Abstract: The present work covers irregularity of jitter of accelerating voltage amplitude over the cells of accelerating resonators and $M \times N$ superstructures of linear electron-positron collider TESLA-ILC (Teraelectron volt Energy Super conducting Linear Accelerator-International Linear Collider), caused by random jitter of cells frequency. The author derived analytical formula and provides the results of numerical calculations for mean-square relative irregularity of accelerating voltage amplitude jitter as a function of a number of cells and mean-square relative jitter of their frequencies. $M \times N$ superstructure has less sensitivity of accelerating voltage amplitude jitter over resonator cells to random jitter of cells frequencies comparing traditional type resonators.

Key words: Collider • accelerating system • accelerating resonator • frequency • cell • shunt resistance • coupling constant • intrinsic Q • accelerating voltage • accelerating voltage amplitude • loss parameter • mean-square jitter • standard jitter • irregularity of accelerating voltage amplitude jitter

INTRODUCTION

Linear electron-positron colliders are the most precise instrument for studying of fundamental matter characteristics and they may cause dramatic rethinking of our concept of the Universe. One of the main subsystems of electron-positron collider TESLA-ILC is accelerating system of the main accelerator that consists of tens of thousands superconducting columbic resonators operating under temperature 2°K. Nine-cell resonators with oscillating operating mode comprised by almost similar cells are considered as accelerating resonators [1]. Each resonator of equipped with two units outputting agitated by beam high order modes HOM coupler, as well as one unit for operating mode input FM coupler.

The frequency of operating mode of oscillations of such resonator $f_{op} = 1300$ MHz, wavelength $\lambda = 230.610$ mm and accelerating voltage amplitude distribution over cells has the same sensitivity to random jitter of cells frequencies and random jitter of coupling constants between them that are caused by resonators production and adjustment errors. On the other hand, these resonators have the highest effective shunt resistance.

Stabilized accelerating resonators were developed in 1960s. They consist of coupled cells. A part of these cells works as a coupling between the sections of resonator with operating mode of oscillations [2-4]. Such resonators has high shunt resistance and relatively low sensitivity of accelerating voltage amplitude distribution to random jitter of cells frequencies and coupling constants between cells characteristics for resonators with operating $\pi/2$ mode of oscillations.

Dependency of irregularity of accelerating voltage amplitude distribution of such resonators from random jitter of cells frequencies and random jitter of coupling constants between cells is analyzed in [5]. Analytical formulas for standard deviation of relative irregularity of accelerating voltage amplitude distribution along resonator as a function of a number of cells and normal for this kind of systems deviation of random relative cells frequencies and coupling constants jitter have been derived in it.

For partial compensation of drawbacks of accelerating resonators with operating mode of oscillations and reduction of number HOM and FM couplers $M \times N$ superstructure was proposed [6-11]. Resonators of this type consist of $M$ units each of them consists of $N$ almost similar cells. Neighboring units are coupled with each other with cylindrical tubes $\lambda/2$ long.
Frequency of operating king of oscillations of this such resonator also equals $f_{op} = 1300$ MHz. Cells should be adjusted in such a way that amplitudes of voltages on all cells are equal, oscillations of electromagnetic fields in neighboring sells that relate to neighboring N-cells blocks are co phased and oscillations of electromagnetic field in neighboring cells of each unit are anti phased. Type of oscillations is called operating 0-π mode. This resonator has one FM coupler and in each coupling cylindrical tube there is one HOM coupler.

Requirements to accelerating voltage amplitude distribution over resonator cells of collider are extremely strict so the problem of irregularity of distribution caused by resonator cells production and adjustment errors attracts special attention.

**METHODIC**

Equivalent circuit of resonator [12] (Fig. 1) was used for calculations.

Agitating of fundamental mode in resonator via the last cell is shown in scheme electromotive difference source with complex amplitude $E_g$ and resistive element with impedance of input wave guide $Z_W$. Electromotive difference source frequency is equal to operating frequency $f_{op}$. In Fig. 3 there are also shown loop currents with complex amplitudes $I_n$, $n = 1, 2, 3, ... N$ or $n = 1, 2, 3, ... M×N$.

For analysis purpose it may be convenient to consider radio-technical parameters such as

$$f_{on} = \frac{1}{2\pi \sqrt{C_n C_m}}$$

the frequency of $n$-th cell and

$$K_{n/2} = \sqrt{C_n C_{n+1}} / C_{n+1/2}$$

coupling capacitance factor of nth and $(n+1)$th cells, $K_{n/2} = 0$.

In these formulae

$$c_{2n} = (c_{n-1,n} c_{n+1} + c_{n,n+1})^{1/2}$$

Complex amplitude of accelerating voltage on nth cell $U_n$ relates to value $I_n = \sqrt{E_{op} C_n}$ as

$$I_n = \frac{jw_{op} C_n U_n - 2j f_{op}}{\sqrt{2 \pi f_{op} (r_{shn}/Q_{ou})}}$$

where

$$j = \sqrt{-1}, \phi_{op} = 2\pi f_{op}$$

$\bar{E}_g$, $Z_W$ and the capacity $P_g$ of electromotive difference source while working on matched load are related as

$$\bar{E}_g = \sqrt{8\pi Z_W P_g} \exp(j\phi_g)$$

where $\phi_g$-initial phase of electromotive difference source in a certain section of input waveguide.

Formula

$$j2\pi f_{op} \sqrt{c_{2n} c_{n+1}} E_g = 4f_{op} \sqrt{2 c_{2n} Z_W P_g} \exp(j\phi_g)$$

may be also defines via radio-technical parameters of the last cell, capacity and operating frequency of electromotive difference source.

Where $Q_{ext}$ is external Q of the last cell.

Assuming that all cells have equal ratio

$$(r_{shn}/Q_{ou}) = (r_{sh}/Q_{0})_{eff}$$

combined equations against complex voltage amplitudes in resonator cells may be written as follows

Fig. 1: Equivalent circuit of resonator with electric coupling between cells
1. Traditional N-cell resonator

Consider resonator consisting of N cells with equal values of ratio \((r_{nh}/Q_0)_{cell}\) and equal values of coupling constants between cells \(K_c/2 = 0.00945\).

Under ideal adjustment of cells \(U_n = (-U_n)\) so cells frequencies and complex amplitudes are equal.

\[
\begin{align*}
\delta f_{cn} = f_{cN} - f_{cn} = \frac{f_{op}}{\sqrt{1 + K_c^2}},
\end{align*}
\]

\[
\begin{align*}
U_n = 2j \left( \frac{1}{\sqrt{1 + K_c^2}} \right) P_g Q_{rhn} \left( \frac{Q_{0_n}}{Q_{cell}} \right) \exp(j\phi_0) U_n
\end{align*}
\]

\[
\begin{align*}
(-1)^{N-n} U_n.
\end{align*}
\]

Let relative jitters of cells \(\delta f_{cn}/f_{cn}\) frequencies are independent random values with similar for all cells dispersion

\[
\sigma^2_{\delta f_{cN}/f_{cN}} = (\delta f_{cn}/f_{cn})_{k, n}^2 - 1, 2, 3, \ldots, N - 1
\]

Dispersion of relative irregularity of voltage distribution over resonator cells may be defined as average over all possible samples dispersion of relative irregularity of distribution of voltage over resonator cells.

\[
\sigma^2_{\delta V_{MN}} = \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \left[ \left( \frac{1}{N-1} \sum_{n=1}^{N} U_{n,k} - \bar{U}_{n,k} \right)^2 \right]
\]

\[
- \frac{1}{N-1} \sum_{n=1}^{N} \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \left( U_{n,k} - \bar{U}_{n,k} \right)^2
\]

Here \(U_{n,k}\)-selective average value of voltage amplitude of cells that is obtained after taking average of voltage amplitudes over all resonator cells. Ratio is derived as a result of expansion over small values \((\delta f_{cn}/f_{cn})_{k, n}\) with keeping only first orders of these values.

As \(0 < K_c < 1\)

Resonator on the base of M×N superstructure

Let us consider resonator consisting of M×N cells with equal ratio \((r_{nh}/Q_0)_{cell}\) and equal values of coupling constants between cells that relate to each M unit. Characteristic values of coupling constants of cells \(K_c/2 = 0.00945\) and coupling constants between cells of neighboring units \(K_{cc}/2 = 0.00101\).

In this case combined equations (1) are written as

\[
\begin{align*}
\delta f_{mn} = f_{mN} - f_{mn} = \frac{f_{op}}{\sqrt{1 + K_c^2}} P_g Q_{rhn} \left( \frac{Q_{0_n}}{Q_{cell}} \right) \exp(j\phi_0) U_n
\end{align*}
\]

\[
\begin{align*}
(-1)^{N-n} U_n.
\end{align*}
\]

The frequency of operating mode of oscillation of such resonator also equals \(f_{op} = 1300\) MHz. Cells should be adjusted in such a way that amplitudes of voltages on all cells are equal, oscillations of electromagnetic fields in neighboring cells that relate to neighboring N-cells blocks are co phased and oscillations of electromagnetic field in neighboring cells of each unit are anti phased. Than for complex voltage amplitudes on resonator cells the following requirements may be written

\[
\begin{align*}
\left\{ \begin{array}{l}
U_{n,1} = -U_{n,2} = \ldots = -U_{n,N-1} = 0;
U_{n,MN-1} = 0;
U_{n,MN} = 2j \frac{f_{op}}{f_{cMN}} P_g Q_{rhn} \left( \frac{Q_{0_n}}{Q_{cell}} \right) \exp(j\phi_0)
\end{array} \right.
\end{align*}
\]

As it follows from combined equations (2) and these requirements cells frequencies should have the following values
3. Results of calculation of relative irregularity of voltage distribution over resonator cells.

Dependencies of ratio $\sigma_{\delta U}/\sigma_{\delta f/f_c}$ of a number of cells for two types of resonators: traditional with operating $\pi$ mode (upper curve) and resonator based on M×N superstructure (lower curve).

Combined equations (2) may be used for calculating dispersion against relative irregularity of voltage amplitude distribution over resonator cells on the base of M×N superstructure. Let us consider that relative mismatch of frequency of each cell $\delta f_{\text{cell}}/f_{\text{cell}}$ is independent random value with zero average and dispersion $\sigma^2_{\delta f/f_c}$.

3. Results of calculation of relative irregularity of voltage distribution over resonator cells.

Dependencies of ratio $\sigma_{\delta U}/\sigma_{\delta f/f_c}$ of a number of cells for two resonator types are shown in Fig. 2. Upper curve relates to traditional resonator with operating $\pi$ mode, lower curve-resonator on the base of M×N superstructure.

Both curves are drawn as a result of taking average on $k = 10^6$ over random samples for solutions of combined equations (1) and (2) for random jitter of cells frequencies.

**CONCLUSION**

According to the results relative irregularity of voltages distribution over resonator cells on the base of M×N superstructure is less sensitive to random mismatch of cells frequencies. In the example difference in relative irregularity of voltage distribution over cells of these types is 1.51, 2.13, 2.69 times and $M = 2, 3$ respectively. In is caused by the fact that operating mode of resonator based of M×N superstructure is not extreme mode by frequency. Operating mode of resonator of resonator based of 4×9 superstructure is fourth mode from upper limit of dispersion dependency of resonator. Dispersion characteristics of resonator based on superstructure and traditional resonator are shown on Fig. 3. Both resonators have the same number of cells with abovementioned values of coupling constants between neighboring cells. Dispersion characteristic of traditional resonator is smooth and dispersion characteristic of resonator based of 4×9 superstructure is fragments into 9 parts of 4 resonances. Two last parts form single group of resonances in which a point in the middle of the group relates to operating mode.
Fourth point from the right relates to operating mode of resonator based on 4×9 superstructure and the rightmost point relates to operating mode of traditional resonator.

**RESUME**

Obtained analytical formulas for ratio $\sigma_{RU}/\sigma_{RF/c}$ of traditional resonator with operating $\pi$ mode provide relatively accurate result for small relative mismatch of resonator cells frequencies and may be used for calculating accelerating fields in collider structures.

**REFERENCES**


