

Fluid Flow and Heat Transfer Characteristics of Natural Convection in Square 3-D Enclosure Due to Discrete Sources

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Abstract: A numerical analysis was carried out to study the laminar natural convection in a three-dimensional enclosure. Main efforts were focused on the Rayleigh number (Ra), position of hot and cold sources and size of them on the flow structure and heat transfer characteristics are investigated in detail. Results reveal that by increasing in Ra from 10^3 to 10^6 the average Nusselt will be increase and the rate of heat transfer from hot and cold sources will be increase, also the maximum component of velocity will increase, The increase in the velocity component along the acceleration of gravity is higher than other component of velocity. According to the results by increasing the distance between heat sources rate of heat transfer between heat sources (average Nusselt) significantly decreased, also average temperature and maximum component of velocity in all direction will be increase. Studies on changes in aspect ratio of sources and its effect on the rate of heat transfer indicative that by increase length of cold source ratio hot source rate of heat transfer is soaring, Also, increasing the aspect ratio of cold sources to warm significantly decreasing average temperature in the enclosure.

Key words: Natural convection • Enclosure • Heat transfer • Velocity component • Flow structure

INTRODUCTION

The convection due to buoyancy forces i.e natural convection in a rectangular enclosures is a fundamental problem due to the wide range of applications in thermal insulation, heating and cooling of buildings, energy drying processes, cooling of electronics, lakes and geothermal reservoirs, solidification of casting, underground water flow, solar collector etc. [1]. Associated industrial applications include secondary and tertiary oil recovery, growth of crystals [2]. During these last three decades, numerous theoretical and experimental studies on natural convection in enclosures have been investigated. Understanding the dynamical evolution of heat and fluid flow due to natural convection, has received a considerable attention from many researchers. An extensive bibliography Natural convection in cavities up to 1988 may be found in the review article by Ostrach [3].

In the framework of the present paper, we refer to the literature pertinent to natural convection in an 3D enclosure whose all walls are adiabatic and sources have constant temperature. In the broad sense, the above class of problem can be classified into two major categories: (I) uniform heated enclosures, (ii) differentially heated enclosures. Above type of problems have been studied extensively by many researchers. A numerical investigation on the natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides has been performed by Ganzarolli and Milanez [4]. Ramos and Milanez [5] studied the natural convection in cavities heated from below by a thermal source which dissipated energy at a constant rate. A study of three-dimensional natural convection from a discrete flush-mounted rectangular heat source on the bottom of a horizontal enclosure whose upper wall surface maintained at a cold temperature along with the two kinds of boundary conditions on

the sidewalls has been presented by Sezai and Mohamad [6]. A steady laminar natural convection in 2D enclosures heated from below and cooled from above for a wide variety of thermal boundary conditions at the sidewalls has been carried numerically by Corcione [7]. An analysis on the combined effects of thermal and mass diffusion of viscous incompressible fluid through a finitely long vertical irregular wall and a smooth flat wall in the presence of internal heat source or sink is performed by Fasogbon, [8]. In his study walls are maintained at constant but different temperatures and species concentrations.

Recently, an experimental and numerical study of free convective heat transfer in a square enclosure characterized by a discrete heating at lower wall and cooled from the vertical walls have been analyzed by Calcagni *et al.* [9]. A numerical investigation of natural convection of air in a vertical square cavity with localized isothermal heating from below and symmetrical cooling from the side walls was investigated by Aydin and Yang [10]. The same problem by replacing a constant flux heat source instead the localized isothermal heat source at the bottom wall has been analyzed by Sharif and Mohammad [11]. They investigated the effect of aspect ratio and inclination of the cavity on the heat transfer process. Natural convection in a square enclosure heated periodically from part of the bottom wall has been investigated by Lakhali *et al.* [13]. The effect of heater and cooler locations on natural convection in square cavities has been reported by Turgoklu and Yucel [12]. Natural convection in rectangular tanks heated locally from bellow has been studied numerically by Sarris *et al.* [14] and found that For small Rayleigh numbers, the heat transfer is dominated by conduction, while at higher Ra convection becomes dominant. Increase of the tank aspect ratio and the heated strip width intensifies the fluid flow and increases the temperature of the fluid. This makes the glass-melt more homogeneous resulting in better quality of the final product.

There are good number of papers which deal with natural convection with nonuniform temperature boundary conditions, for example, in connection with fluid flow in enclosures with one side wall heated using a pulsating heat flux and other side wall cooled at constant temperature Lage and Bejan [15], natural convection in rectangular enclosure with sinusoidal temperature on the upper wall and adiabatic boundary condition on rest walls Sarris *et al* [16], natural convection in a square cavity with the different boundary conditions: uniform as well as nonuniform heating of

bottom as well as side walls Roy *et al.* [17] and cooling by sinusoidal temperature profiles on equally divided active side wall with other sides are insulated Bilgen and Yedder [18].

Natural convection in air-filled 2D square enclosure heated with a constant source from below and cooled from above is studied numerically by Nader *et al.* [19]. They have considered a variety of thermal boundary conditions at the top and sidewalls. Simulations are performed for two kinds of lengths of the heated source, i.e. a small and a large source corresponding to 20 percent and 80 percent of the total length of the bottom wall, respectively. Their results are presented in the form of streamline and isotherm plots as well as the variation of the Nusselt number and maximum temperature at the heat source surface. Also, they have reported the comparisons among the different thermal configurations. A numerical study to investigate the steady laminar natural convection flow in a square cavity with uniformly and non uniformly heated bottom wall and adiabatic top wall maintaining constant temperature of cold vertical walls has been performed by Basak *et al.* [20] with the help of penalty finite element method. In the same geometry, the numerical study deals with natural convection flow in a closed square cavity when the bottom wall is uniformly heated and vertical wall(s) are linearly heated whereas the top wall is well insulated has been reported by Sathiyamoorthy *et al.* [21]. They have found that the average Nusselt number is almost constant unto $Ra = 10^4$ due to dominant heat conduction mode and smoothly increases as Rayleigh number increases further but the smoothness breaks at $Ra = 7.10^4$ for $Pr = 0.7$. In contrast to linearly heated left wall and cooled right wall, at $Ra = 10^5$, local Nusselt number at the left wall exhibits oscillatory behavior due to the presence of secondary circulation near the top edge of the left wall. Further, the average Nusselt numbers smoothly increase as Rayleigh number increases with an exception for $Pr = 10$ at the left wall due to the presence of strong secondary cell near the top edge of the left wall.

In the present paper a numerical analysis was carried out to study the laminar natural convection in a three-dimensional enclosure. Main efforts were focused on the Rayleigh number Ra, position of the hot and cold sources and size of them on the flow structure and heat transfer characteristics are presented. In this investigation The Special attention is given to understand the effect of aspect ratio and heat source intensity, Rayleigh number, Ra, on the fluid flow configuration as well as on the local and average heat transfer rates. The results are presented

Table 1: Comparing average number resulting from computer program and the others works in 3-dimensional enclosure.

Ra	Nu_m			
	Fusegi <i>et al.</i> [12]	Leutree and Lauriat[13]	Ho and Tu [27]	Present research
10^3	1.0851	-	1.057	1.066
10^4	2.001	-	2.074	2.039
10^5	4.361	4.348	4.367	4.308
10^6	8.770	8.651	8.755	8.545

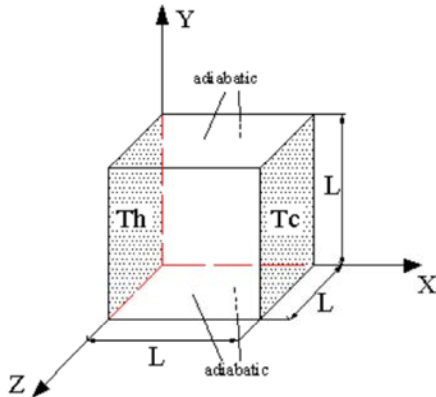


Fig. 1: Problem geometry for evaluating validity of computer code

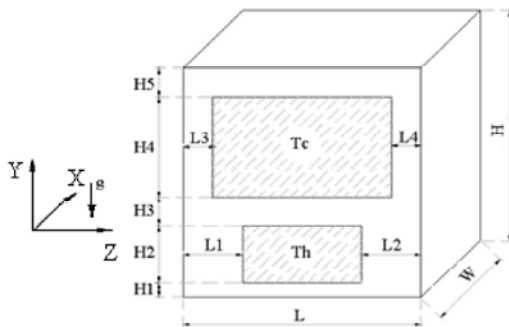


Fig. 2: Scheme of 3D enclosure

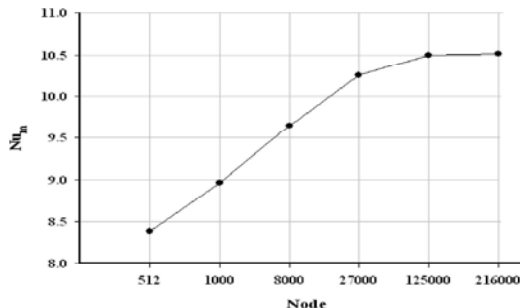


Fig. 3: Average Nusselt of hot source based on number of grid points.

in terms of stream function?, temperature?, vertical velocity component and Nusselt numbers (local Nusselt number, Nu_l and average Nusselt number, Nu_m).

Validation: For studying operation of computer code in 3-dimensional state, a comparison was performed with done work in references [22-24]. In these papers, a square - cubic enclosure with cold wall on the right and hot wall on the left with boundary conditions as fixed temperature has been considered, the other walls of enclosure are insulated Fig.1. The (Table 1) shows a sample of results of the others work and present study in which average Nusselt has been provided in lieu of different Rayleigh numbers. As you see, results of this study have very little difference with whatever has been provided from references. After assurance from program operation, first we describe a suitable grid with regard to boundary conditions change for computer calculations.

Grid Independence: For choosing a solvable network, first according to enclosure dimensions, effect of changing flow and temperature parameters and the number of grids were studied.

We consider a state of enclosure based on Fig. 2 with geometrical dimensions; $L=1, H=1, W=0.875, H_1=0.125, H_2=0.20, H_3=0.25, H_4=0.30, L_1=L_2=0.30, L_3=L_4=0.2$. Rayleigh number is considered 10^5 .

After finding out optimal parameters, average Nusselt of hot source and maximum vertical component of velocity were calculated in lieu of different grid points.

Results of this study have been shown in the Fig.3 and Fig.4. Although, there is a trivial difference (Result of grid study are contain; average Nusselt numbers and maximum vertical component of velocity) between 50.50.50 grid with 60.60.60 grid points therfor Non-uniform grid 60.60.60 had some acceptable errors for performing program.

After assurance from operation of computer code and grid selection, different performances were done in different states.

In all calculations, fluid properties and enclosure dimensions ($L=1, H=1, W=0.875, L_1=L_2=0.30, L_3=L_4=0.2$) were considered fixed.

First with changing Rayleigh number from 10^3 to 10^6 effect of isothermal lines are studied.

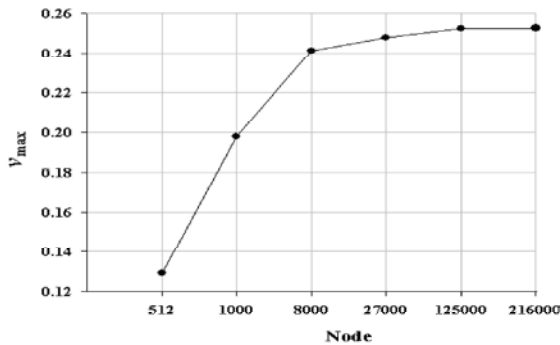


Fig. 4: Maximum vertical component of velocity based on number of grid points

Mathematical Formulation: We consider a three dimensional fluid flow in a rectangular enclosure of length L and height H and W as shown schematically in Fig. 2. The flow is induced by the fix temperature of sources and other walls are adiabatic. The Boussinesq approximation is involved for the fluid properties to relate density changes to temperature changes and so to couple in this way the temperature field to the flow field. Our main focus is on influence of aspect ratio and heat source intensity on the fluid flow configuration as well as on the heat transfer rates. The governing equations for the steady-state flow as well as heat transfer in the Cartesian coordinate system are given by using non-dimensionalequations:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{2-1}$$

$$\frac{(u_i u_j)}{x_j} = -\frac{p}{x_i} + \frac{1}{Gr_l^{1/2}} P^2 u_i - \theta g_i^* \tag{2-2}$$

$$\frac{Y'}{x Y'_j} (u_j \theta) = \frac{1}{Pr Gr_l^{1/2}} P^2 \theta \tag{2-3}$$

$$Nu = -\frac{\partial \theta}{\partial x} \tag{2-4}$$

$$v = \frac{\partial \psi}{\partial y} u = \frac{\partial \psi}{\partial y} \tag{2-5}$$

$$v_{mean} = \frac{1}{n} \sum_{i=1}^n (u_i^2 + v_i^2)^{\frac{1}{2}} \tag{2-6}$$

$$\theta_{mean} = \frac{1}{n} \sum_{i=1}^n \theta_i \tag{2-7}$$

$$Nu = -\frac{\partial \theta}{\partial x} \tag{2-8}$$

$$Gr_l = \frac{g \beta (T_h - T_c) H^3}{\nu^2}, Pr = \frac{\nu}{\alpha}, Ra = Gr_l Pr \tag{2-9}$$

With the boundary conditions, on the walls $u=v=w=0$, on the surface of hot source $\theta=1$ and cold source $\theta=0$ and for adiabatic walls $\frac{\theta}{x}|_{x=0} = 0, \frac{\theta}{y}|_{y=0} = 0, \frac{\theta}{z}|_{z=0} = 0$.

Where x, y and z are dimensionless coordinates along horizontal and vertical directions, respectively; u,v,w, dimensionless velocity components in the x-y and Z directions, respectively; $f\bar{A}$ is the dimensionless temperature; P is the dimensionless pressure; Ra and Pr are Rayleigh and Prandtl numbers, respectively.

Solution Procedure: The governing equations, Esq.; (2-1)- (2-3), are discretized by the finite volume method (FVM) on non-uniform grid system [34]. The third-order QUICK scheme and the second-order central difference scheme are, respectively, implemented for the convection and diffusion terms. The set of discretized equations for each variable are solved by a line-by-line procedure, combining the tri-diagonal matrix algorithm (TDMA) with the successive over-relaxation iteration(SOR) method. The coupling between velocity and pressure is solved by SIMPLE algorithm [34]. The solution is terminated until the convergence criterion is reached, i.e. the maximal residual of all the governing equations is less than 10^{-8} .

RESULTS AND DISCUSSIONS

Studying Effect of Rayleigh Number: In this part, we just change Rayleigh number. Geometrical parameters $H_1=0.125, H_2=0.20, H_3=0.25, H_4=0.30$ are considered fixed. Isothermal lines have been presented in this state in lieu of different Rayleigh numbers in the middle planes $x=0.4375, z=0.50$, see Fig.5. As you can see, flows resulting from buoyancy force are very weak for small Rayleigh numbers, Isothermal lines are more regular and it shows that heat transfer mostly happens with conduction. But gradually with increasing Rayleigh number due to increasing velocity of fluid flow near thermal surface which causes to increase heat transfer by convection, curvature of isothermal lines becomes more. In addition, as we can see from figures, there is a cold area near floor of enclosure between Rayleigh numbers 10^5 - 10^6 . Existence of this area is because of velocity components in third dimension.

Then, we study effect of changing Ra on rate of heat transfer and flow parameters. In the Fig. 6 and Fig. 7,

Table 2: Compare thermal and flow parameter in lieu of different Ra in 3-D enclosure

Ra	Nu _{m,h}	Nu _{m,c}	θ_{mean}	W _{max}	u _{max}	V _{max}
10 ³	5.722	-2.517	0.3690	0.0086	0.0220	0.0270
10 ⁴	6.084	-2.677	0.3655	0.0577	0.0766	0.1069
10 ⁵	10.500	-4.621	0.3324	0.0739	0.0819	0.2531
10 ⁶	16.493	-7.255	0.3102	0.1180	0.1792	0.2591

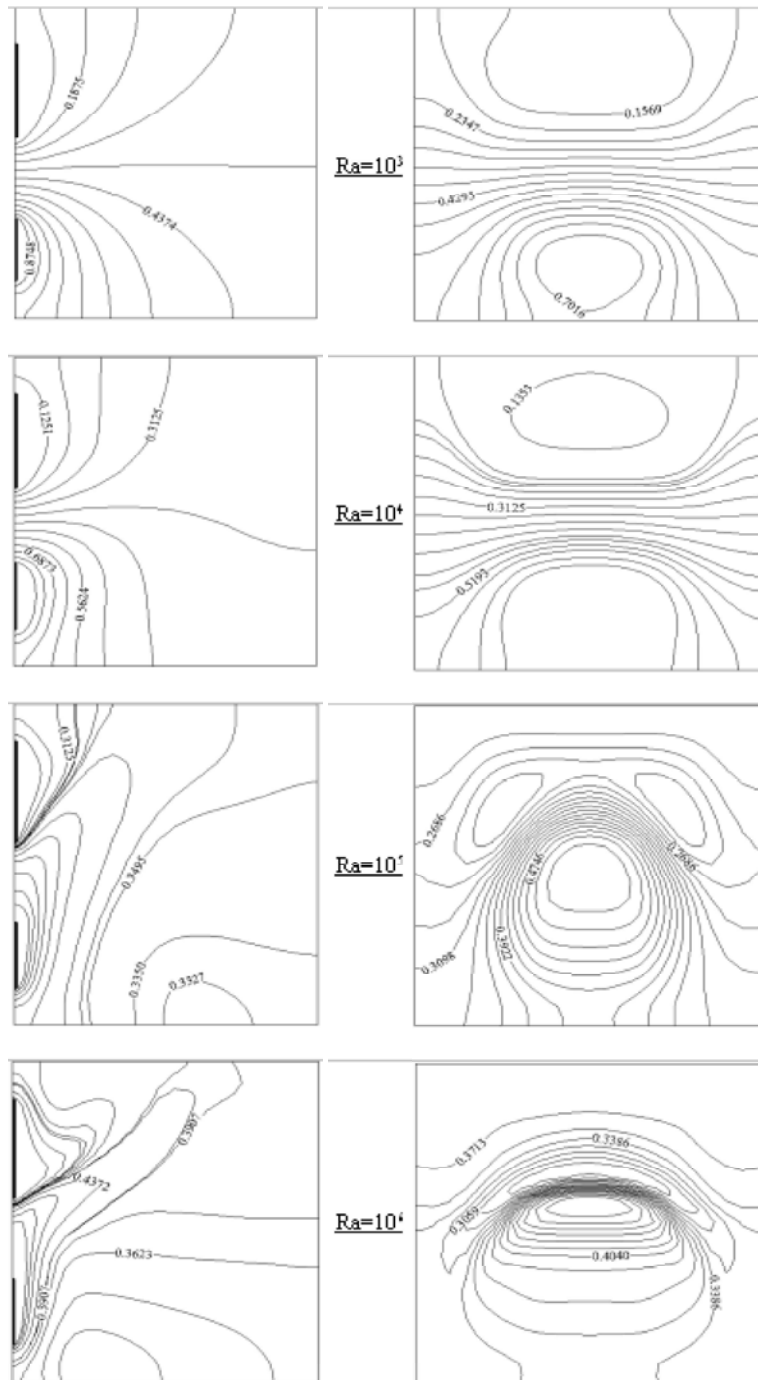


Fig. 5: Isothermal lines (right) in section $x=0.4375$ and isothermal lines (left) in section $z=0.50$ in lieu of different Ra.

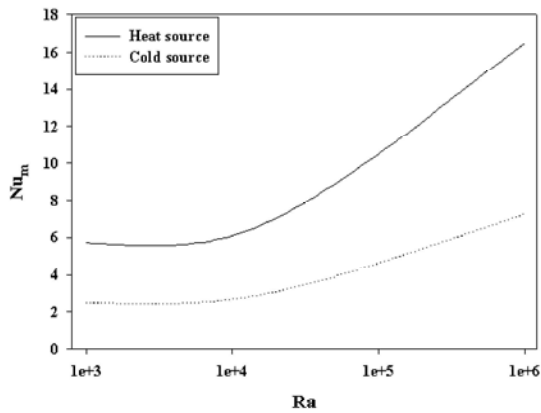


Fig. 6: Average Nusselt changes of hot and cold sources based on changes of Rayleigh number

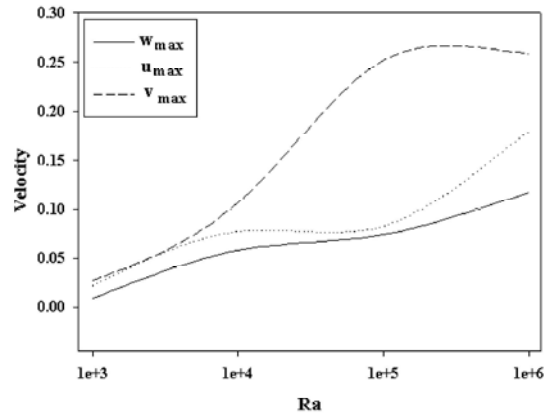


Fig. 7: Maximum changes of velocity components based on changes of Rayleigh number

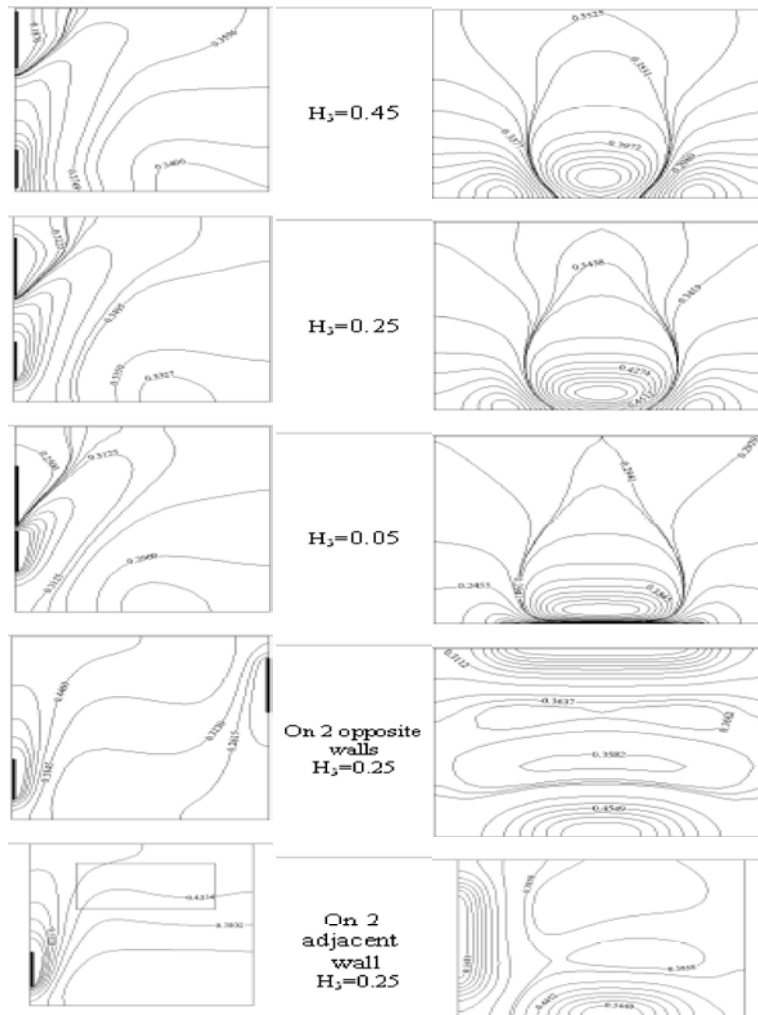


Fig. 8: Isothermal lines of the sources at $y=0.5$ (right) and isothermal lines of the sources at $z=0.5$ (left) in different situations of sources at $Ra=10^5$

average changes of thermal sources and maximum velocity components have been provided based on charges of Ra. The figures show that increasing Ra causes to increase Nu_m that shows increasing rate of heat conduction on cold and hot sources.

In small Ra which are under control of heat conduction mechanism, increasing Ra does not influence on rate of heat transfer, but from $Ra=10^4$ to higher that natural convection is important, increasing Ra influences on Nu_m significantly. Also, we see that maximum velocity components increase with increasing Ra. This increasing in velocity component along gravity acceleration v is more significant.

Therefore, it is concluded that increasing maximum velocity components in the enclosure of a critical parameter is due to increasing heat transfer from a hot source to a cold source, which its consequence can be observed in increasing Nu_m in the enclosure.

For more accurate comparison of above results Table 2 has been provided. In this table we can see not only average values of hot and cold sources and maximum velocity components but also average temperature of fluid in the enclosure is seen, too.

Increasing Rayleigh number causes to decrease mean temperature, it is because of temperature distribution in enclosure. As we know, in low Ra, role of heat transfer by conduction more important than high Ra, If effect of heat transfer by conduction is more, then mean temperature will increase in enclosure (because of linear distribution of temperature).

Studying Situation of Hot and Cold Sources: We consider enclosure of the Fig.2 again. In this part, in addition to changing distance between hot and cold sources H_3 , we put sources on opposite and adjacent walls in 2 states. Geometrical parameters are; $H_1=H_5$, $H_2=0.20$, $H_4=0.30$ and computations are done in lieu of $Ra=10^5$.

In this state, isothermal lines have been shown in lieu of situation change of cold and hot sources in the middle planes $z=0.50$, $y=0.50$ in the Fig.8. As we can see from the figures, when thermal sources are on a wall, with decreasing distance of two sources and inclining two sources towards center of plane, isothermal lines will be more condensed around thermal sources.

When thermal sources are on 2 opposite and adjacent walls, then form of temperature profile will completely be different and when thermal sources are in two walls opposite each other, hot fluid accumulates above enclosure and cold fluid accumulates bottom of enclosure.

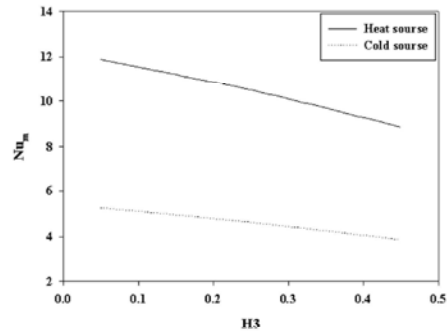


Fig. 9: Nu_m changes of cold and hot sources based on distance changes between thermal sources.

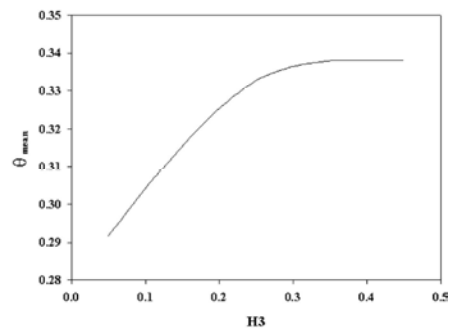


Fig. 10: Average temperature changes in enclosure based on distance changes between thermal sources.

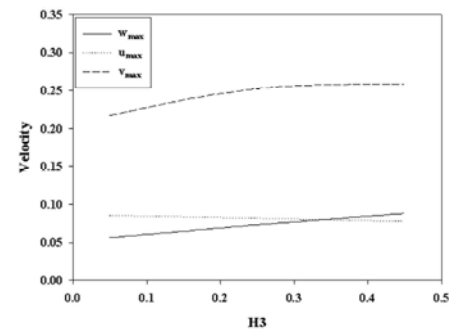


Fig. 11: Maximum changes of velocity components in enclosure based on distance changes between thermal sources

When thermal sources are placed in 2 adjacent walls, then form of temperature profile loses its symmetric state than the plane. Next we will study situation change of thermal sources in rate of heat transfer and flow parameters.

Fig. 9, Fig. 10 and Fig. 11 show rate of heat transfer on cold and hot thermal sources in the form of average Nusselt, average temperature and maximum velocity components in enclosure based on distance between thermal sources.

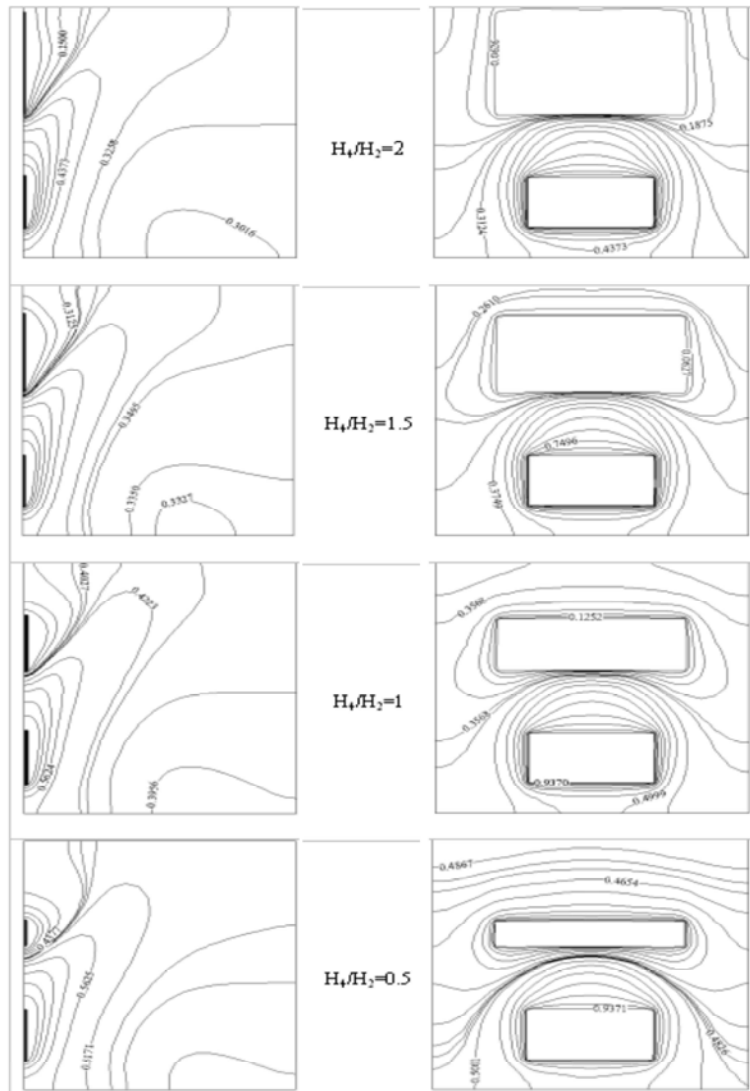


Fig. 12: Isothermal lines on the section $x=0.125$ (right) and isothermal lines on the section $z=0.5$ (left) in dimensional ratio of different

Table 3: Comparing heat conduction in different situations of thermal sources in three dimensional enclosure

Situations of thermal sources	$Nu_{m,h}$	$Nu_{m,c}$	θ_{mean}	W_{max}	u_{max}	v_{max}
One wall $H_3=0.45$	8.859	-3.861	0.3381	0.0885	0.0779	0.2588
One wall $H_3=0.25$	10.501	-4.621	0.3328	0.0747	0.0822	0.2530
One wall $H_3=0.05$	11.877	-5.259	0.2921	0.0565	0.0860	0.2170
0.25 two walls opposite each other $H_3=$	11.255	-4.952	0.4520	0.0417	0.1359	0.2466
Adjacent walls $H_3=0.25$	9.620	-4.622	0.3792	0.1205	0.1255	0.2520

Table 4: Comparing heat conduction and current parameters in different dimensional ratio of thermal sources in 3 dimensional enclosure

Situations of thermal sources	$Nu_{m,h} \times Ah$	$Nu_{m,c} \times Ah$	θ_{mean}	W_{max}	u_{max}	v_{max}
$H_4/H_2=2$	0.8820	-0.8820	0.2980	0.0830	0.0833	0.2622
$H_4/H_2=1.5$	0.8345	-0.8345	0.3325	0.0734	0.0820	0.2526
$H_4/H_2=1$	0.7410	-0.7410	0.3955	0.0676	0.0782	0.2344
$H_4/H_2=0.5$	0.5955	-0.5955	0.4840	0.0621	0.0739	0.2066

As you can see from the figures, increasing distance between thermal sources causes to significantly decrease heat transfer in the form of average Nusselt. In addition, average temperature and maximum components of fluid velocity in y and z directions increase with increasing distance between thermal sources.

Table 3 has been provided for more comparison between above results. In addition, a comparison has been done between heat transfer in cold and hot thermal sources in two opposite and adjacent walls. In this table, we can see increasing Nu_m in two opposite walls than their place on the two adjacent walls.

It is because of regular vortex movement of fluid in which fluid adjacent hot sources goes up and the fluid adjacent cold sources goes down in enclosure. being thermal sources on two opposite walls these two flow of fluid reinforce each other, Therefore, average velocity on thermal sources be increase, increasing velocity parameters on thermal sources caused to increase heat transfer due to fluid convection too.

Studying Effect of Dimensions Ratio of Hot and Cold Sources: Again, we consider enclosure in Fig. 2. In this part; we change height of cold source, H_4 , with fixing dimensions of hot source. The other geometrical parameters are considered constant, $H_1=0.125$, $H_2=0.2$, $H_3=0.25$.

Rayleigh number is considered $Ra=10^5$ in all conditions. Isothermal lines in this state have been presented in lieu of changing height of cold source in sections $x=0.125$, $z=0.5$ (Fig.12). As we can see from this figures, charges in isothermal lines are not very significant in different sections it is just clear that increasing dimensional ratio causes to increase density of isothermal lines in areas between thermal sources. Therefore, form of temperature profile becomes more uniform above cold sources.

After that we will consider effect of dimensional ratio of sources on rate of dimension less heat transfer, average temperature and maximum velocity components.

In Fig. 13, Fig. 14, Fig. 15, value of heat transfer on cold and hot thermal sources has been provided in the form of Nu numbers multiplying by sources area. Studies on changes of dimensional ratio of sources and their effect on heat transfer show that increasing ratio of cold sources length to hot sources causes to increase heat transfer.

This happens because of increasing maximum velocity components around thermal sources with regard to flow amplification of natural convection.

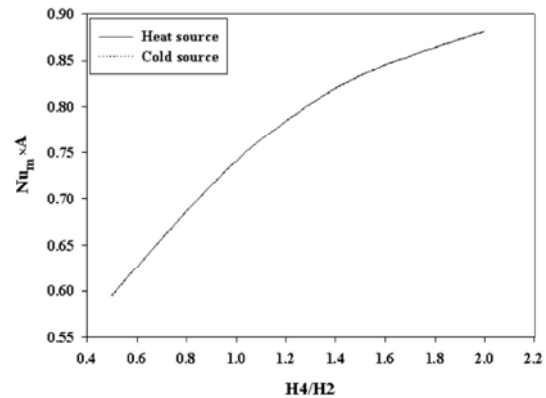


Fig. 13: Changes Num of thermal sources multiplied by sources area based on dimensional ratio of sources.

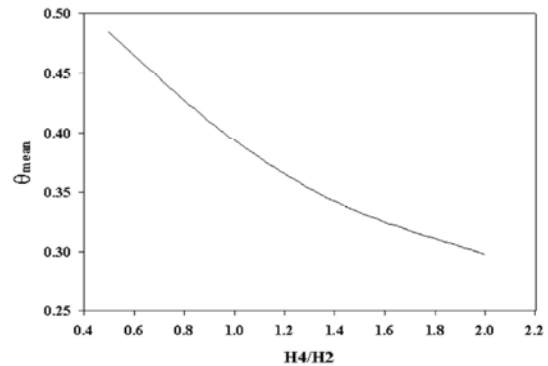


Fig. 14: Changes of average temperature in enclosure based on dimensional ratio of sources

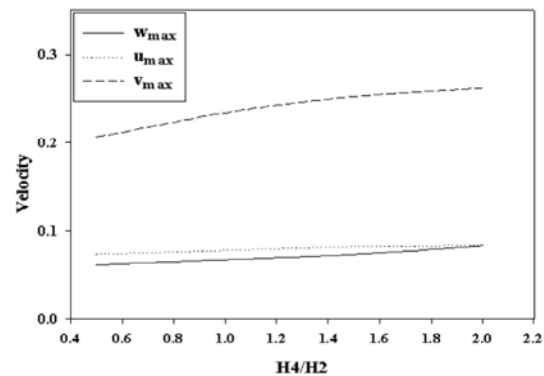


Fig. 15: Maximum charges of velocity components in enclosure based on dimensional ratio of sources

Also, increasing dimensional ratio of cold thermal sources to hot ones causes to decrease average temperature in the enclosure. It is because of manner or way of heat penetration from thermal sources into the enclosure.

A table has been show for more accurate comparison of the above results.

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