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Analysis of an Adjustable Equiripple Window and its Application to Fir Filter Design

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Abstract: In this paper a new class of adjustable equiripple windows based on the exponential function is proposed. The proposed window is rooted in the same way as the Kaiser window, but it has computational cost advantage due to having no power series expansion in its time domain representation. Simulation results show that the proposed window performs better sidelobe ripple ratio than the Kaiser window for the same window length and normalized mainlobe width. The results show that the filters designed by the proposed window provide better far end stopband attenuation than the filters designed by the Kaiser window.

Key words: Window functions · Kaiser window · Cosh window · Ultraspherical window · FIR filter design

NTRODUCTION

Window function are widely used in digital signal processing for the applications in signal analysis and estimation, digital filter design and speech processing [1]. In manuscript many windows have been proposed [2,3]. Due to their flexibility properties, the adaptable windows such as Kaiser and Dolph-Chebyshev windows are very smart in signal processing applications. Since the windows are suboptimal solutions, the best window is depended on the applications. The basic idea behind the window is choosing a proper ideal frequency-selective filter which always has a non causal, infinite-duration impulse response and then truncate or window its impulse response $h_d[n]$ to achieve a linear-phase and causal FIR filter [1].

$$h[n]= [n]w[n];\ w[n] = \begin{Bmatrix} f(n) & 0 \le n \le M \\ 0 & otherwise \end{Bmatrix} \quad (1)$$

Two enviable specifications for a window function are producing smaller main lobe width and good side lobe rejection. However these two requirements are incongruous, since for a given length, a window with a narrow main lobe has a poor side lobe rejection and contrariwise. So by tapering the window effortlessly to zero, side lobes are greatly reduced in amplitude [2]. The Kaiser window is a kind of two parameter windows that have maximum energy concentration in the main lobe, it control the mainlobe width and ripple ratio [4]. A window, *w(nT)*, with a length of *N* is a time domain function which is defined by:

$$w(nT) = \begin{Bmatrix} nonzero & |n| \le (N-1)/2 \\ 0 & other \end{Bmatrix} \quad (2)$$

Windows are generally compared and classified in terms of their spectral characteristics. The frequency spectrum of *w(nT)* can be introduced as [5].

$$W\left(e^{jwT}\right) = W_0\left(e^{jwT}\right)e^{-\frac{iw(N-1)T}{2}} \quad (3)$$

Where $W(e^{jwt})$ is called the amplitude function, N is the window length and T is the space of time between samples.

Two parameters of windows in common are the null to null width B_N and the main-lobe width B_R. These quantities are defined as $B_N = 2\grave{u}_N$ and $B_R = 2\grave{u}_R$, where $\grave{u}_N$ and $\grave{u}_R$ are the half null-to-null and half main-lobe widths, respectively, as shown in Fig. 1.

There are two important performance measures for window which are called as [3]:

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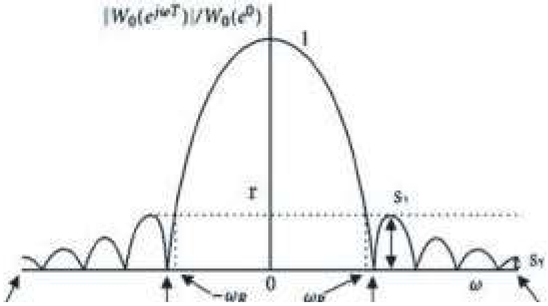

Fig. 1: A typical window normalized amplitude spectrum

-*R*= the ripple ratio in dB.
S = the side-lobe roll-off ratio

In this paper an improved two parameter window based on the exponential function is proposed, that performs better ripple ratio and lower sildelobe (at least 7 db) compared to the Kaiser and Cosine hyperbolic windows, where as having equal mainlobe width. Also its computation reduced because of having no power series.

Derivation of Proposed Window

Kaiser Window: In discrete time domain, Kaiser Window is defined by [6]

$$w_k(n)=\begin{cases}\dfrac{I_0(ax)}{I_0(a)} & |n|\le\dfrac{N-1}{2}\\ 0 & other\end{cases} \tag{4}$$

Where α is the shape parameter, N is the length of window and $I_0(x)$ is the modified Bessel function of the first kind of order zero[3].

$$x=\sqrt{1-\left(\frac{\gamma n}{N-1}\right)^2} \tag{5}$$

The Exponential of x is expressed as:

$$e^x=\sum_{i=0}^{+\infty}\frac{x^n}{n!} \tag{6}$$

The Exponential function and $I_0(x)$ have the same Fourier
series characteristics [5].

Proposed Window: The proposed window based on exponential [5], is made by inserting two new parameters which called γ and τ.

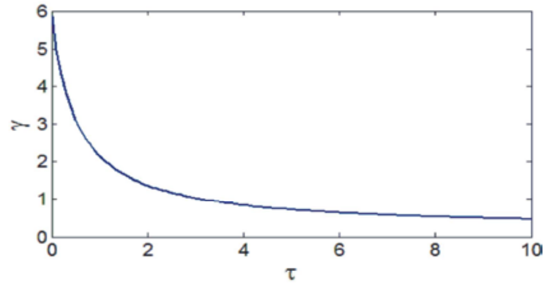

Fig. 2: Relation between γ and τ for constant mainlobe width

γ changes between 0 and 2, because if this quantity become larger than 2, x becomes imaginary value. In Kaiser Window, γ and τ are equal with 2 and 1 respectively. In proposed window, γ is less than 2 and the mainlobe width decreases. So by increasing the τ quantity mainlobe becomes constant. Fig. 2 shows the optimized γ and τ value. By sweeping γ between 0 and 2 as τ changes between (1-α) and ∞, an approximate relationship for optimized γ and τ quantity in terms of R can be found by using curve fitting method as:

$$\gamma=\frac{0.1836\tau+3.1028}{\tau+0.5278} \tag{7}$$

Fig. 2 illustrates the relation between γ and *τ* for constant mainlobe width, the maximum starting point given the most appropriate γ and *τ* for minimum ripple ratio and the coefficients caused an equiripple window.

$$\alpha'=\tau\alpha \tag{8}$$

$$S,P=\frac{e^{ax}}{e^{\alpha'}} \tag{9}$$

By substituting Eq. (7) in Eq. (9), the maximum of S.P function occurred in γ equal with 1.73 and τ equal with 1.42.

Fig. 3 shows the result of changing ellipse's focal by inserting γ and τ in Eq. (5)

In Fig. 4 the relationships between the adjustable shape parameter and ripple ratio for the proposed, Kaiser and Cosh window with N=51 are shown. It is shown that by increasing R, the proposed window has lower than -6db in compare with the Kaiser window and it is true compare with Cosh window too, but for R larger than -17.5 db the Cosh window has lower quantity. By fig.5 an approximate relationship can be found by using a curve fitting method as:

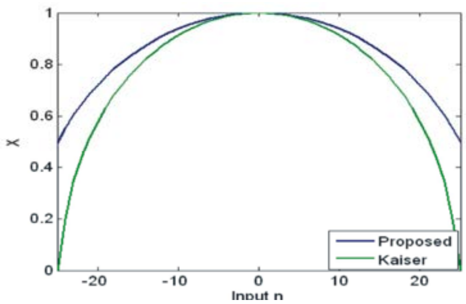

Fig. 3: Effects of inserting γ and τ in Eq.(5) in Proposed Window compared with Kaiser Window

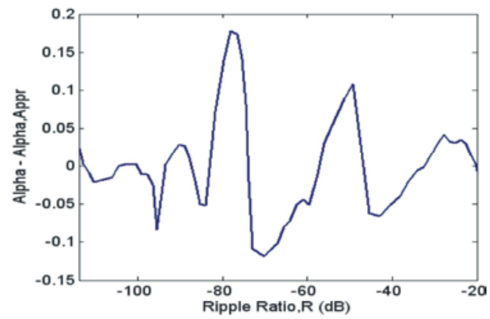

Fig. 5: Error curve of approximated a given by Eq. (10) versus R for N = 51

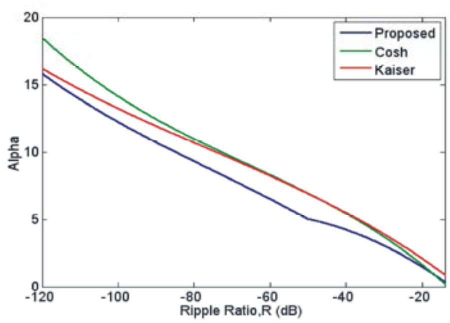

Fig. 4: Relation between a and R with approximation model for the Cosh, Kaiser and Proposed Windows with N = 51

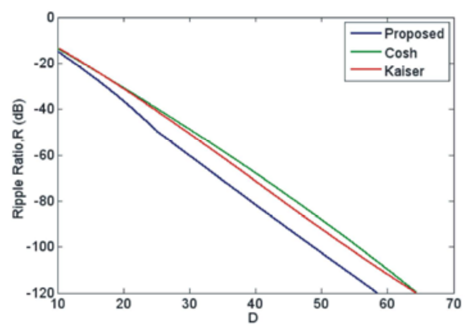

Fig. 6: Relation between D and R with approximation model for the Cosh, Kaiser and "Proposed window with N = 51

$$\alpha = \begin{cases} 0 & R > -13.26 \\ -0.001912R^2 - 0.2508R \\ -2.759 & -50 < R \le -13.26 \\ -1.073\times10^{-5}R^3 - 0.002407R^2 \\ -0.3179R - 6.217 & -120 < R \le -50 \end{cases} \quad (10)$$

The model which plotted in Fig. 4 provides a good approximation with an error designed in Fig. 5. The largest deviation in alpha is 0.18 which corresponds to an error of 0.3dB in actual ripple ratio. As for the Kaiser and cosh models the largest deviation in alpha is 0.07 and 0.1 respectively, but these correspond to an error of 0.44dB and 0.4 db respectively in actual ripple ratio [4, 7].

The second design equation is the relation between the window length and the ripple ratio. To envisage the window length (N) for a given quantities of the ripple ratio(R) and half mainlobe width (w_R), the normalized width $D_w = 2w_R (N - 1)$ is used [6]. The relation between the normalized width and the ripple ratio for the proposed window with $N = 51$ is given in Fig. 6. By using the curve fitting method for Fig. 6, an approximate design relationship between the normalized width (D_w) and the ripple ratio(R) can be established as

$$D_w = \begin{cases} 0, & R > -15 \\ -0.002076R^2 - 0.5663R \\ +2.088, & -50 < R \le -15 \\ 0.00171R^2 \\ +2.583 & -120 < R \le -50 \end{cases} \quad (11)$$

The approximation model for the normalized width given by Eq.(11)is plotted in Fig. 6. The relative error of approximated normalized width in percent versus the ripple ratio for N =51 is plotted in Fig. 7. The percentage error in the model modifies between 0.042 and -0.028. This error variety satisfies the error criterion in [3] which states that the predicted error in the normalized width must be smaller than 0.1%. digit value of the window length (N) can be predicted from [3].

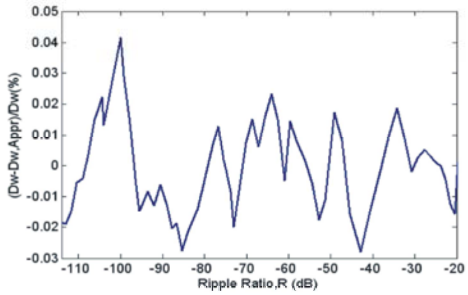

Fig. 7: Relative error of approximated D given by Eq . (11) In percent versus R for N = 51

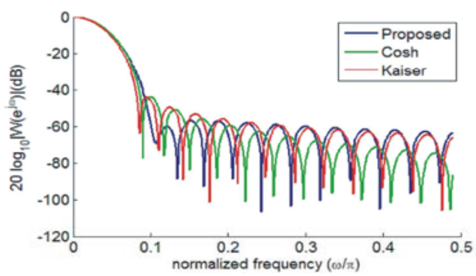

Fig. 8: Comparison of the proposed, Cosh and Kaiser windows for N=51 and a=6

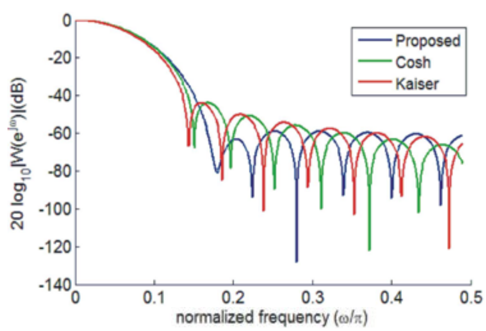

Fig. 9: Comparison of the proposed, Cosh and Kaiser windows for N=31 and a=2

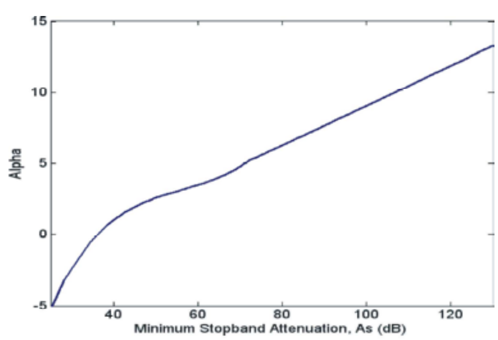

Fig. 10: Relation between a and the minimum stopband attenuation with approximated model for the Proposed window with N =51

Spectrum Comparisions: Fig. 8 and Fig. 9 show the frequency domain plots of Proposed, Cosh and Kaiser windows with N=31 and 51 respectively. By these figures, it is easily observed that as N increases the mainlobe width decreases as the sidelobe increases.

Application to Fir Filter Design: A.Filter design using the window method

FIR filter design are almost entirely restricted to discrete time implementations. The design techniques for FIR filters are based on directly approximating the desired frequency response of the discrete time system [2,8].

Filter Design Equations for Proposed Window: To find a suitable window which satisfies the given prescribed filter specification, it is necessary to obtain the relation between the window parameters and filter parameters. the near stopband attenuation, which also gives the minimum stopband attenuation (A_s). The attenuation of the near stopband is important for some applications [7-9]. Fig. 10 shows the relation between the α and the minimum stop band attenuation (As) for N = 51. It is seen that as the window parameter increases the minimum stop band attenuation also increases. By using the curve fitting method, an approximate expression as a first filter design equation can be found as

$$\alpha = \begin{cases} 0, & As < 21 \\ 0.347(As-21)^{0.4} + 0.000179As^3 & \\ -0.03019As^2 + 1.755As & \\ -33.508, & 21 < As \le 50 \\ 6.623\times10^{-5}As^2 + 0.172As & \\ -4.325, & 50 < As \le 120 \end{cases} \tag{12}$$

The approximation model for the α given by Eq. (12) is plotted in Fig. 10. It is seen that the model provides a good approximation with an error plotted in Fig. 11.

For the sake of another comparison with the Kaiser window, the filters are designed to achieve the cutoff frequency w_{ct}= 0.4π rad/sample and transition width Δw = 0.2 rad/sample with N = 51. The comparison result is shown in Fig. 12. It can be seen that the filter designed by the proposed window performs well A_s, where as the filter designed by the Kaiser window performs better far-end stopband attenuation.

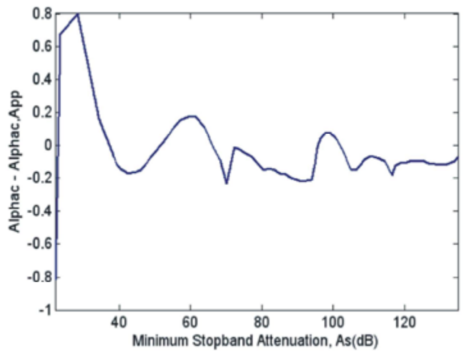

Fig. 11: Error curve of approximated a given by Eq. (12) versus As for N = 51

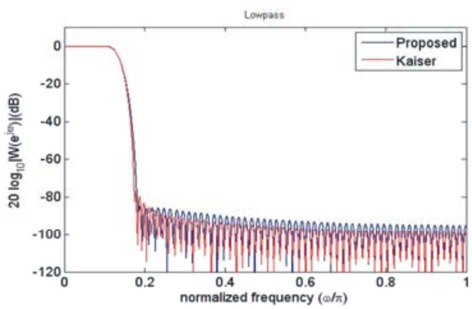

Fig. 12: The filters designed by the Proposed and Kaiser windows for wct = 0.4 and w = 0.2 rad /sample with N = 51

CONCLUSION

This window improved the ripple ratio concretely which the amount of changes in ripple ratio provides better ripple ratio, but worse sidelobe roll-off ratio for the same window length and mainlobe width compared with Kaiser Window. It has the advantage of having no power series expansion in its time domain function, so the proposed window has less computation compared with Kaiser one. Also this window is almost equi-ripple. It gives better results in terms of minimum stopband attenuation, filter length and near stopband attenuation with the optimum choice of the adjustable parameters.

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