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Discrete-Time Sliding Mode Control of a Class of Saturated Systems

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Abstract: This paper presents a design methodology of sliding mode control of a class of discrete-time linear saturated systems. The constraint of saturation is reported on the control vector. Firstly, we present the structure of saturation and the new formulation of the saturated state system. Secondly, we propose a method for sliding surface selection. This latter is formulate as a problem of root clustering, which leads to the development of a discrete and non-linear control law that ensures the reaching condition of the sliding mode. Finally we present a numerical application to validate the theoretical concepts.

Key words: Discrete-Time · LMI · Saturated Systems · Sliding Mode · Variable Structure Control

INTRODUCTION

The variable structure control has received increasing attention because of its inherent insensitivity to disturbances and parametric variations and ease of use for a quick and accurate response [1-3]. Design approaches for continuous-time control systems in sliding mode are already well established [4].

Most industrial processes operate in the areas delineated by many physical and technological constraints (saturation, limit switches...). The implementation of the control law designed without considering these limitations can have dire consequences for the system.

The recent development of the concept of sliding mode control (SMC) of saturated systems has led to linear continuous systems for constructing a robust controller which satisfies the constraints imposed on the control [5].

However, in the recent years with the rapid development of computer technologies and DSP chips, it is imperative to realize a digital system controller by computer. Therefore, it is more significant to extend the design method of sliding mode control in continuous systems into the discrete-time control system. A primary reason is that most control strategies nowadays are implemented in discrete-time.

Research in discrete-time sliding mode control has been intensified in recent years; and many interesting techniques are available [6-7]. However, there is a lack of design approaches that consider the saturated systems. The problem of saturation remains one of the obstacles to provide properties of guarantee on the stability of systems. Nevertheless with the fast evolution of industrial technology, especially in the actuators, it is necessary to envisage methods of resolution for this problem.

Used in early days, let, quote of these methods, the anti-windup design [8-9] and many other methods which introduce conditions on systems containing saturation functions [10-11].

Nerveless, the performance of these approaches in term of robustness is less effective than sliding mode ones. In fact, the latters are a very significant transitory mode for the Variable Structure Control (VSC) [12-14]. The main robustness advantage of VSC was mainly done by Soviet control scientists. In recent years, we find more research and many successful applications [15-19].

In this work we propose a new design methodology of discrete-time sliding mode control for a class of linear saturated systems. Then, we extend the classical SMC method of continuous systems to discrete-time saturated systems. The sliding surface design is formulated as an LMI discrete-time root clustering problem of an order reduced system.

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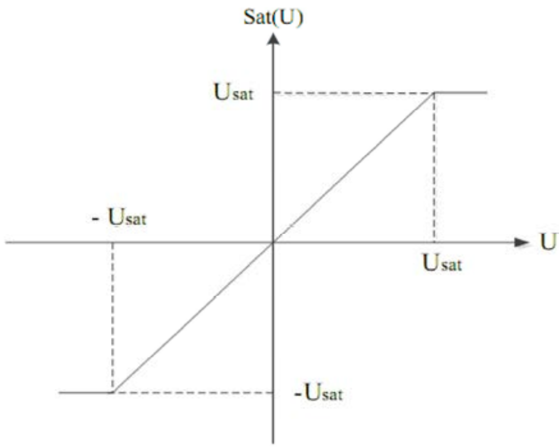

Fig. 1: The Structure of the saturation constraint

This paper is organized as follows: in first section, we give the form of the saturation structure reported on the control vector. Then, in the second section, we present a design procedure of robust saturated discrete-time-sliding mode control. This controller development procedure contains the classical steps of sliding mode design. The first one is to build an optimal sliding surface using the technique of pole placement in LMI region and the second one is to choose a control law to enforce the system behavior to reach and stay in the desired sliding surface. Finally, we apply the proposed approach to a numerical example.

System Description and Preliminaries: Let us consider that the structure of the saturation constraint is described by Figure 1:

Assumption: The control vector is subjected to constant limitations in amplitude. It's defined by:

$$u \in \Re^m = \left\{ u \in \Re^m / -U_{sat} \le u(k) \le U_{sat} \; ; U_{sat} > 0 \right\} \tag{1}$$

For $0 < \psi < 1$ such as $sat(u(k)) = \Psi\, u(k)$, the term of saturation $sat(u(k))$ and Ψ are given by, [20].

$$sat\big(u(k)\big) = \begin{cases} U_{sat} & if \quad U > U_{sat} \\ u(k) & if \quad -U_{sat} < U < U_{sat} \\ -U_{sat} & if \quad U < U_{sat} \end{cases} \tag{2}$$

With

$$\Psi = \begin{cases} \dfrac{U_{sat}}{u(k)} & if \quad U > U_{sat} \\ 1 & if \quad -U_{sat} < U < U_{sat} \\ -\dfrac{U_{sat}}{u(k)} & if \quad U < U_{sat} \end{cases} \tag{3}$$

The saturated system can be written as:

$$x(k+1) = \Phi x(k) + \Gamma\Psi u(k)$$
$$\Phi \in \Re^{n\times n}, \Gamma \in \Re^{n\times m}, x \in \Re^n \text{ and } u \in \Re^m \tag{4}$$

Assumption: The pair (Φ,Γ) is controllable, Γ has full rank m and n > m.

The sliding mode condition can be represented by the following equation:

$$s(k) = s(k+1) = \ldots = s(k+i) \;\; \forall k \ge k_g \tag{5}$$

k_g represent the instant when the sliding mode is reached. With i=1, 2, …
Also, we have:

$$Gx(k+1) = Gx(k) = 0 \tag{6}$$

G: matrix which defines the sliding surface.
It can be written as:

$$G\big(\Phi x(k) + \Gamma\Psi u_{eq}(k)\big) = Gx(k) \tag{7}$$

If $(G\,\Psi\,\Gamma)^{-1}$ exists, Then

$$u_{eq}(k) = -\big(G\Gamma\Psi\big)^{-1}\big(\Phi - I\big)Gx(k) = -K_{eq}x(k) \tag{8}$$

With

$$K_{eq} = \big(G\Gamma\Psi\big)^{-1}\big(\Phi - I\big)G \tag{9}$$

The dynamics $x(k+1) = \left[\Phi - \Gamma\Psi\big(G\Gamma\Psi\big)^{-1}G\big(\Phi - I\big)\right]x(k)$, describes the motion on the sliding surface which depends only on the choice of the matrix G.

To determine the matrix G of the sliding surface, we made the call to the principle of pole placement in LMI region in the complex plane.

Design of the Sliding Surface: In this section, we will prove the existence of the sliding mode. Indeed the canonical form used by for VSC design can be extended to saturated systems to select the gain matrix G.

Assumption: There exists an (nxn) orthogonal transformation matrix T such that $y(k+1) = Tx(k+1)$ and $T\Gamma = \begin{bmatrix} 0 \\ \Gamma_2 \end{bmatrix}$ where Γ has full rank m and Γ_2 is (m x m) and non-singular.

The transformed state equation

$$y(k+1) = T\Phi T^T y(k) + T\Gamma\Psi u(k) \tag{10}$$

Such as $y_k{}^T = \begin{bmatrix} y_{1k}^T & y_{2k}^T \end{bmatrix}$, with $y_{1k} \in \Re^{n-m}$ and $y_{2k} \in \Re^m$, can be rewritten as:

$$\begin{cases} y_1(k+1) = \Phi_{11} y_1(k) + \Phi_{12} y_2(k) \\ y_2(k+1) = \Phi_{21} y_1(k) + \Phi_{22} y_2(k) + \Gamma_2 \Psi u(k) \end{cases} \tag{11}$$

Since the sliding condition is $Gx(k) = GT^T y(k) = 0$, with:

$$T\Phi T^T = \begin{bmatrix} \Phi_{11}\Phi_{12} \\ \Phi_{21}\Phi_{22} \end{bmatrix},\ T\Gamma = \begin{bmatrix} 0 \\ \Gamma_2 \end{bmatrix} \text{ and } GT^T = \begin{bmatrix} G_1 & G_2 \end{bmatrix} \tag{12}$$

We can obtain the new defining sliding condition:

$$G_1\, y_1(k) + G_2\, y_2(k) = 0 \tag{13}$$

By assumption $G\Gamma$ is non-singular then G_2 must be non-singular.
The sliding mode condition becomes:

$$y_2(k) = -G_2^{-1} G_1\, y_1(k) = -F y_1(k) \tag{14}$$

With $F = G_2^{-1} G_1$, F being an [m x (n-m)] matrix.

The reduced system is (n-m) order. y_2 becomes a state feedback control. The sliding mode is then governed by:

$$\begin{cases} y_1(k+1) = \Phi_{11} y_1(k) + \Phi_{12} y_2(k) \\ y_2(k) = -F y_1(k) \end{cases} \tag{15}$$

The closed loop system will have the dynamics:

$$y_1(k+1) = (\Phi_{11} - \Phi_{12} F)\, y_1(k) \tag{16}$$

This indicates that the design of a stable sliding mode requires the selection of a matrix F such that ($\Phi_{11} - \Phi_{12} F$) has (n-m) eigenvalues in the unit circle.

If the stabilizing matrix F has been determined, G is given by:

$$G = [F\ 1_m]T \tag{17}$$

Determination of the Gain of the Reduced System: To determine the matrix G defining the sliding surface and the gain F, the method of the LMI seems to us very effective. Indeed to improve the performances of the control law and the response of system, we select to place the poles in a defined area [21], called area LMI. For that, we propose to choose all the eigenvalues of the matrix ($\Phi_{11} - \Phi_{12} F$) in an area defined by a disc of center q and ray r in the unit circle.

Poles must be placed in a circle with center on the positive real axis in order to obtain a reasonably damped response (damping ratio $\gamma < 1$), [22].

The system ($\Phi_{11} - \Phi_{12} F$) is asymptotically stable such as:

The eigenvalues of system ($\Phi_{11} - \Phi_{12} F$) are all in area Ω circle of center (q, 0) and of ray r of the complex plan so if $\exists Q > 0$:

$$f_\Omega = \begin{pmatrix} -rQ & qQ + Q\Phi_{11}{}^T + L^T \Phi_{12}{}^T \\ qQ + \Phi_{11} Q + \Phi_{12} L & -rQ \end{pmatrix} < 0 \tag{18}$$

With L: optimal gain given by the solution of LMIs. Then the gain is given by $F = LQ$

Saturated Control Law Design: To reach the sliding surface and ensures that trajectories are directed towards the switching surface from any point in the state space, we select a saturated feedback nonlinear control function $u = u_L + u_N$, where u_L and u_N are the linear and nonlinear control law parts. The general form is the following

$$u(k) = Kx(k) + \beta sig(s(k)) \tag{19}$$

Where K and β are appropriate matrix.

To determine the sliding mode control with state feedback, we proceed in the following way [23]:

$$s(k+1) = (1 - qT)s(k) - \varepsilon T\, sig(s(k)) \tag{20}$$

With $\varepsilon > 0$, $q > 0$ and $1 - qT > 0$

Theorem 1: *If the reaching law in Eq (19) is respected and at any* $|s(k)| < \varepsilon T/1 - qT$, *then* $|s(k+1)| < |s(k)|$, [23].

Remark 1: *If the reaching law in Eq (19) is respected in the design of a suitable control law, the reachability condition of* $|s(k+1)|<|s(k)|$ *can be satisfied under the condition that* $|s(k)|<\varepsilon T/1-qT$.

Remark 2: $|s(k)|<\varepsilon T/1-qT$ *Define a switching boundary in which the state trajectory will cross the ideal switching surface s(k)=0 at the next sampling instance.*

Referring to the control law defined [23]: for unsaturated systems, we developed a control law for saturated systems, by integrating the term of saturation Ψ this integration is given by the following equation:

$$G^T\Phi\, x(k) + G^T\Gamma\Psi u(k) - G^T x(k) = -qT\, s(k) - \varepsilon T\, sig(s(k)) \tag{21}$$

The resolution in u(k) gives the control law expressed by:

$$u(k) = -\left(G^T\Gamma\Psi\right)^{-1}\left[\left(G^T\Phi - G^T + qTG^T\right)x(k) + \varepsilon T\, sig(s(k))\right] \tag{22}$$

By identification one obtains:

$$K = -\left(G^T\Gamma\Psi\right)^{-1}\left[G^T\Phi - G^T + qTG^T\right] \tag{23}$$

And

$$\beta = -\left(G^T\Gamma\Psi\right)^{-1}\varepsilon T \tag{24}$$

Numerical Application: In this study, a quarter-vehicle MR suspension system is established to evaluate the control performance of the manufactured MR damper. Fig. 2 shows the quarter-vehicle model of the semi-active MR suspension system, which has two degrees of freedom. Here, m_1 and m_2 represent the sprung mass and unsprung mass respectively. The spring for the suspension is assumed to be linear and the tire is also modeled as linear spring component and MR damper. Now, by considering the dynamic relationship, the state-space control model is expressed for the quarter-vehicle MR suspension system as follows.

$$\dot{x}(t) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{C_1+C_2}{m_1} & \frac{C_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{C_2}{m_2} & -\frac{C_2}{m_2} \end{pmatrix} x(t) + B\begin{pmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{pmatrix} u(t)$$

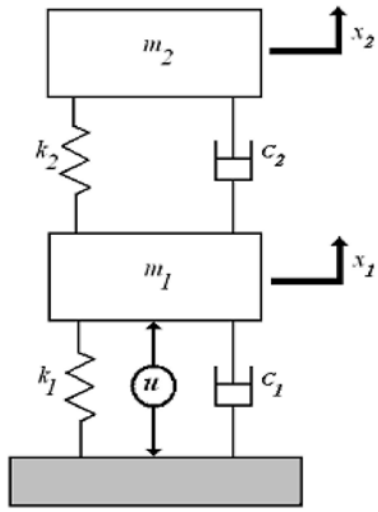

Fig. 2: Two degree of freedom vibrating system with one actuator

Simulation is achieved under the following condition:

$$m_1 = m_2 = 1kg,\ k_1 = k_2 = 1N/m$$
$$C_1 = C_2 = 0.01Ns/m,\ -1 \le u(k) \le 1$$

The initial condition is given by $x_0 = [0\ \ 0\ \ 0\ \ 1]'$
The discrete-time model is given as follows:

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$

With $\Phi = e^{AT},\ \Gamma = \int_0^T e^{A(T-\tau)}Bd\tau$

In this section we have the results of simulations with a sampling period T=0.3, one obtains:

$$\Phi = \begin{bmatrix} 0.9119 & 0.0439 & 0.2902 & 0.0049 \\ 0.0439 & 0.9558 & 0.0049 & 0.2951 \\ -0.5756 & 0.2854 & 0.9061 & 0.0467 \\ 0.2854 & -0.2902 & 0.0467 & 0.9529 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.0442 \\ 0.0004 \\ 0.2902 \\ 0.0049 \end{bmatrix}$$

After 3 iterations, the algorithm gives the stabilizing gain F of the reduced discrete-time system:

F= [-3.5497 3.1844 -7.2307]

The Figure 3 represents the poles of the reduced discrete-time system in an area defined by $\Omega(q,r)$ in the complex plan. Indeed, this area offers mainly a minimal damping of the poles and an absolute degree of stability minimum:

The matrix G which definite the sliding surface is given by:

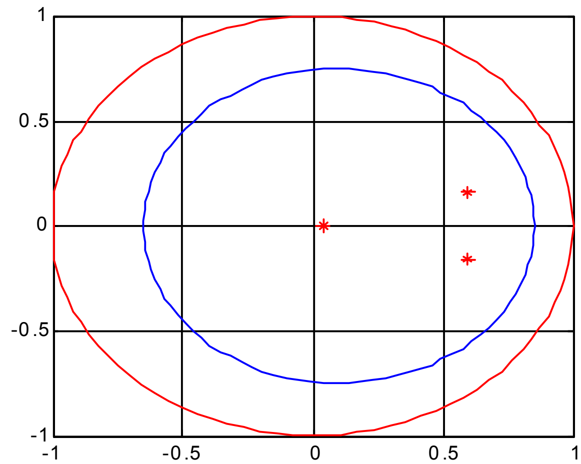

Fig. 3: Poles of the reduced discrete system

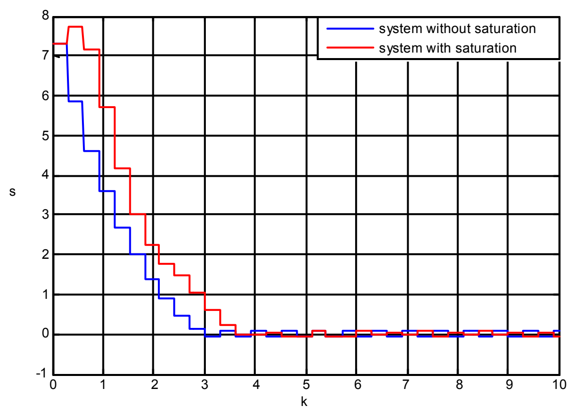

Fig. 4: Evolution of switching surface

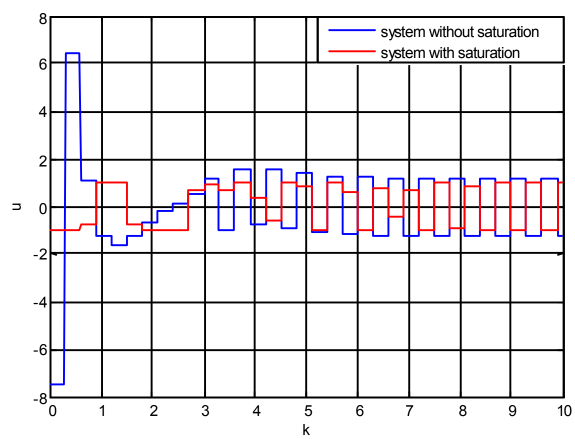

Fig. 5: Evolution of the control law

$$G = \begin{bmatrix} 3.1738 & 3.5543 & 0.4010 & 7.2908 \end{bmatrix}$$

The following results are obtained:

$$K = \begin{bmatrix} -2.2339 & 1.4510 & -1.3056 & -1.9059 \end{bmatrix}$$
$$\beta = -1.0217$$

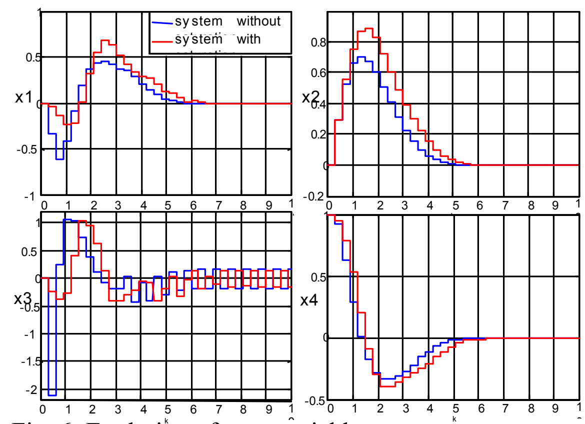

Fig. 6: Evolution of state variables

Simulations enable us to obtain the results given in Figure 4, Figure 5 and Figure 6 which presents, respectively, the evolution of the switching surface, control input and state variables (□□□ □: system without saturation, □□□ □: system with saturation constraint). These figures show a typical stable sliding mode convergence of the system in the two cases. However, that the introduction of saturation level of the control law is slightly degrade system performance. As consequence, the state variables dynamics of the saturated system have a more slowly transient mode than that of the system without saturation. The control is saturated and always inferior to its maximal value and able to reach S in a small time.

CONCLUSION

In this paper, we presented a new design approach for discrete-time sliding mode of a class of linear time invariant saturated system. The control input is saturated and is being of constant limitations in amplitude. In the first step, we have exposed the design of the stable sliding surface by solving linear matrix inequalities by means of the LMI. In the second step, a non-linear control schema is introduced, which drives and maintain system state trajectories in to a switch band in limit time. Numerical simulations have been presented showing the applicability, the efficiency and the robustness of the proposed method.

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