear analysis of castellated beams that includes the effects of elastic lateral restraints at midspan and large lateral deflections is developed by Mohebkhah and Showkati [2]. The model is used to investigate the effects of central elastic lateral restraints attached to the top flange on the inelastic flexural-torsional strengths of simply supported castellated beams under pure bending loading. It is found that a central elastic lateral restraint generally increases the inelastic strength of the beam, but that the effect of the restraint depends not only on the stiffness of the restraint but also on the modified slenderness of the beam [2] and a series of six tests on full-scale simply supported castellated beams with a centrally concentrated load and an effective lateral brace at the midspan of the compression flange was performed by Showkati and Zirakian, mainly with the aim of experimentally verifying the web distortion in these structural members [1].

MODAL ANALYSIS

Determination of natural frequencies and mode shapes of oscillation is the first step in dynamic analysis. In this section, using ANSYS 5.4, a modal analysis is performed on both castellated and plain-webbed cantilever beams. The length of the beams is 310 cm. for each beam two different types of lateral restraints are considered. In the first case, lateral restraints used along the beam, are located only at the centerline of the beam and at the top and bottom of the web plate of the beam (imperfect bracing). In the second case, lateral restraints are used at whole nodes along the beam (perfect bracing).

Some mode shapes of castellated beams with imperfect bracing are presented as examples in Fig. 1-5. Figure 1 shows the third mode shape of oscillation which is the third flexural mode shape. Figure 2 shows the fourth mode shape of oscillation which is the axial mode. Figure 3 shows the fifth mode shape of oscillation which is the combination of the first longitudinal torsional mode and the second web buckling mode. Figure 4 shows the sixth mode shape of oscillation which is the combination of the second

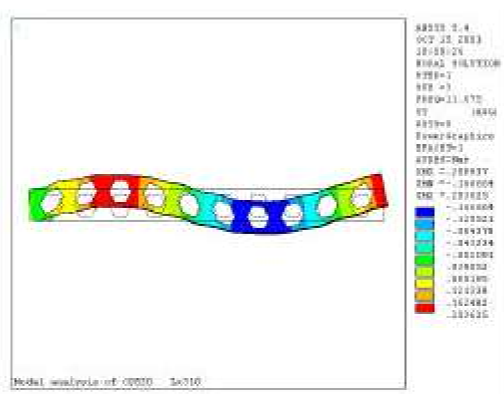

Fig. 1: 3rd mode shape of castellated beam with imperfectly lateral bracing

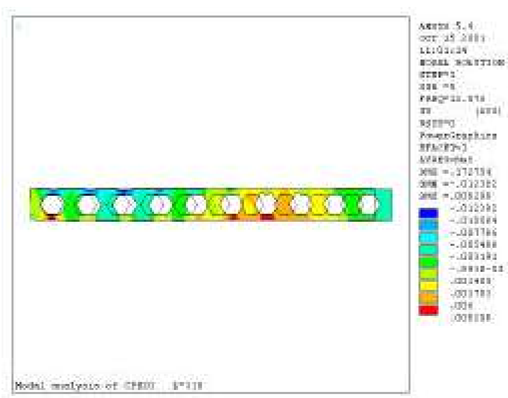

Fig. 2: 4th mode shape of castellated beam with imperfectly lateral bracing

Table 1: Natural frequencies of oscillation for plain-webbed beam with imperfectly lateral bracing

Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.
20.926	16	17.894	11	14.832	6	1.1477	1
21.651	17	18.517	12	15.587	7	6.4007	2
22.432	18	18.611	13	15.852	8	13.247	3
23.288	19	19.636	14	17.101	9	13.724	4
24.112	20	20.049	15	17.525	10	14.104	5

Table 2: Natural frequencies of oscillation for castellated beam with imperfectly lateral bracing

Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.
18.294	16	15.678	11	12.192	6	1.1495	1
19.080	17	15.740	12	13.073	7	5.3327	2
19.947	18	16.851	13	14.286	8	11.675	3
20.839	19	17.359	14	14.388	9	11.716	4
21.766	20	17. 995	15	14.844	10	12.076	5

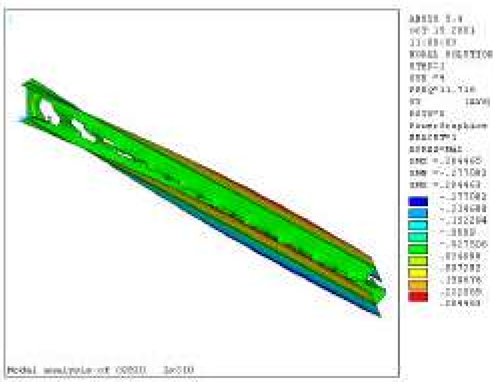

Fig. 3: 5th mode shape of castellated beam with imperfectly lateral bracing

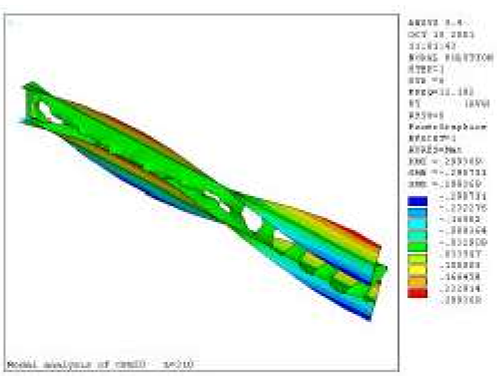

Fig. 4: 6th mode shape of castellated beam with imperfectly lateral bracing

longitudinal torsional mode and the second web buckling mode. Figure 5 shows the eighth mode shape of oscillation which is the combination of the fourth longitudinal torsional mode and the second web buckling mode.

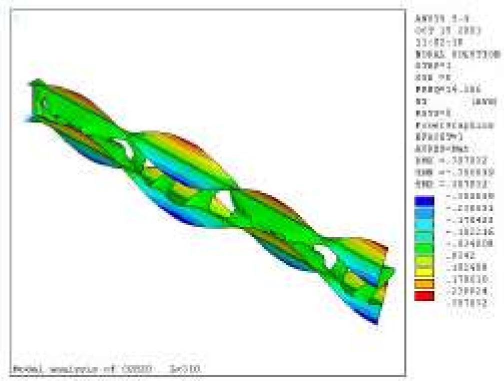

Fig. 5: 8th mode shape of castellated beam with imperfectly lateral bracing

Natural frequencies of oscillation for plain-webbed and castellated beams with imperfect bracing are presented in Table 1 and 2, respectively. Also the natural frequencies of oscillation for plain-webbed and castellated beams with perfect bracing are presented in Table 3 and 4, respectively. Frequencies of common mode shapes between imperfect and perfect bracing cases are compared in Table 5 and 6 for plain-webbed and castellated beams, respectively.

According to Table 5, there are only 4 common mode shapes between imperfectly and perfectly laterally braced plain-webbed beams. The other mode shapes of imperfectly laterally braced plain-webbed beams which are not presented in Table 5 are complicated combined mode shapes which have high frequencies. According to Table 6, there is 5 common mode shapes between imperfectly and perfectly laterally braced castellated beams.

It can be seen that both the plain-webbed and castellated perfectly laterally braced beams are stiffer and have higher frequencies in comparison with

Table 3: Natural frequencies of oscillation for plain-webbed beam with perfectly lateral bracing

Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.
36.389	10	35.717	7	15.940	4	1.1864	1
36.389	11	35.943	8	26.661	5	6.5786	2
37.051	12	35.943	9	35.717	6	13.565	3

Table 4: Natural frequencies of oscillation for castellated beam with perfectly lateral bracing

Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	Mode No.
32.621	10	30.208	7	12.490	4	1.1868	1
32.805	11	31.597	8	18.221	5	5.4464	2
33.849	12	32.534	9	24.412	6	11.867	3

Table 5: Frequencies of common mode shapes between imperfectly and perfectly laterally braced plain-webbed beams

Mode No.	Mode Type	Frequencies of perfectly laterally braced beam (Hz)	Frequencies of imperfectly laterally braced beam (Hz)	Difference (%)
1	First flexural mode	1.1864	1.1477	3.0
2	Second flexural mode	6.5786	6.4007	2.7
3	Axial mode	13.565	13.247	2.0
4	Third flexural mode	15.940	15.587	2.0
5	Fourth flexural mode	26.661	-	-

Table 6: Frequencies of common mode shapes between imperfectly and perfectly laterally braced castellated beams

Mode No.	Mode Type	Frequencies of perfectly laterally braced beam (Hz)	Frequencies of imperfectly laterally braced beam (Hz)	Difference (%)
1	First flexural mode	1.1868	1.1495	3.0
2	Second flexural mode	5.446	5.3327	2.0
3	Third flexural mode	11.867	11.675	1.6
4	Axial mode	12.490	12.076	3.0
5	Fourth flexural mode	18.221	17.995	1.2
6	Fifth flexural mode	24.412	-	-
7	Sixth flexural mode	30.208	-	-

imperfect laterally braced beams. Also the maximum decrement of frequencies in imperfectly laterally braced beams is 3%.

RESPONSE TO RANDOM LOADING

The loads that arise from natural phenomena can not adequately be described by sinusoidal or other periodic functions. The pattern of loading with time does not repeat itself at regular intervals. In such cases it is necessary to resort to a statistical analysis, in which use is made of certain properties of the randomly varying load. These properties are chosen such that they remain constant or stationary over the time period for the analysis [10, 11]. The problem is to relate the response of a structure, x (t) to the random input excitation F (t) (Fig. 6).

The underlying principle of random vibration analysis is the concept of the Fourier integral and the most direct and engineering way of understanding this is to first consider the application of the Fourier series to a system of large period. It is often more convenient to make use of the complex forms for sine and cosine and express the Fourier series in its complex equivalent as:

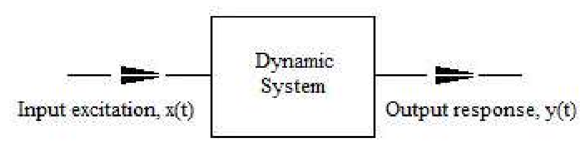

Fig. 6: Input excitation and output response in a dynamic system

$$F(t) = \sum_{-\infty}^{+\infty} c_r e^{jr\omega t} \quad (1)$$

$$c_r = \frac{1}{T}\int_{-T/2}^{T/2} F(t)e^{-jr\omega t}dt \quad (2)$$

Thus equation 1 becomes (with the expression for coefficient c_r written in full)

$$F(t)=\sum_{-\infty}^{+\infty}\left\{\frac{\Delta\omega}{2\pi}\int_{-\pi/\Delta\omega}^{\pi/\Delta\omega}F(t)e^{-jr\Delta\omega t}dt\right\}e^{jr\Delta\omega t} \quad (3)$$

If the random process is considered to be periodic but of infinite period, the summation is replaced by an integral and the Fourier series, equation 3 becomes:

$$F(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left\{\int_{-\infty}^{+\infty}F(t)e^{-j\omega t}dt\right\}e^{j\omega t}d\omega \quad (4)$$

Defining the term in parenthesis by

$$F(j\omega)=\int_{-\infty}^{+\infty}F(t)e^{-j\omega t}dt \quad (5)$$

Then

$$F(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}F(j\omega)e^{j\omega t}d\omega \quad (6)$$

The near symmetry of equations 5 and 6 has led to the terminology of referring to them as the Fourier transform pair and it should be noted that the Fourier integral equation 5, enables a time varying quantity, F(t), to be expressed in its frequency components, F(jω), whereas equation 6 is an inverse transform, from the frequency domain, F(jω), to the time domain, F(t).

If the frequency is expressed in Hertz instead of radius/sec in equations 5 and 6, such that ω = 2πf, the asymmetry caused by the 1/2π term is removed and the equations possess even more symmetry, differing only by the sign in the exponential term

$$F(jf)=\int_{-\infty}^{+\infty}F(t)e^{-j2\pi ft}dt \quad (7)$$

$$F(t)=\int_{-\infty}^{+\infty}F(jf)e^{j2\pi ft}df \quad (8)$$

From equation 8 it may be shown that

$$\int_{-\infty}^{+\infty}F^2(t)dt=2\int_0^{\infty}\left|F(jf)\right|^2 df \quad (9)$$

$S_{FF}(f)$ Which is known as the spectral density (sometimes called, although aptly, power spectral density) is defined as below:

$$S_{FF}(f)=\frac{2\left|F(jf)\right|^2}{T} \quad (10)$$

For a single degree of freedom system, using the Duhamel (or Convolution) integral technique, it can be shown that (Fig. 7) [11, 12].

$$S_{XX}(f)=\left|H(jf)\right|^2 S_{FF}(f) \quad (11)$$

Where H (jf) is given as below

$$\left|H(jf)\right|^2=\frac{1}{(K-M(2\pi f))^2+(C.2\pi f)^2} \quad (12)$$

COMPUTER MODELLING AND ANALYSIS

In this article, ANSYS 5.4, powerful FEM software, is used for computer modelling and analysis of the beams. ANSYS 5.4 presents various types of elements for structural modelling. Shell 63 which is used for modelling of shell and flexural members is selected here in order to create the model. It has three transitional and three rotational degrees of freedom in each node [13]. Modulus of elasticity of steel is 2.1×10^6 kgf/cm^2. Poison's ratio is 0.3. Damping ratio is 0.02 and weight per volume is 7800 kgf/m^3. Also MATLAB is used for drawing the PSD diagrams of various responses of the beams [14].

THE RESPONSE OF STRUCTURE TO WHITE NOISE EXCITATION

The processes which are activated in a narrow range of frequencies are called narrow banded processes and which are activated in a broad range of

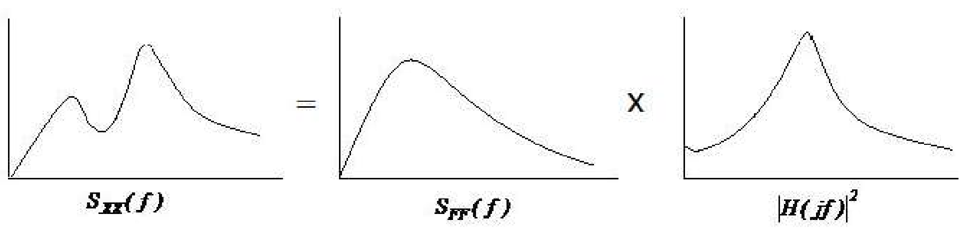

Fig. 7: Schematic representation of spectral technique

frequencies are called broad banded processes [15]. White noise is a broad banded forcing function which its range of frequency is 0 to ∞ and its spectral density has a constant value at all frequencies. It denotes a totally random process. Practically, a forcing function which has a constant spectral density in an adequately broad range of frequencies can be called white noise.

Using white noise as an input force is very useful in order to analyze the dynamic systems because all of the effective mode shapes of oscillation can be activated under the white noise excitation. Also since the spectral density of white noise has a constant value at all frequencies, the importance and effectiveness of various modes can be determined.

It must be noted that loading pattern has a considerable effect on the excited modes. In other words, in order to excite certain modes of oscillation in a specific structure, white noise must be exerted in a suitable pattern on the structure [16]. Since the analysis is performed in frequency domain, it is better to show the responses by PSD diagrams.

In this article, some uniform distributed white noise dynamic loads with different loading patterns and appropriate frequency range are exerted on both plain-webbed and castellated beams with two different types of lateral bracing (imperfectly and perfectly lateral restraints) and their various dynamic responses are studied. Under the white noise dynamic loading, various responses of the beams such as displacement, flexural moment and acceleration are monitored for different loading frequencies along the beams and PSD diagrams of these responses are investigated.

PSD OF RESPONSES FOR IMPERFECTLY LATERALLY BRACED BEAMS

In this section, a uniform downward distributed white noise dynamic load is exerted on both plain-webbed and castellated imperfectly laterally braced beams and their various dynamic responses are studied.

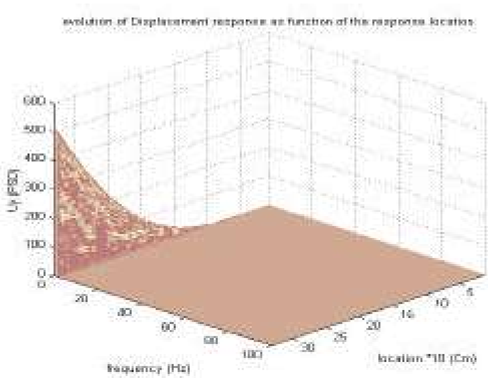

Fig. 8: PSD of vertical displacement for plain-webbed beam with imperfectly lateral bracing

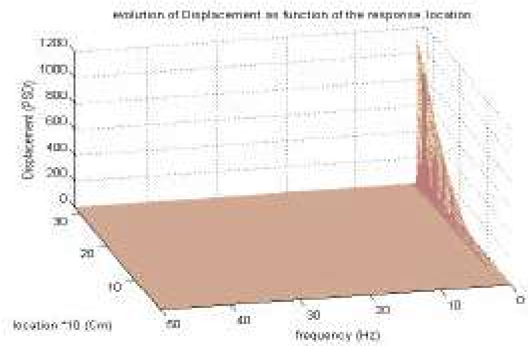

Fig. 9: PSD of vertical displacement for castellated beam with imperfectly lateral bracing

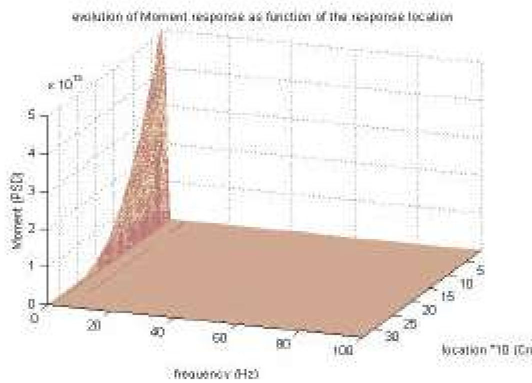

Fig. 10: PSD of flexural moment for plain-webbed beam with imperfectly lateral bracing

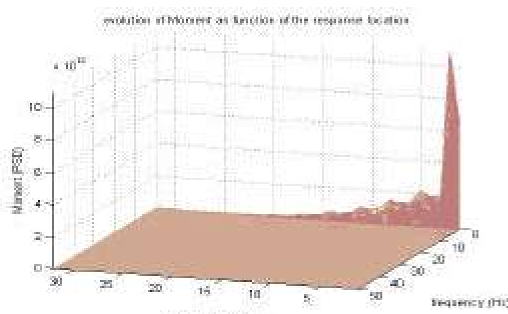

Fig. 11: PSD of flexural moment for castellated beam with imperfectly lateral bracing

PSD of vertical displacement: PSD of vertical displacement for plain-webbed and castellated beams are presented in Fig. 8 and 9, respectively. It can be seen that only the first mode is excited and the values of PSD in castellated beam are higher than the values in plain-webbed beams.

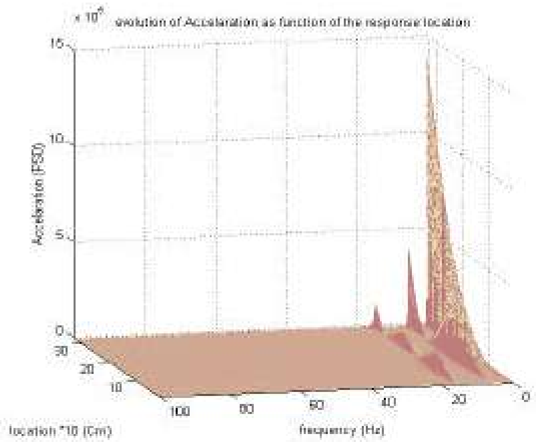

Fig. 12: PSD of acceleration for plain-webbed beam with imperfectly lateral bracing

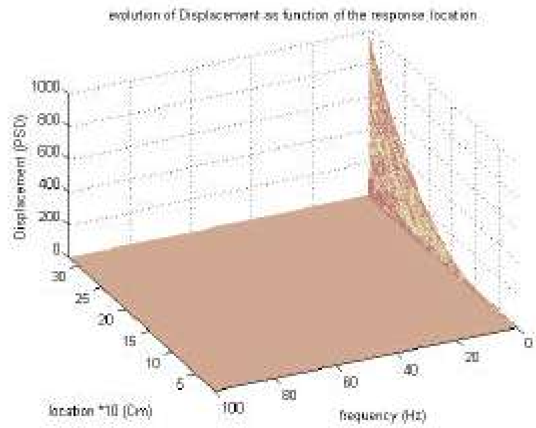

Fig. 15: PSD of vertical displacement for castellated beam with perfectly lateral bracing

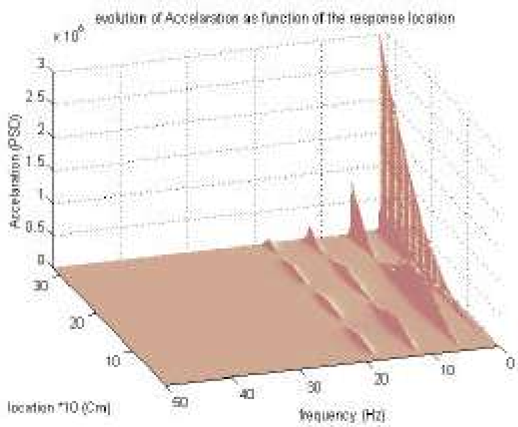

Fig. 13: PSD of acceleration for castellated beam with imperfectly lateral bracing

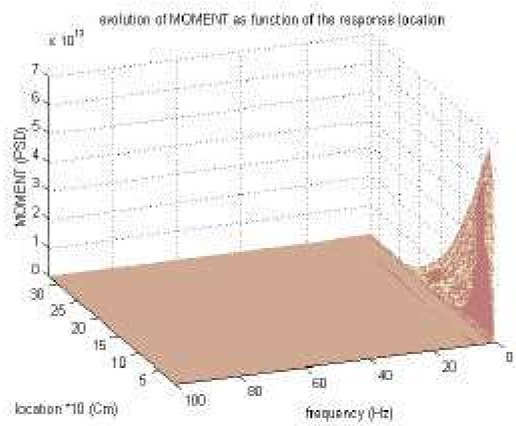

Fig. 16: PSD of flexural moment for plain-webbed beam with perfectly lateral bracing

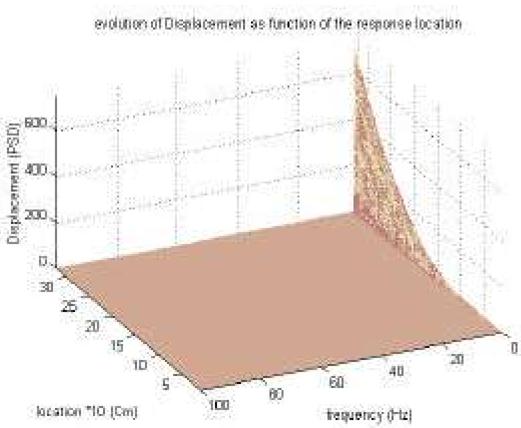

Fig. 14: PSD of vertical displacement for plain-webbed beam with perfectly lateral bracing

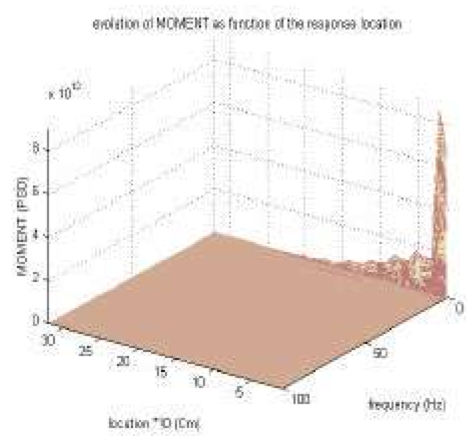

Fig. 17: PSD of flexural moment for castellated beam with perfectly lateral bracing

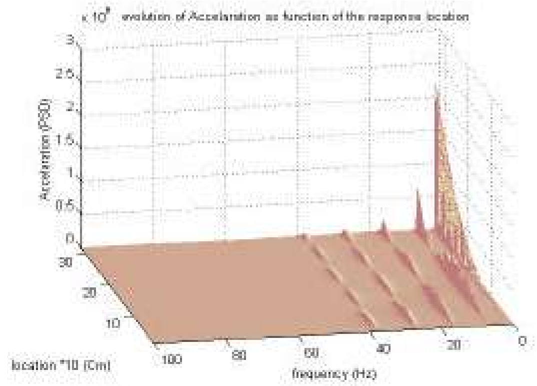

Fig. 18: PSD of acceleration for plain-webbed beam with perfectly lateral bracing

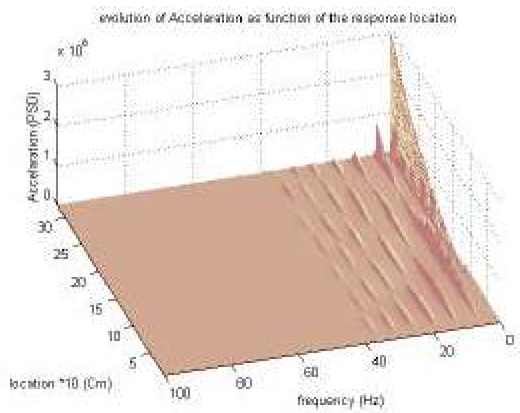

Fig. 19: PSD of acceleration for castellated beam with perfectly lateral bracing

PSD of flexural moment: PSD of flexural moment for plain-webbed and castellated beams are presented in Fig. 10 and 11, respectively. It can be seen that only the second mode is excited and the values of PSD in castellated beam are higher than the values in plain-webbed beams. Also some breaks are observed at the location of openings. So PSD of flexural moment can be used in order to identify the location of defects such as cracks in a structural member.

PSD of acceleration: PSD of acceleration for plain-webbed and castellated beams are presented in Fig. 12 and 13, respectively. The first mode is excited and Moreover, the higher modes can be observed in both beams. The influence of the 1^{st}, 2^{nd} and 3^{rd} modes can be clearly observed in both beams. Also the 4^{h} mode is excited in castellated beam. Since the applied load is downward so the frequencies related to torsional and axial deformation and web buckling are not observed in the PSD of acceleration response [17].

PSD OF RESPONSES FOR PERFECTLY LATERALLY BRACED BEAMS

In this section, a uniform downward distributed white noise dynamic load is exerted on both plain-webbed and castellated perfectly laterally braced beams and their various dynamic responses are studied.

PSD of vertical displacement: PSD of vertical displacement for plain-webbed and castellated beams are presented in Fig. 14 and 15, respectively. It can be seen that only the first mode is excited and the values of PSD in castellated beam are higher than the values in plain-webbed beams.

PSD of flexural moment: PSD of flexural moment for plain-webbed and castellated beams are presented in Fig. 16 and 17, respectively. It can be seen that only the second mode is excited and the values of PSD in castellated beam are higher than the values in plain-webbed beams. Also some breaks are observed at the location of openings.

PSD of acceleration: PSD of acceleration for plain-webbed and castellated beams are shown in Fig. 18 and 19, respectively. Activity of the 1^{st}, 2^{nd}, 3^{rd}, 4^{th} and 5^{th} modes is clear in both beams and, moreover, the 6^{th} and 7^{th} modes are excited in castellated beam.

It can be seen that both odd and even modes are observed in PSD of acceleration. It must be noted that even modes were not observed in simply supported beams [18].

CONCLUSIONS

1. The frequency of flexural, axial and torsional modes of oscillation in cantilever castellated beam with imperfectly lateral bracing are always less than frequencies of these modes in plain-webbed beam. So the flexibility of castellated beam is higher than the flexibility of plain-webbed beam with the same height.
2. Both the plain-webbed and castellated perfectly laterally braced beams are stiffer and have higher frequencies in comparison with imperfect laterally braced beams. Also the maximum decrement of frequencies in imperfectly laterally braced beams is 3%.
3. According to PSD of vertical displacement it can be seen that only the first mode is excited and the values of PSD in castellated beam are higher than the values in plain-webbed beams and according to PSD of flexural moment it can be seen that only

the second mode is excited and the values of PSD in castellated beam are higher than the values in plain-webbed beams.

4. Some breaks are observed at the location of openings in PSD of flexural moment. So PSD of flexural moment can be used in order to identify the location of defects such as cracks in a structural member.
5. Contribution of higher modes in PSD of acceleration response is clearer than their contribution in PSD of vertical displacement and flexural moment.
6. Since the applied load is downward so the frequencies related to torsional and axial deformation and web buckling are not observed in the PSD of acceleration response.

REFERENCES

1. Mohebkhah, A., 2004. The moment-gradient factor in lateral-torsional buckling on inelastic castellated beams. J. Constructional Steel Res., 60: 1481-1494.
2. Nethercot, D.A. and D. Kerdal, 1982. Lateral-torsional buckling of castellated beams. The Struct. Eng., 60B (3): 53-61.
3. Hancock, G.J., M.A. Bradford and N.S. Trahair, 1980. Web Distortion and Flexural-Torsional Buckling. J. Struct. Division.
4. Trahair, N.S., 1993. Flexural-Torsional Buckling of Structures. E&F N Spon London, UK.
5. Trahair, N.S., 1995. Stability and Design of Steel Beams and Arches. Balkema, Rotterdam.
6. Redwood, R. and W. Zaarour, 1996. Web Buckling in Thin Webbed Castellated Beams. J. Struct. Eng., (ASCE).
7. Kerdal, D. and D.A. Nethercot, 1984. Failure Mode for Castellated Beams. J. Construct Steel and Res., 4 (4): 295-315.
8. Mohebkhah, A. and H. Showkati, 2005. Bracing requirements for inelastic castellated beams. J. Constructional Steel Res., 61: 1373-1386.
9. Showkati, H. and T. Zirakian, 2006. Distortional buckling of castellated beams. J. Constructional Steel Res., 62: 863-871.
10. Newland, D.E., 1993. An Introduction to Random Vibrations, Spectral and Wavelet Analysis. 3rd Edn. Longman Inc. London. Distributed in the USA by John Wiley, New York.
11. Hallam, M.G. *et al*., 1978. Dynamics of Marine Structures. Atkins Research and Development. Report UR8. 2nd Edn. Great Britain.
12. Chatfield, C., 1989. The Analysis of Time Series, an Introduction. 4th Edn. Chapman and Hall Ltd., London.
13. ANSYS User's Guide. Revision 5.5. Swanson Analysis System. 9th Edn. 1998.
14. Krauss, T.P. *et al*., 1994. Signal Processing Toolbox, For Use with MATLAB. The Math Works Inc.
15. Ochi, M.K., 1990. Applied Probability and Stochastic Processes. In Engineering and Physical Sciences. John Wiley and Sons Inc. New York.
16. Clough, R.W. and J. Penzin, 1993. Dynamics of Structures. 2nd Edn. Mc Graw Hill.
17. Lotfollahi-Yaghin, M.A., 2002. Dynamic analysis of castellated beams using finite elements method. Final Report. National Program of Researches. Urmia University, Iran, (In Persian).
18. Lotfollahi-Yaghin, M.A., 2002. Investigation of the changes in dynamic mode shapes of castellated beams in comparison with plain-webbed beams. 1st conference on thin-walled structures. Urmia University, Iran. (In Persian).