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The Usefulness and Viability of Systems: Assessment Methodology Taking into Account Possible Damages

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Abstract: An analytical approach to the assessment of risk for the system elements at implementation of attacks is proposed. The risk analysis is carried out in terms of: an analytical assessment of damage based on the utility function integration; the search of expression in terms of finding its extremum and ranges of the random variable for a given level of risk. The corresponding analytical expressions obtained in the work are the methodological basis for risk analysis of the systems with different profiles in terms of their possible death. Specifically in this paper, we obtain the following analytical expressions: for the integrated assessment of damage on a given utility function, simplified expressions of risk of system death in a given time period, as well as the coordinates of its peak. To solve the resulting equations the researchers used the chord method and Taylor series and applied assessment of the accuracy of the proposed models. The proposed methodology of risk analysis is invariant to the type of the considered systems and therefore can be widely applied to assess the utility and survivability of information and other systems, including distributed systems and networks.

Key words: Risk · Damage · Utility function · Log-logistic distribution · Peak · Mode

INTRODUCTION

Assessment of systems viability is not only of theoretical but also of real practical interest [1-2]. Obviously, this study should be based on the methodology of risk analysis [1], as only assessing probable (possible) damage [3] it is possible to find the true risk of the system "death". An adequate description of the characteristics of the studied threats and their use in the future risk analysis require obtaining the mathematical models of the rise of the distribution of damages [4] from these threats.

Therefore, when analyzing the viability it is the damage that should be considered as a random value. In this case, it is conventional to make a description [1-5] using different distribution laws, among which the log-logistic distribution is the most popular for the analysis of systems viability [6]. We use the most common approach to measure risks through the assessment on two factors: the probability of occurrence and the severity of possible consequences.

Key Part: The function of damage can be obtained from the normalized utility function (utility per unit of time), which is presented by the following expression:

$$\omega(t)=\sqrt[\alpha_?]{\frac{t}{T_{cp}}}\left[1-\left(\frac{t}{T_{av}}\right)^{\alpha_?}\right] \qquad (1)$$

where $a_B>1$ and $\alpha_\varepsilon > 1$ are coefficients of nonlinearity determining steepness of utility function cuts and T_{av} - average up (life) time.

The graph of the function of utility is presented in Fig 1.

Area of the figure S_2 represents a loss of profit, which the system could have got if in the moment of time t_0 it did not lose its efficiency forever. In the case of a fatal failure the utility function is integrated from the point of failure, namely, to the average lifetime of the system.

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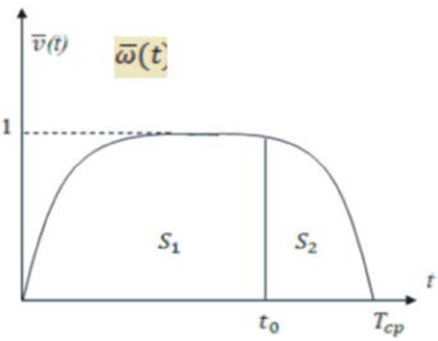

Fig. 1: Graph of utility function

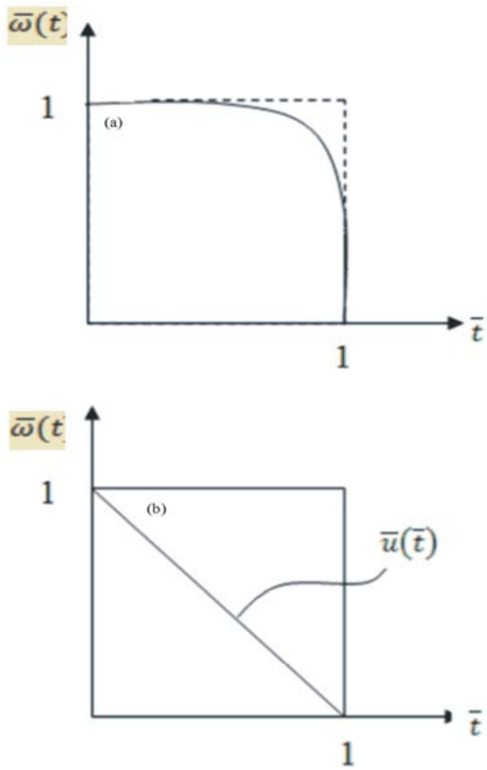

Fig. 2: Simplification of utility curve

$$\overline{u}(t_0) = S_2 = \int_{t_0}^{T_{av}} v(t)dt = \int_{t_0}^{T_{av}} \left(\alpha_? \sqrt{\frac{t}{T_{av}}} \left[1 - \left(\frac{t}{T_{av}} \right) \right] \right) dt = \tag{2}$$

$$T_{av} \left(\frac{1}{\frac{1}{a_?}+1} - \frac{1}{a_? + \frac{1}{a_?} + 1} + \frac{\left(\frac{t_0}{T_{av}}\right)^{a_? + \frac{1}{a_?} + 1}}{a_? + \frac{1}{a_?} + 1} - \frac{\left(\frac{t_0}{T_{av}}\right)^{\frac{1}{a_?}+1}}{\frac{1}{a_?}+1} \right)$$

where t_0 is the moment of failure of the analyzed system. Equation (2) should be apparently normalized on T_{av}, beyond which the consideration does not make sense, that is,

$$\overline{t}_0 = t_0 / T_{cp} \tag{3}$$

Considering that $\overline{t}_0 < \frac{1}{2}$ is quite a rare occasion the utility curve can be modified to a first approximation, discarding the first factor $\overline{\omega}(t) = 1 - \left(\overline{t}\right)^{\gamma}$ that is unimportant in this case, which leads to Figure 2a, where $\gamma = \alpha_{\varepsilon}$.

Obviously, at $\gamma \to \infty$ the utility function will degenerate into an angle of the unit square and the function of the normalized loss - in its diagonal (Fig 2b). Taking into account (2) and (3) the equation of damage will take the following form

$$\overline{u}(\overline{t}) = \frac{\gamma}{\gamma+1} - \overline{t}_0 + \frac{\overline{t_0}^{\gamma+1}}{\gamma+1} \tag{4}$$

The estimating calculations show that 90% of life goes in normal mode. At $\gamma = 9$ the last term of equation (4) brings in the error less than 5% even for $\overline{t}_0 = 0{,}9$. Thus for practical use the equation of damage is allowable

$$\overline{u}(\overline{t}) \cong \frac{\gamma}{\gamma+1} - \overline{t}_0,$$

that should be taken into account in the subsequent mathematical calculations.

The analytical expression for the risk function with preset function of damage is presented as follows

$$\mathrm{Risk}\left(\overline{t}_0 \pm \frac{?\,\overline{t}}{2} \right) = \left(\frac{?}{?+1} - \overline{t}_0 \right) \times \frac{(\beta/a)\left(\overline{t}_0/a\right)^{\beta-1}}{\left[1+\left(\overline{t}_0/a\right)^{\beta}\right]^2} ?\,\overline{t} \tag{5}$$

where $\overline{u}(\overline{t}_0) = \left(\frac{\gamma}{\gamma+1} - \overline{t}_0 \right), \overline{u}(\overline{t}_0) \in (0;1)$ is normalized damage from the death of DS component at the moment of time $\overline{t}_0$,

$\gamma > 1$ - Nonlinearity coefficient,

$\overline{t}_0 \in (0;1)$ - normalized time of component failure,

$\alpha > 0$, $\beta > 1$ - parameters of distribution of probability density of damage occurrence,

$f(\overline{t}_0, \alpha, \beta) = \frac{(\beta/a)\left(\overline{t}_0/a\right)^{\beta-1}}{\left[1+\left(\overline{t}_0/a\right)^{\beta}\right]^2}$ - the probability density of the time of system death [6], $\Delta\overline{t}$ - the sampling interval.

Find the extremum of risk taking the derivative of the probability density and equating it to zero:

$$\frac{df(\overline{t})}{dt} = \frac{\beta\left(\overline{t}/\alpha\right)^{\beta-2}\left(\beta - 1 - \beta\left(\overline{t}/\alpha\right)^{\beta} - \left(\overline{t}/\alpha\right)^{\beta}\right)}{\alpha^2\left[1+\left(\overline{t}/\alpha\right)^{\beta}\right]^3} = 0$$

The probability density function assumes a maximum value at the point $\bar{t}_{\max} = \alpha^{\beta}\sqrt{\frac{\beta-1}{\beta+1}}$.

Substitute $\bar{t}_{\max}$ to the probability density function. As a result obtain the extremum:

$$f_{\max} = \frac{(\beta/\alpha)\left(\left(\alpha\sqrt[\beta]{\frac{\beta-1}{\beta+1}}\right)/\alpha\right)^{\beta-1}}{\left[1+\left(\left(\alpha\sqrt[\beta]{\frac{\beta-1}{\beta+1}}\right)/\alpha\right)^{\beta}\right]^2} = \frac{(\beta-1)^{\frac{\beta-1}{\beta}}(\beta+1)^{\frac{\beta+1}{\beta}}}{4\alpha\beta}.$$

Hence obtain

$$\Delta t = \frac{4\alpha\beta}{n(\beta-1)^{\frac{\beta-1}{\beta}}(\beta+1)^{\frac{\beta+1}{\beta}}} \tag{6}$$

To investigate the risk in relation to the problems of viability it is necessary to evaluate the risk that an entity fails fatally in the vicinity of $\Delta\bar{t}$ of the moment of system death $\bar{t}_0$. The resulting risk function will be a function of a continuous random value of risk.

To solve the problems of viability in general it is necessary to determine the parameters of risk. Calculate the risk parameters for the obtained expression of damage and the log-logistic distribution law [6].

In this context, we define the mode and the peak of risk, which can be found by examining the risk function at extremes. In this regard find its derivative.

$$Risk'\left(\bar{t}_0 \pm \frac{\Delta\bar{t}}{2}\right) = \Delta\bar{t}\left(\bar{U}(\bar{t}_0)\cdot f(\bar{t}_0)\right) = \Delta\bar{t}\left[\left(\frac{\gamma}{\gamma+1}-\bar{t}_0\right)\times\frac{(\beta/\alpha)(\bar{t}_0)^{\beta-1}}{\left[1+(\bar{t}_0/\alpha)^{\beta}\right]^2}\right]' =$$
$$= \Delta\bar{t}\times\frac{\beta(\bar{t}_0/\alpha)^{\beta-2}}{\left[1+(\bar{t}_0/\alpha)^{\beta}\right]^3}\left[\frac{\beta}{\alpha^{\beta}}\bar{t}_0^{\beta+1}+\left(-\frac{\gamma(\beta+1)}{\alpha^{\beta}(\gamma+1)}\right)\bar{t}_0^{\beta}+(-\beta)\bar{t}_0+\frac{\gamma(\beta-1)}{(\gamma+1)}\right]. \tag{7}$$

Equate the resulting expression (7) to null:

$$\Delta\bar{t}\times\frac{\beta(\bar{t}_0/\alpha)^{\beta-2}}{\left[1+(\bar{t}_0/\alpha)^{\beta}\right]^3}\left[\frac{\beta}{\alpha^{\beta}}\bar{t}_0^{\beta+1}+\left(-\frac{\gamma(\beta+1)}{\alpha^{\beta}(\gamma+1)}\right)\bar{t}_0^{\beta}+(-\beta)\bar{t}_0+\frac{\gamma(\beta-1)}{(\gamma+1)}\right]=0 \quad 1$$

Critical points of function $Risk(\bar{t}_0 \pm \frac{\Delta\bar{t}}{2})$ in the studied range [0,1] are as follows.

1) $(\bar{t}_0/\alpha)^{\beta-2} = 0 \Rightarrow \bar{t}_0 = 0;$

2) $\frac{\beta}{\alpha^{\beta}}\bar{t}_0^{\beta+1}+\left(-\frac{\gamma(\beta+1)}{\alpha^{\beta}(\gamma+1)}\right)\bar{t}_0^{\beta}+(-\beta)\bar{t}_0+\frac{\gamma(\beta-1)}{(\gamma+1)}=0$

For simplification introduce the following designations.

$$N = \frac{\beta}{\alpha^{\beta}},$$
$$M = -\frac{\gamma(\beta+1)}{\alpha^{\beta}(\gamma+1)},$$
$$K = -\beta,$$
$$P = \frac{\gamma(\beta-1)}{(\gamma+1)},$$

Then the equation takes the following form.

$$N\cdot\bar{t}_0^{\beta+1}+M\cdot\bar{t}_0^{\beta}+K\cdot\bar{t}_0+P=0$$

This equation is impossible to solve by radicals. Therefore it is expedient to apply the numerical method of approximate determination of roots. Using the chord method obtain

$$\bar{t}_{0i+1} = \bar{t}_{0i} - \frac{(\bar{t}_{0i}-\bar{t}_{0i-1})\cdot\left(-\frac{N}{K}\cdot\bar{t}_{0i}^{\beta+1}-\frac{M}{K}\cdot\bar{t}_{0i}^{\beta}-\frac{P}{K}\right)}{-\frac{N}{K}\cdot\bar{t}_{0i}^{\beta+1}-\frac{M}{K}\cdot\bar{t}_{0i}^{\beta}+\frac{N}{K}\cdot\bar{t}_{0i+1}^{\beta+1}+\frac{M}{K}\cdot\bar{t}_{0i+1}^{\beta}+\frac{P}{K}}.$$

To find an approximate value $\bar{t}_0$ with a predetermined accuracy 0 it is necessary to use the obtained iterative formula until $|\bar{t}_n-\bar{t}_{n-1}|\le\xi$. The initial approximation is 0.5. The value obtained as a result of $\bar{t}_n$ calculation is the desired critical point.

To calculate the risk mode consider that the point $\bar{t}_0=0$ is not the sought maximum. The point $\bar{t}_n$, while in the interval [0,1] divides it into two sections: $[0,\bar{t}_n)$, where the function values increase and $(\bar{t}_n,1]$, where the function decreases. Consequently, the point $\bar{t}_n$ is the desired maximum point of risk function $Risk\left(\bar{t}_0 \pm \frac{\Delta\bar{t}}{2}\right)$.

Then, the mode is approximately equal to:

$$\bar{t}_m \cong \frac{\gamma}{\gamma+1}-\bar{t}_n. \tag{8}$$

Hence the risk peak takes the following form:

$$Risk(\bar{t})_{\max} = \Delta\bar{t}\frac{\beta\left(\frac{\gamma}{\alpha(\gamma+1)}(\bar{t}_n/\alpha)^{\beta-1}-(\bar{t}_n/\alpha)^{\beta}\right)}{\left[1+(\bar{t}_n/\alpha)^{\beta}\right]^2}. \tag{9}$$

To find the values of damage at the preset risk value it is necessary to solve the following equation:

$$Risk(t)_{\max} \cdot k = Risk(t),$$

where $Risk(t)_{\max}$ is a peak value of risk, k - coefficient ($k \in (0,1)$) setting the level of reading from $Risk(t)_{\max}$.

Considering the abovementioned, obtain:

$$\Delta\bar{t} \cdot \frac{\beta\left(\frac{\gamma}{\alpha(\gamma+1)}\left(\bar{t}_n/\alpha\right)^{\beta-1} - \left(\bar{t}_n/\alpha\right)^{\beta}\right)}{\left[1+\left(\bar{t}_n/\alpha\right)^{\beta}\right]^2} \cdot k =$$

$$= \Delta\bar{t} \cdot \left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right) \times \frac{(\beta/\alpha)\left(\bar{t}_0/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_0/\alpha\right)^{\beta}\right]^2}$$

or

$$\left(\frac{\gamma}{\gamma+1} - \bar{t}_n\right) \times \frac{\left(\bar{t}_n/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_n/\alpha\right)^{\beta}\right]^2} \cdot k = \left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right) \times \frac{\left(\bar{t}_0/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_0/\alpha\right)^{\beta}\right]^2}$$

To find the solution to the latter equation find its logarithm:

$$\ln\left[\left(\frac{\gamma}{\gamma+1} - \bar{t}_n\right) \times \frac{\left(\bar{t}_n/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_n/\alpha\right)^{\beta}\right]^2} \cdot k\right] = \ln\left[\left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right) \times \frac{\left(\bar{t}_0/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_0/\alpha\right)^{\beta}\right]^2}\right]$$

And introducing the designation

$$c = \ln\left[\left(\frac{\gamma}{\gamma+1} - \bar{t}_n\right) \times \frac{\left(\bar{t}_n/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_n/\alpha\right)^{\beta}\right]^2} \cdot k\right],$$

obtain the following equation:

$$\ln\left[\left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right) \times \frac{\left(\bar{t}_0/\alpha\right)^{\beta-1}}{\left[1+\left(\bar{t}_0/\alpha\right)^{\beta}\right]^2}\right] = c$$

or

$$\ln\left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right) + \ln\left(\left(\bar{t}_0/\alpha\right)^{\beta-1}\right) - \ln\left[1+\left(\bar{t}_0/\alpha\right)^{\beta}\right]^2 = c$$

or

$$\ln\left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right) + (\beta-1)\cdot\ln\left(\bar{t}_0/\alpha\right) - 2\cdot\ln\left(1+\left(\bar{t}_0/\alpha\right)^{\beta}\right) = c$$

To find the roots of this equation expand the natural logarithms $\ln\left(\bar{t}_0/\alpha\right)$, $\ln\left(\frac{\gamma}{\gamma+1} - \bar{t}_0\right)$ and $\ln\left(1+\left(\bar{t}_0/\alpha\right)^{\beta}\right)$ into Taylor series in the vicinity of the point $\bar{t}_n$. For the expansion limit to five first members of the series, then the error will be negligible. The result is an equation of the fourth degree, which is solvable by radicals.

CONCLUSION

The obtained analytical expressions (1)-(9) will allow assessing the risks of system "death" and determining their viability in the range of normalized damages. This gives an opportunity of adequate assessment of security, which will allow determining the damage from efficiency loss in advance and taking effective managerial decisions to optimize the risk at various attacks [7].

Findings. The proposed mathematical apparatus in contrast to its analogues [7-11] takes into account not only the probability of death, but also the value of damage arising in this case, which in different periods of the system life will be obviously different. Such an approach should be considered more appropriate and can be recommended for practical use in solving problems of evaluating the usefulness and survivability of systems at dos- attacks [9], the attack by forging cached DNS records [10] and attacks using the TCP-de-synchronization method [11].

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