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Wavelet-Kernel Estimation of Regression Function for Uniformly Mixing Process

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Abstract: We propose estimation of the regression function r for uniformly mixing processes with common probability density function wavelets and some asymptotic properties of the proposed estimator are investigated.

Key words: Wavelets . uniformly mixing . smooth estimation . regularity . orthogonal . multiresolution . holder space . asymptotic properties

1. INTRODUCTION

Methods of estimation of density and regression function are quite common in statistical applications.

Wavelet theory has the potential to provide statisticians with powerful new techniques for nonparametric inference. It combines recent advances in approximation theory with insights gained from applied signal analysis. Nonparametric curve estimation by wavelets has been treated in numerous articles in various setups see, Antoniadis [1], Donoho [2] and Hardle [3].

The problem of interest is the estimation of nonparametric regression function based on the observations $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$. There are many interesting examples where applications of regression smoothing methods have yielded analysis essentially unobtainable by other techniques. Eubank [4] and Muller [5]. In contrast with most existing works Antoniadis [6], Delyon [7], Kovac and Silverman [8], Vidakovic [9] and Sardy [10], Antoniadis and Fan [11].

In this paper we consider wavelet estimator of regression function for uniformly mixing processes when the random design model is given as the

$$Y_i = r(X_i) + \varepsilon_i, \quad i = 1,2,3,\ldots,n \tag{1.1}$$

Where (X_i, Y_i), $i = 1, 2, \ldots, n$ be identically distributed as a two-dimensional random vector with $E(Y^2)\leq\infty$ and the error δ_i, conditionally on X_i are assumed to be independent with zero expectation and a bounded conditional variance. Some asymptotic properties of propose estimator is investigated.

2. PRELIMINARIES

Let $\{X_n, n\geq 1\}$ be a sequence of random variables on the probability space $(\Omega,\aleph,P)$. We suppose that X_i has a bounded and compactly supported marginal density f(x), with respect to Lebesgue measure, which does not depend on i A natural estimator of F(x) is the piecewise constant empirical dis tribution function:

$$\hat{F}(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x)$$

Since f(x) is defined to be derivative of F(x), the natural estimator of f(x) for suitably h can be written:

$$\begin{aligned}\hat{f}(x) &= \frac{1}{2h}\left(\hat{F}(x+h) - \hat{F}(x-h)\right)\\ &= \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right) = \frac{1}{n}\sum K_h(x, X_i)\end{aligned} \tag{2.1}$$

Where K(x) is the kernel function form as:

$$K(x) = \begin{cases}\frac{1}{2} & x \in (-1,1)\\ 0 & \text{otherwise}\end{cases}$$

The series expansion of function in terms of a set of orthogonal basis functions is familiar in statistics.

Let the nested sequence of closed subspaces; …… $V_{j-1}\subset V_j \subset V_{j+1}\subset\ldots$, $j\in Z$, be a multiresolutuon approximation to $L^2(R)$. Define W_j, $j\in Z$ to be orthogonal complement of V_j in V_{j+1}.

The term wavelets are used to refer to a set of basis functions with very special structure. The special of wavelets basis for function $f\in L^2(R)$ as scaling function φ and mother wavelet ψ such that $\{\varphi(x-k)\}_{k\in Z}$ forms an orthogonal basis for V_0 and $\{\psi(x-k)\}_{k\in Z}$ forms an orthonormal basis for W_0. Other wavelets in the basis are then generated by translation of the scaling function and dilations of the mother wavelet by using the relationships:

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$$\varphi_{m_0 k}(x) = 2^{m_0/2}\varphi(2^{m_0}x - k), \psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k) \quad (2.2)$$

Given above Wavelet basis, a function $f \in L^2(R)$ can be written a formal expansion:

$$f = \sum_{k\in Z}\alpha_{m_0,k}\varphi_{m_0,k} + \sum_{j=m_0}^{\infty}\sum_{k\in Z}\beta_{j,k}\psi_{j,k} \quad (2.3)$$

Where

$$\alpha_{j_0,k} = \int f(x)\varphi_{j_0 k}(x)dx\ ,\ \delta_{j,k} = \int f(x)\psi_{j,k}dx$$

We suppose that both φ and ψ have compact supports included in [0,1] and r^- regular multiresolution analysis belong to the *Holder space* C^{r+1}, $r\in N$.

Consider the density which has the same form as (2.3). As for general orthogonal series estimator, Daubechies [11], density estimator can be writhen as:

$$\begin{aligned}\hat{f} &= \sum_{k\in Z}\hat{\alpha}_{m_0,k}\varphi_{m_0,k}(x) + \sum_{j\geq m_0}\sum_{k\in Z}\hat{\beta}_{j,k}\psi_{j,k}(x)\\ &= \mathbf{P}_{m_0}f + \sum_{j\geq m_0}\sum_{k\in Z}\hat{\beta}_{j,k}\psi_{j,k}\end{aligned} \quad (2.4)$$

Where the obvious coefficient estimator can be written:

$$\begin{aligned}\hat{\alpha}_{m_0,k} &= E[\varphi_{m_0,k}(X)] = \frac{1}{n}\sum_{i=1}^{n}\varphi_{m_0,k}(X_i)\\ \hat{\beta}_{j,k} &= E[\psi_{j,k}(X)] = \frac{1}{n}\sum_{i=1}^{n}\psi_{j,k}(X_i)\end{aligned} \quad (2.5)$$

The projection of f in $L^2(R)$ on to the space V_{m_0} in equation (2.4) is a special case of a kernel density estimator with kernel,

$$\begin{aligned}K_{m_0,k}(x,y) &= \sum_{k\in Z}\varphi_{m_0,k}(x)\varphi_{m_0,k}(y)\\ &= 2^{m_0}\sum_{k\in Z}\varphi_{m_0,k}(2^{m_0}x - k)\varphi_{m_0,k}(2^{m_0}y - k)\end{aligned}$$

In terms of this kernel, this can be expressed as:

$$\hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n}K_h(x, X_i) = \frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{x}{h}, \frac{X_i}{h}\right) \quad (2.6)$$

Where $h = 2^{-m_0}$ and the orthogonal projection kernels are $K_{m_0 k}(x,y) = 2^{m_0}K_{m_0,k}(2^{m_0}x, 2^{m_0}y)$ It is easy to see that $K_0(x,y) = K_0(x+k, y+k)$ for $K\in z$

Obviously, K_0 is not a convolution kernel, but the regularity of φ and ψ implies that is bounded above by convolution kernel, that is $|K_0(x,y) \leq c(x-y)|$, where c is some positive, bounded integrable function satisfying moment condition [12]. In particular, the bounded

$$\mathrm{Supp}_{x,y}\left|K_{m_0}(x,y)\right| = O\left(\frac{1}{h}\right)$$

is often needed. Obviously we cannot estimate an infinite set of $\beta_{j,k}$ from the finite sample, so it is usually assumed that f belong to a class of function with certain regularity. The corresponding norm of the sequence of $\beta_{j,k}$ is finite and therefore $\beta_{j,k}$ must be zero. The resulting density estimate is:

$$\begin{aligned}\hat{f} &= \sum_{k\in Z}\tilde{\alpha}_{m_0,k}\varphi_{m_0,k}(x) + \sum_{j=m_0}^{J-1}\sum_{k\in Z}\tilde{\beta}_{j,k}\psi_{j,k}(x)\\ &= \mathbf{P}_J f + \sum_{j=m_0}^{J-1}\sum_{k\in Z}\tilde{\beta}_{j,k}\psi_{j,k}(x)\end{aligned} \quad (2.7)$$

Then these empirical Coefficients are calculated for resolution level m_0 up to some large value j, which is chosen that $P_J f$ approximates very well.

3. MAIN RESULT

One of the basic approaches to nonparametric regression is to consider unknown function r expanded as a generalized Fourier series and then to estimate the generalized Fourier coefficients from the data. The original nonparametric problem is thus transformed to a parametric problem, although the potential number of parameters is finite. An appropriate choice of basis of the expansion is therefore a key point in relation to the efficiency of such an approach. A good basis should be parsimonious in the sense that a large set of possible response functions can be approximated well with only a few terms of the generalized Fourier expansion employed. Wavelet series allow a parsimonious expansion for a wide variety of functions, including inhomogeneous cases. It is therefore natural to consider applying the generalized Fourier series approach by using a wavelet series. The field of nonparametric regression has developed to fit a curve to data with out assuming any particular parametric structure on the underlying function r. Techniques in nonparametric regression each come with their own sets of assumptions, typically regarding the smoothness of r, such as specifying that r has at least one continues derivative.

Definition 3.1: Let $\{X_n, n\geq 1\}$ be a stochastic process defined on the $(\Omega, \aleph, P)$ and $\mathbf{N}_k^m$ denote the σ-algebra generated by the events $\{X_k\in A_k, \ldots, X_m\in A_m\}$. The process $\{X_n, n\geq 1\}$ is said to satisfy the uniform mixing condition if

$$\sup_{m} \sup_{A\in N_1^m, P(A)>0, B\in N_{m+s}^{\infty}} \frac{|P(AB)-P(A)P(B)|}{P(A)} = \phi(s) \to 0$$

$$\text{as } s\to\infty \text{ and } \phi(0)=1$$

Uniformly mixing, also called ϕ-mixing.

Know we consider random design mode (1.1), in which (X_i, Y_i) are independent and distributed as (x, Y). For this model Antoniadis *et al*. [1], suggest the estimator:

$$\hat{r}(x_i) = \frac{n^{-1}\sum_{i=1}^{n} Y_i k_h(x, X_i)}{n^{-1}\sum_{i=1}^{n} k_h(x, X_i)} = \frac{\hat{g}}{\hat{f}} \tag{3.1}$$

We want to find bias and variance of $\hat{f}$ and $\hat{g}$ for uniformly mixing processes by the following theorems and using these results for finding the convergence rate of our proposed estimator $\hat{r}$.

Theorem 3.1: [13]. Assume that the density f belongs to the Holder space $C^{m+\alpha}$, $0\le\alpha\le1$ and the wavelet-kernel $K(x, y)$ satisfies the localization property:

$$\int_{-\infty}^{+\infty} \left|K(x,y)(y-x)^{m+\alpha}\right| dy \le c$$

for some positive c. Let $j\to\infty$ and $n2^{-j}\to\infty$ as $n\to\infty$, then for fixed x:

$$\mathbf{E}\hat{f}(x) - f(x) = \frac{-1}{m!} f^{(m)}(x) b_m(2^j x) 2^{-mj} + O(2^{-j(m+\alpha)})$$

where

$$b_m = x^m - \int_{-\infty}^{+\infty} K(x,y) y^m dy$$

Theorem 3.2: Let the process $\{X_n, n\ge1\}$ be ϕ-*mixing* and X, Y be measurable random variables with respect to N_1^m and N_{m+s}^{∞} and $\|X\|_p<\infty$, $\|Y\|_q<\infty$, for $p,q<\infty$, then

$$|Cov(X,Y)| \le 2[\phi(s)]^{\frac{1}{p}} \|X\|_p \|Y\|_q$$

for any

$$p,q>1,\ \frac{1}{p}+\frac{1}{q}=1$$

Proof: Suppose that X and Y represented by finite sums as

$$X = \sum_j \alpha_j I_{X\in A_j}$$

$$Y = \sum_i \beta_i I_{Y\in A_i}$$

Where A_j and B_i are disjoint events in N_1^m and N_{m+s}^{∞}. By using Holder inequality we can write:

$$\begin{aligned}
|Cov(X,Y)| &= \left|\sum_i\sum_j \alpha_j\beta_i P(A_jB_i) - \sum_i\sum_j \alpha_j\beta_i P(A_j)P(B_i)\right| = \left|\sum_j \alpha_j\left[P(A_j)\right]\sum_i \beta_i\left[P(B_i|A_j)-P(B_i)\right]\right| \\
&= \left|\sum_j \alpha_j\left[P(A_j)\right]^{\frac{1}{p}} \sum_i \beta_i\left[P(B_i|A_j)-P(B_i)\right]\left[P(A_j)\right]^{\frac{1}{q}}\right| \le [\sum_j |\alpha_j|^p P(A_j)]^{\frac{1}{p}} \left\{\sum_j P(A_j)\left|\sum_i \beta_i\left[P(B_i|A_j)-P(B_i)\right]\right|^{\frac{q}{p}}\right\}^{\frac{1}{q}} \\
&\le \left\{E|X|^p\right\}^{\frac{1}{p}} \sum_j P(A_j)\left\{\sum_i |\beta_i|^q \left(P(B_i|A_j)-P(B_i)\right)\times\left[\sum_i \left|P(B_i|A_j)-P(B_i)\right|\right]^{\frac{q}{p}}\right\}^{\frac{1}{q}} \le 2^{\frac{1}{p}}\left\{E|X|^p\right\}^{\frac{1}{p}}\left\{E|Y|^q\right\}^{\frac{1}{q}} \max_j\left\{\sum_i \left|P(B_i|A_j)-P(B_i)\right|\right\}^{\frac{1}{p}}
\end{aligned} \tag{3.2}$$

Denoting the summation $\sum_i$ over positive and negative terms $\sum^+, \sum^-$, then we have:

$$\begin{aligned}
\sum_i \left|P(B_i|A_j)-P(B_i)\right| &= \sum_i{}^{+}\left\{P(B_i|A_j)-P(B_i)\right\} + \sum_i{}^{-}\left\{P(B_i|A_j)-P(B_i)\right\} \\
&= \left\{P\left(\bigcup_i{}^{+} B_i|A_j\right) - P\left(\bigcup_i{}^{+} B_i\right)\right\} + \left\{P\left(\bigcup_i{}^{-} B_i|A_j\right) - P\left(\bigcup_i{}^{-} B_i\right)\right\} \le 2\phi(s)
\end{aligned} \tag{3.3}$$

By substituting (3.3 in (3.2), the theorem is proved.

Theorem 3.3: Let $\{X_n, n\ge1\}$ be stochastic process defined on the $(\Omega,\aleph,P)$, with density function f(x) such as satisfying the following condition:

1) $\{X_n, n\ge1\}$ be a ϕ-mixing.
2) $\sum_s \phi^{\frac{1}{2}}(s) \le K < \infty$
3) $f\in C_1$, f and f' (the first derivative off), be uniformly bounded, then for fixed x,

$$\mathrm{Var}\left(\hat{f}(x)\right)=\frac{C}{nh}f(x)V\left(\frac{x}{h}\right)+O\left(\frac{1}{n}\right)$$

Where

$$V(x)=\int_{-\infty}^{+\infty}k^2(x,y)dy=k(x,x)$$

Proof

$$\mathrm{Var}\hat{f}(x)=\mathrm{Var}\left\{\frac{1}{n}\sum_{i=◆}^{n}K_h(x,X_i)\right\}=\frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}\{K_h(x,X_i)\}+\frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mathrm{Cov}\left(K_h(x,X_i),K_h(x,X_j)\right)=\mathtt{I}+\mathtt{II} \quad (3.4)$$

Know we obtain upper bound for (I) and (II).

$$\begin{aligned}\mathtt{I}&=\frac{1}{n}\int_{-\infty}^{\infty}K_h^2(x,y)f(y)dy-\frac{1}{n}\left(\int_{-\infty}^{\infty}K_h^2(x,y)f(y)dy\right)^2\\&=\frac{1}{n}f(x)\int_{-\infty}^{\infty}K_h^2(x,y)dy+\frac{1}{n}\int_{-\infty}^{\infty}K_h^2(x,y)\left(f(y)-f(x)\right)dy-\frac{1}{n}\left(\int_{-\infty}^{\infty}K_h^2(x,y)f(y)dy\right)^2\\&=\frac{1}{nh}f(x)V(\frac{x}{h})+\frac{1}{n}\int_{-\infty}^{\infty}K_h^2(x,y)\left(f(y)-f(x)\right)dy-\frac{1}{n}\left(\int_{-\infty}^{\infty}K_h^2(x,y)f(y)dy\right)^2\end{aligned}$$

Below, we show that the second and the third terms in last equality are order of 1/n.

$$\begin{aligned}\left|\frac{1}{n}\int_{-\infty}^{\infty}k_h^2(x,y)\left(f(y)-f(x)\right)dy\right|&\le\frac{1}{n}\sup_x\left|f'(x)\right|\frac{1}{h^2}\int_{-\infty}^{\infty}k^2\left(\frac{x}{h},\frac{y}{h}\right)|y-x|dy\\&\le\frac{1}{n}\sup_x\left|f'(x)\right|\sup_{s,t\in R}\left|k(s,t)\right|\int_{-\infty}^{\infty}\left|k\left(\frac{x}{h},t\right)\left(t-\frac{x}{h}\right)\right|=O\left(n^{-1}\right)\end{aligned}$$

By the uniform bounded ness of f(x), it is easy to see that

$$\frac{1}{n}\left(\int_{-\infty}^{\infty}k_h(x,y)f(y)dy\right)^2=O(\frac{1}{n})\ \text{Thus, }\mathtt{I}\le\frac{c}{nh}f(x)V\left(\frac{x}{h}\right)+O\left(n^{-1}\right) \quad (3.5)$$

To complete the proof it is enough to prove

$$\mathtt{II}=\frac{c}{nh}f(x)V\left(\frac{x}{h}\right)$$

$$\mathtt{II}=\frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mathrm{Cov}\left(k_h(x,X_i),k_h(x,X_j)\right)=\frac{2}{n^2}\sum_{l=1}^{n}(n-1)\mathrm{Cov}\left(k_h(x,X),k_h(x,X_s)\right)\le\frac{2}{n}\sum_{l=1}^{n}\left|\mathrm{Cov}\left(k_h(x,X_1),k_h(x,X_s)\right)\right| \quad (3.6)$$

Know by using theorem (3.2) we can write:

$$\begin{aligned}\left|\mathrm{Cov}\left(k_h(x,X_1),k_h(x,X_s)\right)\right|&\le2\phi^{\frac{1}{2}}(s-1)\left\|K_h(x,X_1\right\|_2\left\|K_h(x,X_s\right\|_2=2\phi^{\frac{1}{2}}(s-1)\left\|K_h(x,Y\right\|_2^2\\&\le2\phi^{\frac{1}{2}}(s-1)\left\|K_h(x,X_1\right\|_2\int_{-\infty}^{+\infty}K_h^2(x,Y)f(Y)dY\le2\phi^{\frac{1}{2}}(s-1)\frac{1}{h^2}\|f\|\int_{-\infty}^{+\infty}K_h^2(\frac{x}{h},\frac{Y}{h})dY \quad (3.7)\\&=2\phi^{\frac{1}{2}}(s-1)\frac{1}{h}\|f\|\int_{-\infty}^{+\infty}K_h^2(\frac{x}{h},t)dt=2\phi^{\frac{1}{2}}(s-1)\frac{1}{h}V(\frac{x}{h})\|f\|\end{aligned}$$

By substituting (3.7) in (3.6) we have:

$$\mathtt{II}\le\frac{2}{n}\sum_{s=1}^{n}\frac{\phi^{\frac{1}{2}}(s-1)}{h}V(\frac{x}{h})\|f\|=\frac{c}{nh}V(\frac{x}{h})\sum_{s=1}^{n}\phi^{\frac{1}{2}}(s-1)\le\frac{cK}{nh}V(\frac{x}{h})\le\frac{C}{nh}V(\frac{x}{h}) \quad (3.8)$$

Substituting (3.8) and (3.5) in (3.4), complete the proof.

Theorem 3.4:[1] Assume that the g(x) belong to the Holder space $C^{m+\alpha}$, $0\le\alpha\le 1$ and the wavelet-kernel K(x, y) satisfies the localization property:

$$\int_{-\infty}^{+\infty} K(x,y)(y-x)^{m+\alpha}dy \le C$$

for some positive C. Let $j\to\infty$ and $n2^j\to\infty$, as $n\to\infty$. Then for fixed x,

$$E\hat{g}(x)-g(x) = -\frac{1}{m!}g^{(m)}(x)b_m(2^j x)2^{-mj}+O(2^{-j(m+\alpha)})$$

Theorem 3.5: Let $\{X_n,\ n\ge 1\}$ be stochastic process defined on the $(\Omega,\aleph,P)$, with density function f(x) and regression function r(x) such that f(x) and r(x) locally bounded. Suppose that the process is ϕ-mixing and

$$\sum_s \phi^{\frac{1}{2}}(s)\le K<\infty$$

Then $\mathrm{Var}[\hat{g}(x)] = O(\frac{1}{nh}) + V(\frac{x}{h})$ where $h=2^{-m_0}$.

Proof

$$\begin{aligned}\mathrm{Var}\hat{g}(x) &= \mathrm{Var}\left\{\frac{1}{n}\sum_{i=1}^{n} Y_i K_h(x,X_i)\right\}\\ &= \frac{1}{n^2}\mathrm{Var}\sum_{i=1}^{n}\mathrm{Var}\{Y_i K_h(x,X_i)\}\\ &\quad+\frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mathrm{Cov}(Y_iK_h(x,X_i),Y_jK_h(x,X_j))\\ &= \mathtt{I}+\mathtt{II}\end{aligned} \tag{3.9}$$

Know we want to obtain upper bound for (I) and (II). By Antoniadis [14], we have

$$\mathtt{I}\le\frac{K}{nh} \tag{3.10}$$

Next, we can write

$$\begin{aligned}|\mathtt{II}| &= \left|\frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\mathrm{Cov}(Y_iK_h(x,X_i),Y_jK_h(x,X_j))\right|\\ &\le\frac{2}{n}\sum_{l=1}^{n}\left|\mathrm{Cov}(YK_h(x,X_l),Y_jK_h(x,X_l))\right|\end{aligned}$$

Where $\mathrm{Cov}\{(Y_i,\ Y_j)|X_i,\ X_j\}\}$ denoting the conditional covariance of Y_i, Y_j given X_i, X_j, this will be locally bounded by assumption. Because of Uniform mixing stochastic process, we know that remains uniform mixing process, therefore by theorem (3.2) we have

$$|\mathtt{II}|\le\frac{c}{nh}V(\frac{x}{h}) \tag{3.11}$$

Hence, equation (3.10) and (3.11) complete the proof. The following theorem allows the convergence rate of estimator $\hat{r}(x)$.

Theorem 3.6: Under conditions theorem (3.3), we have:

$$\mathrm{bias}(\hat{\gamma}(x)) = O\left(2^{-j_0 m}\right)+O\left(\frac{2^{j_0}}{n}\right),\ \mathrm{Var}(\hat{\gamma}(x))\le O\left(\frac{2^{j_0}}{n}\right)$$

Proof: Using Rosenblatt's expansion [15], we have

$$\begin{aligned}\hat{\gamma}(x) &= \frac{E\hat{g}}{E\hat{f}}+\frac{\hat{g}(x)-E\hat{g}(x)}{E\hat{f}(x)}-\frac{\hat{f}(x)-E\hat{f}(x)}{\left[E\hat{f}(x)\right]^2}\\ &\quad+O\left(\left[\hat{g}(x)-E\hat{g}(x)\right]^2\right)+O\left(\left[\hat{f}(x)-E\hat{f}(x)\right]^2\right)\end{aligned}$$

By using theorem (3.1), (3.2), (3.3) ,(3.5)and[11] it follows that:

$$E\hat{\gamma}(x) = \frac{E\hat{g}}{E\hat{f}}+O\left(\mathrm{Var}\hat{g}(x)\right)+O\left(\mathrm{Var}\hat{f}(x)\right)\le\frac{E\hat{g}}{E\hat{f}}+O\left(\frac{2^{j_0}}{n}\right)$$

Now, by using Equation (2.7) of Rosenblatt [15], we have

$$\begin{aligned}\frac{E\hat{g}}{E\hat{f}} &= \gamma(x)+\frac{E\hat{g}(x)-g(x)}{f(x)}-\frac{E\hat{f}(x)-f(x)}{f(x)}\gamma(x)\\ &\quad+O\left(\left[g(x)-E\hat{g}(x)^2\right]\right)+O\left(\left[f(x)-E\hat{f}(x)\right]^2\right)\end{aligned}$$

so

$$\frac{E\hat{g}}{E\hat{f}}\le\gamma(x)+O\left(2^{-j_0 m}\right)$$

Then we get

$$\mathrm{bias}(\hat{\gamma}(x)) = O\left(2^{-j_0 m}\right)+O\left(\frac{2^{j_0}}{n}\right)$$

For variance of $\hat{r}(x)$ we have:

$$\begin{aligned}\mathrm{Var}(\hat{\gamma}(x)) &\le\frac{\mathrm{Var}\hat{g}(x)}{\left[E\hat{f}(x)\right]^2}+\frac{\left[E\hat{g}(x)\right]^2}{\left[E\hat{f}(x)\right]^4}\mathrm{Var}\hat{f}(x)\\ &\quad+O\left(E\left[g(x)-E\hat{g}(x)\right]^4\right)+O\left(E\left[f(x)-E\hat{f}(x)\right]^4\right)\end{aligned}$$

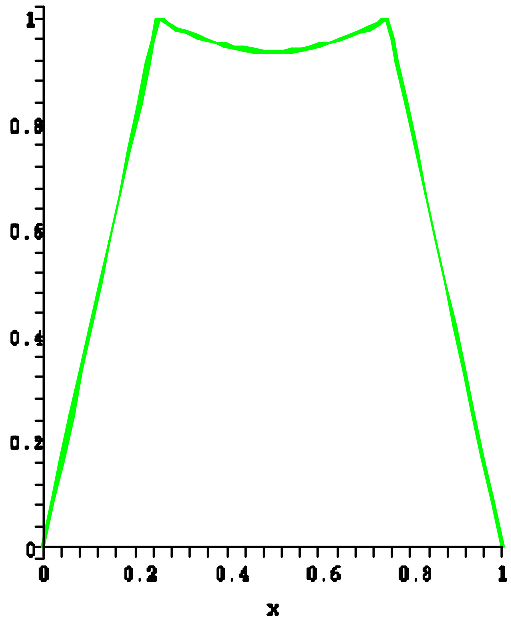

Fig. 1:

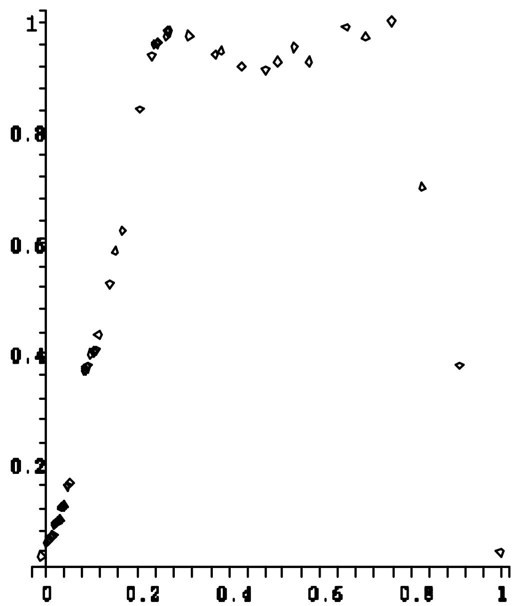

Fig. 2:

Assuming that f(x)>0 for all x and using the results on the asymptotic bias and variance of $\hat{g}$ and $\hat{f}$ we conclude that

$$\operatorname{Var}\left(\hat{\gamma}(x)\right)\le O\left(\frac{2^{j_\circ}}{n}\right)$$

Now we present an example and verify the performance of our wavelet estimators.

Example: Suppose that regression function r(x) as following:

$$\frac{1}{3}\le x<\frac{2}{3}\gamma(x)=\begin{cases}3x & 0\le x<\frac{1}{3}\\ x^2-x+\frac{11}{9} & \\ 3(1-x) & \frac{2}{3}\le x<1\end{cases}$$

By perturbing r(x) with ε_i neighborhood noise with zero mean and using MAPLE9, we produce the data set $X_{Data}^{(n)}$ from AR(1) model $X_t = 0.95X_{t-1}+\varepsilon_i$ for n=128 and ε=0.02.

Figure 1 is the graph of the regression function r(x) and Fig. 2 is the graph of estimated $\hat{r}(x)$. As we see, $\hat{r}(x)$, and the convergence rate for bias and variance of our proposed estimator is very well.

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