

Noise Removal Using Mixtures of Projected Gaussian Scale Mixtures

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Abstract: Denoising of natural images is the fundamental and challenging problem of Image processing. As this problem depends upon the type of noise, amount of noise and the type of images it is not truly a practical approach. Wavelet transform method is used for localized in both frequency and spatial domain. The general de-noising wavelet transform method involves three steps 1) Compute the wavelet decomposition of the image 2) Threshold detail coefficients 3) Compute wavelet reconstruction. Dimension reduction methods search for the manifolds in the high-dimensional space on which the data resides. The technique used in this paper for dimension reduction is Principle Component Analysis (PCA). PCA is a well-known technique to map n-dimensional vectors into k-dimensional vectors. a new statistical model for image restoration in which neighbourhoods of wavelet subbands are modeled by a discrete mixture of linear projected Gaussian Scale Mixtures (MPGSM). In each projection, a lower dimensional approximation of the local neighbourhood is obtained, thereby modeling the strongest correlations in that neighbourhood. The algorithm used is Expectation Maximisation (EM) algorithm.

Key words: Image denoising • Wavelet transform • Principle Component Analysis • Gaussian Scale Mixtures • Wavelet subbands

INTRODUCTION

An image may be defined as a two-dimensional function $f(x,y)$ where x and y are spatial coordinates and the amplitude of f at any pair of coordinates (x,y) is called the intensity or gray level of the image at that point. When x , y and the amplitude values of are all finite, discrete quantities, we call the image a digital image. The field of digital image processing refers to processing digital images by means of a digital computer. Digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels and pixels. Pixel is the term most widely used to denote the elements of digital image [1]. Digital images play an important role both in daily life applications such as satellite television, magnetic resonance imaging, computer tomography as well as in areas of research and technology such as geographical information systems and astronomy. Digital image processing applications are becoming increasingly important and they all start with a mathematical representation of the image. Image

processing modifies pictures to improve them (enhancement, restoration), extract information (analysis, recognition) and change their structure (composition, image editing). Images can be processed by optical, photographic and electronic means, but image processing using digital computers is the most common method because digital methods are fast, flexible and precise. An image can be synthesized from a micrograph of various cell organelles by assigning a light intensity value to each cell organelle. The sensor signal is “digitized” converted to an array of numerical values, each value representing the light intensity of a small area of the cell. The digitized values are called picture elements, or “pixels,” and are stored in computer memory as a digital image. A typical size for a digital image is an array of 512 by 512 pixels, where each pixel has value in the range of 0 to 255. The digital image is processed by a computer to achieve the desired result [2].

Many noise reduction techniques have been developed for removing noise and retaining edge details. Most of the standard algorithms use a defined filter window to estimate the local noise variance of a noise

image and perform the individual unique filtering process. The result is generally a greatly reduced noise level in areas that are homogeneous. But the image is either blurred or over smoothed due to losses in detail in non-homogenous areas like edges or lines. This creates a barrier for the use of Satellite based remote sensing images to classify, interpret and analyze terrain accurately especially in sensitive applications like military and scientific research. The primary goal of noise reduction is to remove the noise without losing much detail contained in an image. To achieve this goal, we make use of a mathematical function known as the wavelet transform to localize an image into different frequency components or useful subbands and effectively reduce the noise in the subbands according to the local statistics within the bands. The main advantage of the wavelet transform is that the image fidelity after reconstruction is visually lossless. We combine the wavelet shrinkage denoising techniques with different wavelet bases and decomposition levels on the individual subbands to achieve the best acceptable noise reduction while maintaining the fidelity of the image. Image denoising is an important image processing task, both as a process itself and as a component in other processes. Very many ways to denoise an image or a set of data exists. The main properties of a good image denoising model are that it will remove noise while preserving edges. Traditionally, linear models have been used. One common approach is to use a Gaussian filter, or equivalently solving the heat-equation with the noisy image as input-data, i.e. a linear, 2nd order PDE-model. For some purposes this kind of denoising is adequate. One big advantage of linear noise removal models is the speed. But a back draw of the linear models is that they are not able to preserve edges in a good manner: edges, which are recognized as discontinuities in the image, are smeared out. Nonlinear models on the other hand can handle edges in a much better way than linear models can [3-5].

Noise is an unwanted perturbation caused during image acquisition and transmission Image denoising is used to removing unwanted noise in order to restore the original image. In Bayesian restoration methods, the image manifold is encoded in the form of prior knowledge that expresses the probabilities that given combinations of pixel intensities can be observed in an image. Because image spaces are high-dimensional, one often isolates the manifolds by decomposing images into their components and by fitting probabilistic models on it [6-10]. Thus, denoising is often a necessary and the first step to be taken before the images data is analyzed. Image denoising

has remained a fundamental problem in the field of image processing. This paper describes different method for noise reduction (or denoising) giving an insight as to which algorithm should be used to find the most reliable estimate of the original image data given its degraded version. Different algorithms are used depending on the noise model [11-15].

The need for efficient image restoration methods has grown with the massive production of digital images and movies of all kinds, often taken in poor conditions. No matter how good cameras are, an image improvement is always desirable to extend their range of action. A digital image is generally encoded as a matrix of grey-level or color values. In the case of a movie, this matrix has three dimensions, the third one corresponding to time. Each pair $(i, u(i))$, where $u(i)$ is the value at i , is called a pixel, short for "picture element." In the case of grey-level images, i is a point on a two-dimensional (2D) grid and $u(i)$ is a real value. In the case of classical color images, $u(i)$ is a triplet of values for the red, green and blue components. All of what we shall say applies identically to movies, three-dimensional (3D) images and color or multispectral images. For the sake of simplicity in notation and display of experiments, we shall here be content with rectangular 2D grey-level images.

With Wavelet Transform gaining popularity in the last two decades various algorithms for denoising in wavelet domain were introduced. The focus was shifted from the spatial and fourier domain to the wavelet transform domain. Marginal histograms of wavelet coefficients are typically leptokurtotic and have heavy tails. Taking advantage of correlations between wavelet coefficients either across space, scale or orientation, additional improvement in denoising performance is obtained. This paper describes different method for noise reduction (or denoising) giving an insight as to which algorithm should be used to find the most reliable estimate of the original image data given its degraded version. Wavelet transforms are broadly divided into three classes: continuous, discrete and multiresolution-based.

Consider an image decomposed into oriented subbands at multiple scales. The coefficient corresponding to a linear basis function at scale, orientation and centered at spatial location A neighborhood coefficients clustered around this reference coefficient. The neighborhood may include coefficients from other subbands (i.e., corresponding to basis functions at nearby scales and orientations), as well as from the same subband. GSM model can account for both the shape of wavelet coefficient marginals and the strong

correlation between the amplitudes of neighbor coefficients. In order to construct a global model for images from this local description, one must specify both the neighborhood structure of the coefficients and the distribution of the multipliers. The definition of (and calculations using) the global model is considerably simplified by partitioning the coefficients into non-overlapping neighborhoods. One can then specify either a marginal model for the multipliers (treating them as independent variables), or specify a joint density over the full set of multipliers.

Unfortunately, the use of disjoint neighborhoods leads to noticeable denoising artifacts at the discontinuities introduced by the neighborhood boundaries. An alternative approach is to use a GSM as a local description of the behavior of the cluster of coefficients centered at each coefficient in the pyramid. Since the neighborhoods overlap, each coefficient will be a member of many neighborhoods. The local model implicitly defines a global (Markov) model, described by the conditional density of a coefficient in the cluster given its surrounding neighborhood, assuming conditional independence on the rest of the coefficients. But the structure of the resulting model is such that performing statistical inference (i.e., computing Bayes estimates) in an exact way is intractable. In this paper, we simply solve the estimation problem for the reference coefficient at the center of each neighborhood independently.

Related Work: The general de-noising procedure involves three steps such as Decomposition, thresholding, reconstruction. In Decomposition choose a wavelet and choose a level N . Compute the wavelet decomposition of the signal s at level N . In Thresholding for each level from 1 to N , select a threshold and apply soft thresholding to the detail coefficients. In Reconstruction, Compute wavelet reconstruction using the original approximation coefficients of level N and the modified detail coefficients of levels from 1 to N .

Gaussian scale mixture model in which the cluster of coefficients are modelled as a product of Gaussian random vector and a positive scaling variable. Under this model, the Bayesian least squares estimate of each coefficient reduces to a weighted average of the local linear (Wiener) estimate over all possible values of the hidden multiplier variable. An extension of the original Bayes Least Squares - Gaussian Scale Mixtures (BLS-GSM) denoising algorithm that also compensated the blur. However, this method had some problems: a) it could not compensate for some blurring kernels; b) its

performance depended critically on having an accurate estimation of the original power spectral density (PSD); and c). A more tractable problem arises when we only consider additive noise in the degradation model (denoising problem). Hence we go for spatially variant Gaussian scale mixture model.

SVGSM model can be viewed as a fine adaptation of the model to the signal variance at each scale, orientation and spatial location. Enhancement of the model is by introducing a coarser adaptation level, where a larger neighborhood is used to estimate the local signal covariance within every subband. We formulate our model as a Bayes least squares estimator using space-variant Gaussian scale mixtures. The model can be also applied to image de-convolution, by first performing a global blur compensation and then doing local adaptive denoising. Advantage is less artifacts and much more texture is recovered. The problem is uniform blurring. It cannot be properly treated by using this method. Another drawback of this method is that, to work properly, it requires a reasonably accurate estimation of the cross-covariance among the original and the filtered image.

OAGSM Model is a statistical model to describe the spatially varying behavior of local neighborhoods of coefficients in a multiscale image representation. Neighborhoods are modeled as samples of a multivariate Gaussian density that are modulated and rotated according to the values of two hidden random variables, thus allowing the model to adapt to the local amplitude and orientation of the signal. A third hidden variable selects between this oriented process and a non-oriented scale mixture of Gaussians process, thus providing adaptability to the local orientedness of the signal. Although orientation is an important property of natural image patches, images may also contain features such as corners and textures that are either unoriented or of mixed orientation. Modeling such non-oriented areas with the OAGSM process may lead to inappropriate behavior, such as the introduction of oriented artifacts during denoising.

In MGSM Model by clustering the local covariance matrices globally, this model can also exploit non-local redundancy (or repetitivity) in images. This results in a denoising performance that is significantly better than the "single" GSM model. Also the MGSM model is able to deal explicitly with correlated noise whereas, the non-wavelet based method of is not. While MGSM is potentially very powerful, there are a number of issues involved: the most severe is the computational cost that

is linear in the number of GSM components and quadratic in neighbourhood size. Moreover, the high number of free parameters can cause problems due to the “curse of dimensionality” especially for smaller wavelet subbands with few neighbourhood vectors.

Proposed Work: Many issues has been addressed by introducing dimension reduction through linear projections in the MGSM model and this model is called mixtures of projected GSM (MPGSM). The use of linear projections not only significantly reduces the number of model parameters but also allows us to design fast training algorithms. MPGSM is a generalized MGSM model that unifies the SVGSM and OAGSM methods. To reduce the number of free parameters of the MGSM model, we use dimension reduction through linear projections. Dimension reduction methods have been commonly used for content-based multimedia indexing and retrieval. Dimension reduction methods search for the manifolds in the high-dimensional space on which the data resides. This can be obtained by fitting a linear subspace through the observations, using a given criterion. If one minimizes the Euclidean distance between the observations and the subspace, this results in Principal Component Analysis (PCA) also known as the Karhunen-Lovre Transform (KLT). Because most of the energy is covered by the first principle components, we achieve a lower dimensional approximation of the local neighbourhood, thereby reducing the number of independent model parameters. Mixture models that embed PCA projections have also been proposed for more general tasks as density modeling, data visualization and data compression. Compared to the MGSM model and the GSM model, the proposed MPGSM model adds a third layer of adaptation as depicted in Fig. 1.

In this conceptual scheme, the first layer is the GSM scaling factor that provides adaptation to the local signal amplitude or variance. The second layer is the MGSM component index, which provides adaptation to signal covariance (textural and edge characteristics). The third layer is added by the proposed model and it encodes the information inside the covariance matrix more efficiently. The model training is performed using the Expectation Maximization (EM) algorithm. The more efficient covariance matrix representation allows us to reduce the computational cost of the training phase. The dimension reduction through a linear projection is quite general. It has two approaches: data-driven and data-independent projection bases. This approach easily allows for variable sized neighbourhoods, which are more efficient for representing edges. When only using data-independent

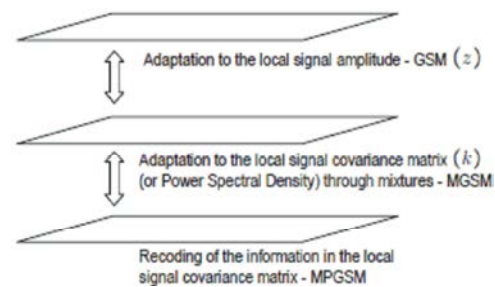


Fig. 1: Three layers of MPGSM Model

projection bases, the EM training can even be completely skipped, resulting in computational savings up to factors 4 even compared to the BLS-GSM method, with limited loss of PSNR. The advantage of MPGSM Method is less cost and reduces the number of free parameters using dimension reduction method.

Dimension Reduction Method: In order to better adapt to the observed data, we can also estimate the projection bases from the observed data, e.g., using PCA.

Principal Component Analysis: Principal component analysis (PCA) is the best, in the mean-square error sense linear dimension reduction technique. Being based on the covariance matrix of the variables, it is a second-order method. In various fields, it is also known as the singular value decomposition (SVD), the Karhunen-Lohe transform, the Hotelling transform and the empirical orthogonal function (EOF) method [16]. In essence, PCA seeks to reduce the dimension of the data by finding a few orthogonal linear combinations (the PCs) of the original variables with the largest variance. The first PC is the linear combination with the largest variance. The second PC is the linear combination with the second largest variance and orthogonal to the first PC and so on. There are as many PCs as the number of the original variables [17]. For many datasets, the first several PCs explain most of the variance, so that the rest can be disregarded with minimal loss of information. Since the variance depends on the scale of the variables, it is customary to first standardize each variable to have mean zero and standard deviation one. After the standardization, the original variables with possibly different units of measurement.

Expectation/Maximization Algorithm: The Expectation/Maximization (EM) algorithm simultaneously segments and fits data generated from multiple parametric models. Collection of data points (x, y) generated from one of two linear models of the form:

$$y(i) = a_1x(i) + b_1 + n_1(i) \text{ or } y(i) = a_2x(i) + b_2 + n_2(i)$$

where the model parameters are a_1 , b_1 and a_2 , b_2 and the system is modeled with additive noise $n_1(i)$ and $n_2(i)$. If we are told the model parameters, then determining which data point was generated by which model, for each data point i , the model k that minimizes the error between the data and the model prediction:

$$r^k(i) = |a^kx(i) + b^k - y(i)|$$

The EM algorithm is an iterative two step algorithm that estimates both the model assignment and parameters. The “E-step” of EM assumes that the model parameters are known (initially, the model parameters can be assigned random values) and calculates the likelihood of each data point belonging to each model. In so doing the model assignment is made in a “soft” probabilistic fashion. That is, each data point is not explicitly assigned a single model, instead each data point i is assigned a probability of it belonging to each model k . For each model the residual error is first computed as:

$$r^k(i) = a^kx(i) + b^k - y(i)$$

from which the likelihoods are calculated [18]. The “M-step” of EM takes the likelihood of each data point belonging to each model and re-estimates the model parameters using weighted least-squares. That is, the following weighted error function on the model parameters is minimized: The intuition here is that each data point contributes to the estimation of each model’s parameters in proportion to the belief that it belongs to that particular model [19] This quadratic error function is minimized by computing the partial derivatives with respect to the model parameters, setting the result equal to zero and solving for the model parameters. The EM algorithm iteratively executes the “E” and “M” step, repeatedly estimating and refining the model assignments and parameters.

RESULTS

In this work a Gaussian noise is added to the original image. Perform multiscale decomposition on the image corrupted by Gaussian noise using wavelet transform. For each level, the sub band is computed. For each subband Compute threshold Apply soft thresholding to the noisy coefficients. Invert the multiscale decomposition to reconstruct the denoised image.

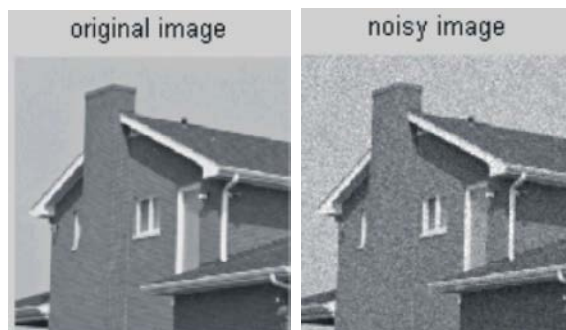


Fig. 2: Noise added image

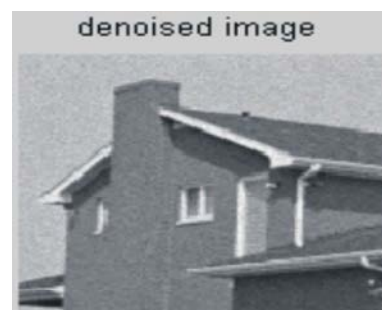


Fig. 3: Reconstructed image



Fig. 4: Decomposition level before and after noise added image



Fig. 4: Comparison to GSM method

The Fig. 2 and 3 shows the image obtained by adding the Gaussian noise and the reconstructed image. The Fig. 4 shows the mean square error estimated during the reconstruction process. It is calculated to limit the PSNR loss.

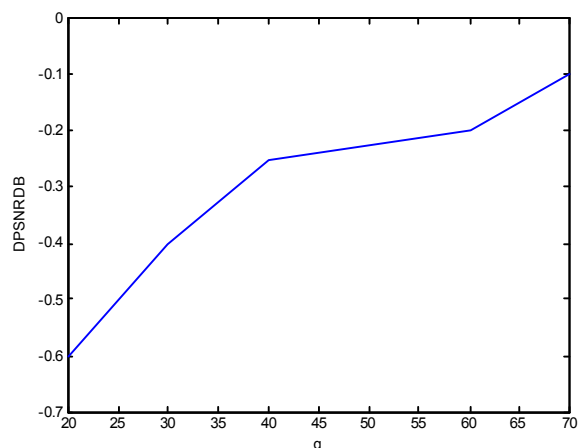


Fig. 5: PSNR Performance

CONCLUSION

In this work, we proposed the Mixtures of Projected Gaussian Scale Mixtures (MPGSM) as a means to further improve upon the recently proposed MGSM model. The new model is a generalization of the existing SVGSM, OAGSM and MGSM techniques and allows for a lot of flexibility and made dependencies between different wavelet subbands. A fast EM algorithm is used, taking advantage of the Principal Component bases. This technique can also be used to speed up the denoising itself and to offer computational savings compared to the GSM-BLS method which can be useful for real-time denoising applications. Finally PSNR improvement is shown [20-23].

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