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Methods for Calculating Life Insurance Rates

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Abstract: This article analyzes the mechanism for modeling the probability of survival and death for a population, with a view to determining the attributes of mortality given different age groups; it looks into methods of calculating life insurance rates for short-term and long-term insurance and attempts to assess the possibility of determining risks for insurance companies under different types of insurance. The article also presents the general workflow of the life insurance system. The author has conducted a theoretical analysis of the mortality process by using the models developed by A. de Moivre (a person's lifespan is distributed evenly over the interval (0, ω), where the parameter ω is the limiting age people don't normally live past), Gompertz (some parameters are determined based on statistical data for a certain population) and Makeham (the force of mortality is calculated using the $\mu_x = A + Be^{\alpha x}$ formula). The author has examined the practical application of all the methods used in the study with respect to different types of insurance: it's customary for an insurance company to enter into a large number of contracts; individual claims are independent quantities; the long-term life insurance model is characterized by that in these cases, one takes into account changes in the value of money occurring over time; the exponential smoothing method is used for short-term and long-term forecasts and is predicated on the mean-weighted value of sales over a certain number of periods passed.

Key words: Model · Probability · Insurance event · Mathematical expectation · Mortality · Survival function · Risk

INTRODUCTION

The demographic situation in a region governs the net cost of life insurance: the lower the population's age-related mortality level, the lower the risk rate and vice versa. Of great significance in implementing life insurance in a region is the demographic situation stability factor, for the invariability of age-related mortality indicators in time lets one realize the major principle behind determining life insurance rates – rate stability with no detriment to the material interests of the policyholder and the insurer.

When it comes to life insurance, of interest is modeling the survival and death probability with a view to detecting the attributes of mortality for different age groups.

This area is known as actuarial mathematics and is characterized by the use of the probability theory, mathematical statistics and mathematical modeling methods [1].

Life insurance models are nominally subsumed into two large groups wherein, respectively:

- Changes in the value of money over time are counted in;
- Changes in the value of money over time are not counted in [2].

We can dispense with money value changes for short insurance policy terms.

For this reason, life expectancy models that do not take into account changes in the value of money over time are called short-term life insurance models [2].

Otherwise, when changes in the value of money over time are taken into account in calculations, we are dealing with long-term life insurance models.

To a commercial insurance company, compared with one in the Islamic insurance system called *takaful* [3], of importance is not an individual claim or A disbursement on that claim, but the total size of payments to policyholders, who file individual claims, i.e. the total risk which should not exceed the company's capital.

Otherwise, there is a probability the company can go bankrupt.

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The assessment of this probability is crucial to the operation of a commercial insurance company.

The Main Part: The general workflow of the life insurance system.

The size of disbursements on an insurance contract is of a fortuitous character, which means that the size of payments on all contracts is likewise a chance quantity.

The size of payments is limited by the insurance fund which is formed of premium payments.

Based on that, the cumulative insurance size is varied over a certain interval, the upper limit whereof equals the size of all disbursements on all contracts taken together. Therefore, the calculation of the premium payment T is of great importance to commercial insurers.

If the probability of an insurance event q occurring is known ahead of time (based on former experience, by analogy, etc.), then theoretically, without taking into account all the other factors (including the time factor), the size of the premium T is calculated using the following formula [4].

$$T = S.q$$

where S is an insurance disbursement;

q is the probability of an insurance event occurring.

The equation above just illustrates the principle of the financial equivalency of policyholder and insurer obligations.

Suppose that T is the size of the premium and q_n is the probability of an insurance event occurring (for instance, the death of a policyholder in n years after the insurance policy comes into effect).

If an insurance event occurs within the first year of insurance, the insurer will get an amount T (assume that the premium is paid at the beginning of the year), while if this event occurs within the second year, the insurer will get an amount equaling $2T$, etc.

The mathematical expectation of this row of premiums is

$$T.q_1 + 2.T.q_n + ... + n$$

Although the resulting quantity sums up all the policyholder's payments, given the credibility of these having been paid, when summing the corresponding quantities we do not take into account the fact that the premiums are paid at different points in time.

With this factor in mind (through discounting the payment amounts), we find the mathematical expectation of the up-to-the-minute cost (actuarial cost) of payments to be

$$E(A) = T \cdot \left[q_1 + (1+v) \cdot q_2 + (1+v+v^2) \cdot q_3 + ... + (1+v+...+v^{n-1}) \cdot q_n \right]$$

where $v = 1/(1 + i)$ is a discount multiplier; i is a percentage rate.

We could admit that it is paid at the end of the year the insurance event occurred in.

Then the mathematical expectation of the disbursement within the first year will be Sq_1, the second year - Sq_2, etc.

The mathematical expectation inclusive of the disbursement time factor (actuarial cost) is

$$E(S) = S \cdot \left[v \cdot q_1 + v^2 \cdot q_2 + ... + v^n \cdot q_n \right]$$

Based on the principle of the equivalence of policyholder and insurer obligations, we get an equation

$$E(S)=E(A)$$

which enables us to derive the sought value of the net premium T.

The calculation of net and gross premiums is based on the insurance company's model – more specifically, on deriving the total insurance amount on all insurance contracts and the credibility of insurance events occurring.

To ensure a 100% guarantee that the size of net premiums will exceed the size of disbursements, the insurer has to create an insurance fund the size of the cumulative insurance amount.

In this case, the premium will equal the insurance amount.

The insurer determines for itself the size of its acceptable risk, which mathematically can be expressed as the following in equation

$$P\left(\sum S_1 < \sum P_1\right) \ge y$$
$$P\left(\sum S_1 - \sum P_1\right) \le b$$

where P is the probability;

y is a safety guarantee established by the insurer;
S_1 is a disbursement on the 1st contract;
P_1 is a premium on the 1st contract;
b is the upper limit of the safety guarantee.

The rationale behind these in equations is that the probability that the size of all premiums will exceed the size of all policyholder payments should be determined upfront.

According to the Lyapunov theorem, insurance events and insurance disbursements are distributed according to the normal law [4].

If the chance quantity distribution law is determined, the above in equation is solved easily.

Firstly, the probability that the continuous quantity X will get a value that belongs to the interval (A, b) equals the definite integral of the frequency of the distribution over the interval from A to b, namely

$$Đ(à < Đ < \mathrm{b}) = \int_a^b f(x)dx$$

Secondly, the normal distribution function equals

$$f(x) = \frac{1}{\sigma \cdot \sqrt{\pi}} \cdot e^{\frac{-(x-a)^2}{2 \cdot \sigma^2}}$$

where x is the mathematical expectation of a chance quantity;

y is the mean square deviation.

Then

$$ifĐ|X - à| \leq b = 2 \cdot \hat{O} \cdot \frac{b}{2} \rightarrow P\left|\sum S_i - \sum P_i\right| \leq b = 2 \cdot \hat{O} \cdot \frac{b}{2}$$

where Φ is the Laplace function.

Based on the financial equivalence principle, the sought size of the net premium can be expressed as the product of the insurance amount and the net rate expressed as a percentage.

The major part of the rate is the net rate which expresses the cost of the insurance risk and ensures the covering of losses [4].

$$T_n = \frac{u \cdot S}{100}$$

where T_{Π} is the net rate;

u is the size of the net rate for 100 rub.;
S is the size of a basic insurance amount.

The rate is the gross rate.

$$T_b = \frac{T_n}{1-f}$$

where T_b is the gross rate;

T_{Π} is the net rate;
f is the load fraction in the gross rate.

The major part of the rate is the net rate which expresses the cost of the insurance risk and ensures the covering of losses.

The load fraction f can be calculated based on accounting data.

$$f = \frac{R}{\sum Ï} + K + V$$

where R is expenses;

$\Sigma\Pi$ is the size of premiums collected on this type ofinsurance;

K is the percentage of commission paid to agents in effecting this type of insurance;

V is a part of profit in the gross rate the insurer looks to get on this type of insurance.

The basis of any actuarial calculations is the study of the mortality process of any given population, determination of certain patterns and further calculation of rates.

In the insurance system, the mortality process is mostly modeled based on certain analytical laws, i.e. the theoretical analysis of the state of a population over a long period of time was conducted [5].

With this theoretical analysis of mortality processes, the primary and simplified study of real-life situations normally employs standard models that let us derive major patterns.

Besides, the mortality process is well approximated by the analytical laws considered below.

One of the founders of the credibility theory, A. de Moivre, suggested in 1729 [6] holding that a person's lifespan is distributed evenly over the interval $(0, \omega)$ where the parameter ω is the limiting age people normally don't live past.

For this model, with $0 < x < \omega$ we get

$$f(x) = \frac{1}{\omega}$$

$$s(x) = 1 - \frac{x}{\omega}$$

Table 1:Input data

Name of parameter	Use
Age (in years)	The primary and most crucial parameter that helps determine corresponding values in the mortality table
Limit (in years)	Used in calculating the cost of rent:postnumerando and pronumerando
Number of disbursements per year	Used in calculating the cost of rent: postnumerando and pronumerando
Disbursement in case of death	Based on the value of this parameter, the cost of rates is calculated
Probability	The parameter ties the mechanisms of actuarial mathematics and the probability theory together: relying on the value of this parameter, the policyholder and the insurer take a decision

Table 2: Input data

Name of parameter	Use
Net premium for short-term and long-term insurance	The rate that expresses the cost of the insurance risk and ensures the covering of losses
Gross premium for short-term and long-term insurance	The rate at which an insurance contract is entered into
Load	Determines what part of the gross premium is determined by the insurer's expenditures associated with the implementation of the insurance process, attraction of new clients, etc.

Table 3: Calculation of the main indicators of the morality table

Indicator	Formula	Parameters
Number of people who survived until a certain age *x*	$l_x = l_0 \cdot S(x)$	l_0 is the initial population; *S(x)* is the survival function; *x* is the age (the table is formed for any age, x= 0...99)
Number of people who died while transiting from age *x* to age *x*+1	$d_x = l_x - l_{x+1}$	l_x is the number of people of age *x*; l_{x+1} is the number of people of age x+1
The probability of dying at age *x*, without surviving until age *x*+1	$q_x = 1 - d_x$	*dx* is the number of people who died while transiting from age *x* to age *x*+1
The probability of surviving until a certain age	$P_x = 1 - q_x$	*qx* is the probability of dying at age*x*, without surviving until age *x*+1

$$\mu_x = \frac{f(x)}{s(x)} = \frac{1}{\omega - x}$$

In the Gompertz model, the force of mortality is calculated via the formula

$$\mu_x = \frac{f(x)}{s(x)} = Be^{\alpha x}$$

where a > *B* > 0 are some parameters determined based on statistical data for a certain population being studied.

The survival function is

$$s(x) = \exp\left[-\int_0^x \mu_u du\right] = \exp\left[-\int_0^x Be^{\alpha u} du\right] = \exp\left[-\int_0^x B\left(\frac{e^{\alpha x}-1}{\alpha}\right) du\right]$$

and the deaths curve is

$$f(x) = \mu_x \cdot s(x) = B \cdot \exp\left[\alpha \cdot x - B \cdot \frac{e^{\alpha x}-1}{\alpha}\right]$$

We have a maximum in the point

$$x = \frac{\ln\alpha - \ln B}{\alpha}$$

Later on, in 1860 [6], Makeham suggested determining the force of mortality using the formula

$$\mu_x = A + Be^{\alpha x}$$

where the parameter *A* takes into account the occurrence of insurance events and the summand BE^{ax} takes into account the effect of age on mortality.

For the Makeham model, the survival function is

$$s(x) = \exp\left[-\int_0^x \left(A + Be^{\alpha u}\right) du\right] = \exp\left[-A \cdot x - \frac{B \cdot \left(e^{\alpha x}-1\right)}{\alpha}\right]$$

and the deaths curve is

$$f(x) = -s'(x) = \left[A + Be^{\alpha x}\right] \exp\left[-A \cdot x - \frac{B \cdot \left(e^{\alpha x}-1\right)}{\alpha}\right]$$

The models presented are used for the theoretical analysis of the population mortality process.

In Table 2, one can see the initial data for the algorithms presented below.

The most commonly used algorithm for solving actuarial mathematics problems works the following way.

For convenience, we take the initial aggregate l_0 to equal

l_0 =100000

We will further model up the life expectancy of an aggregate of people over the next 100 years and will be taking down the times of death.

The modeling will be implemented based on the analytical laws considered above.

Let's calculate the commutation numbers:

$$D_x = l_x \cdot (1+i)^{-x}$$

where x is the current age;

l is the percentage rate;
l_x is the number of people who were still alive by age x.

$$N_x = \sum_{j=x}^{w} Dj$$
$$M_x = \sum_{j=x}^{w} Cj$$
$$C_x = d_x \cdot (1+i)^{-x+1}$$

Mortality tables are used in calculating the size of the insurance rate [7].

Let's take a look at the algorithm for calculating short-term insurance rates.

We'll take the size of an insurance premium as the unit of measurement for amounts of money.

In this case, disbursements on the i-th contract X_i take the values 0 and 1 with the probability (1 – q) and q respectively.

We get

$$EX_i = (1-q)\cdot 0 + q\cdot 1 = q$$
$$EX_i^2 = (1-q)\cdot 0^2 + q\cdot 1^2 = q$$
$$VarX_i = EX_i^2 - (EX_i)^2 = q - q^2$$

Now for the mean value and summary disbursements

$$S = X_i + ... + X_n$$

we have

$$ES = N \cdot EX_i$$
$$VarS = N \cdot VarX_i$$

Using Gaussian approximation for the centered and scaled quantity of summary disbursements, we'll express the probability of a disbursement by a traditional insurance company as follows:

$$P(S \le u) = P\left(\frac{S-ES}{\sqrt{VarS}} \le \frac{u-ES}{\sqrt{VarS}}\right) \approx \hat{O}\left(\frac{u-ES}{\sqrt{VarS}}\right)$$

We find those that satisfy the value of the quintile of the normal distribution x_p, where p is the probability.

We get the equation

$$\hat{O}\left(\frac{u-ES}{\sqrt{VarS}}\right) = x_p$$

Then

$$u = x_p \cdot \sqrt{VarS} + ES$$

We've obtained the value of a net premium for 100 rubles.

Let's find the size of the net premium

$$T_n = \frac{u \cdot S}{100}$$

where S is the amount the client wants to get in the event of his/her death, however paradoxical that may sound.

We find the value of the gross premium

$$T_b = \frac{T_n}{1-f}$$

where f is the load the size of which each insurer determines for itself.

As a result, the algorithm for calculating short-term insurance rates was obtained.

When it comes to long-term insurance, the remaining term of life is characterized by a constant force of mortality [8].

Let's calculate the net premium:

$$t_0 = \overline{A}_x = \int_0^\infty v^t \cdot f_x(t)dt$$

where *fx(t)* is the density of the remaining term of life.

Since the force of intensity is given, we find the survival function using the formula

$$S_x(t) = e^{-\mu t}$$

and the mortality function *fx(t)*

$$f_x(t) = \mu \cdot e^{-\mu t}$$

Then the net premium is

$$\overline{A}_x = \int_0^\infty \mu e^{-(\mu+\delta)\cdot t} dt = \frac{\mu}{\mu+\delta}$$

The up-to-the-minute size of disbursements on an individual contract can be obtained through substituting 2δ for δ.

$$\overline{Z}_x^{\,2} = \frac{\mu}{\mu+\delta}$$

Consequently

$$Var\overline{Z}_x = E\overline{Z}_x^{\,2} - \left(E\overline{Z}_x\right)^2$$

Now we can calculate the relative insurance mark-up:

$$\theta = x_\alpha \cdot \frac{\sqrt{Var\overline{Z}_x}}{\overline{A}_x \cdot \sqrt{N}}$$

Let's determine the size of rates:

$$T_b = \frac{\overline{A}_x \cdot (1+\theta) \cdot b}{100}$$

$$T_n = T_b \cdot (1-f)$$

As a result, the algorithm for calculating long-term insurance rates was obtained.

Table 4: Structure of initial data for algorithmic models

Name of parameter	Value entered
Age (in years)	25
Insurance policy term (in years)	15
Limit (in years)	10
Number of disbursements per year	6
Insurance amount	100000
Probability of disbursement	0,89

Let's consider the practical application of the above methodology using a specific example (Table 4).

We'll first model up the life expectancy parameters, which is an alternative method for obtaining data needed in calculating an insurance company's rates.

We'll use the Gompertz law for the modeling; the initial population is 100000 people [9].

We'll model the mortality process through approximation by the law chosen.

Parameters

$B = 0{,}0019332$
$a = 0{,}03615656$

Thus

$$s(0) = 1$$

$$s(1) = \text{Åõð} - 0{,}0019332 = \frac{e^{0{,}036156561} - 1}{0{,}036156561} = 0{,}998033362$$

and so on.

The number of those who died while transiting from age x to age $x+1$ is determined through the formula

$$l_1 = 100000 \cdot 0{,}998033362 = 99803{,}33624$$

We'll determine the probability of dying at age x without surviving until age $x+1$ the following way:

$$d_0 = l_0 - l_1 = 100000 - 99803{,}33624 = 196{,}663706$$

The probability of surviving until a certain age is determined as follows:

$$\text{ð}_0 = 1 - q_0 = 1 - 0{,}001966638 = 0{,}998033362$$

The commutation numbers are calculated the similar way

$$i = e^{0{,}09} - 1 = 0{,}094174$$

$$D_1 = 99803,33624 \cdot (1+0,094174) = 91213,41$$
$$N_1 = \sum_{j=1}^{98} Dj = 1024898,38029019$$
$$C_1 = 196,663706 \cdot (1+0,094174)^{-1+1} = 196,6636994277$$

We'll calculate short-term insurance rates.

The number of insurance contracts is N=450; the insurance amount is b= 100000 pya.

The probability of death taking place within a year is

q = 0,00484910116854934

In this case, disbursements on the 1st contract X_1 take the values 0 and 1 with the probability (1 – q) and q respectively.

Consequently,

$$EX_1 = (1-q)\cdot 0 + q\cdot 1 = q = 0,00484910116854934$$
$$VarX_1 = EX_1^2 - (EX_1)^2 = q - q^2 \approx 0,04826$$

Now for the mean value and summary disbursements

$$S = X_i + ... + X_n$$

we have

$$ES = N \cdot EX_1 = 300 \cdot 0,0013 = 2,1820955262$$
$$VarS = N \cdot VarX_1 \approx 300 - 0,0013 \ = \ 2,171514324$$

Using Gaussian approximation for the centered and scaled size of summary disbursements, we'll express the probability of a disbursement by the company as follows:

$$P(S \le u) = P\left(\frac{S-ES}{\sqrt{VarS}} \le \frac{u-ES}{\sqrt{VarS}}\right) \approx \hat{O}\left(\frac{u-2,182095526}{1,473605892}\right)$$

If we need the probability to equal 0,89, the quantity $\frac{u-2,182095526}{1,473605892}$ has to equal 0,872 (which satisfies the value of the quantile of the normal distribution), i.e.

$$\grave{e} \ = \ 0,872 \ \cdot \ 1,473605892 \ + \ 2,182095526 \ = \ 3,4670799$$

– the size of the insurance amount for 100 rubles.

For 100000 rub. the net rate will be

$$T_n = \frac{3,4670799 \cdot 100000}{100} = 3467,08 \text{ ðóá}.$$

The gross rate equals

$$T_b = \frac{3467,0799}{1-0,2} = 4333,85 \text{ ðóá}.$$

We can calculate long-term life insurance rates the similar way.

CONCLUSION

We shouldn't, nevertheless, forget that multiple scientific studies reveal the need to keep track of policyholders' gender in determining life insurance rates.

Furthermore, mortality in men depends on the social, economic, political situation in the country [10].

Mortality in women is a relatively steady indicator, thanks to the attributes of women's psychology, behavior, being less stress-prone compared with men.

The residence factor (whether he/she lives in the country or a town/city) doesn't have a tangible effect on the probability of death – we can ignore it in building insurance rates.

Inferences: Thus, the use of mortality tables in calculating net life insurance rates leads to a decrease in the net cost of insurance for the most active group of policyholders.

We get a fair layout of the risk of incurring material losses associated with a person's life expectancy (between all policyholders); the consistency of insurance operations is boosted; the reliability of insurance companies increases; reducing the cost of insurance for the most probable policyholders enables insurance companies to bring in more customers, which brings the insurance rates down for all policyholders.

Let's consider the practical application of the above methods for different types of insurance:

1. It is customary for an insurance company to enter into a large number of contracts.

Therefore, what's important to it is not an individual claim and effecting a disbursement on that claim but the total size of disbursements, S, across all individual claims.

Let's assume that the company has entered into N insurance contracts and ξ is individual claims on these contracts. Then

$$S = \sum_{k=1}^{n} \xi_k$$

We'll call the amount S the summary risk.

If the summary risk S doesn't exceed the company's capital, then the company will succeed in fulfilling its obligations before its clients.

But if $S > u$, the company won't be able to disburse all individual claims.

In this case, the company could go bankrupt.

Thus, the probability (R) of the company going bankrupt can be expressed via the formula

$$R = P\{S > u\}$$

In other words, the probability (R) of the company going bankrupt equals the value of the additional function of the distribution of the summary risk in the point u, where u is the company's capital.

Accordingly, the probability of the company not going bankrupt equals the value of the function of the distribution of the summary risk in the point u.

The calculation of these probabilities is crucial to the company, for it's on the basis of these calculations that the company makes its important decisions.

We'll assume that individual claims are independent quantities.

The number of policyholders insured with a company is normally very high.

Therefore, there is always a need to calculate the distribution of the amount of a large number of deposits.

It's clear that manual calculation methods using generating functions are not helpful here.

Consequently, it's practically impossible to effect an accurate calculation of the attributes of the summary risk of the sum of a large number of summands.

In a situation of this kind, we can resort to the limit theorems of the probability theory.

The rationale behind these theorems is that given some quite general conditions, the function of the distribution of the sum of independent chance quantities at N ¯ 8 overlaps with the function of the distribution of some specific chance quantity.

This enables us to employ this specific distribution function in solving practical problems, instead of using the function of the distribution of the sum, which is hard to compute.

Note that the error resulting from this substitution is quite small and does satisfy practical requirements with respect to the accuracy of calculations.

For the further calculation of the attributes of the summary risk on short-term policies we use the Poisson theorem:

$$P\{\xi \le x_a\} \ge \alpha$$

In other words, the capital of the company u (provided that the probability of the company not going bankrupt is not less than 95%) must satisfy the condition

$$P\left\{\sum_{k=1}^{N} \xi_k \le u\right\} \ge 0,95$$

And according to the Poisson theorem

$$P\{\xi \le u\} \ge 0,95$$

Numeric calculations are done using the table of quintiles of the level a=0,95 of the Poisson distribution.

Example: There are N_1 = 3000 individuals aged 38 and N_1 = 1000 individuals aged 18 insured with the company.

The company pays the policyholder's heirs a disbursement to the tune of b = 250000 rubles in the event of his/her death within one year and pays nothing if he/she lives for over one year after entering into the contract.

Using the Poisson method, we'll calculate the size of the insurance disbursement which guarantees that the probability of the company going bankrupt won't exceed 5%.

Solution: Let's make the size of the insurance disbursement b our unit.

Using the life expectancy tables, we find

q_{38} ˜ 0,003

q_{18} ˜ 0,001

Therefore, according to the Poisson theorem, the summary risk from policyholders aged 38 (18) can be considered as a chance quantity distributed by the Poisson law with the parameter

$$\lambda_1 = N_1 \cdot q_{38}$$
$$\lambda_2 = N_1 \cdot q_{18}$$

Then, the company's summary risk is the Poisson chance quantity with the parameter

$$\lambda = \lambda_1 + \lambda_2 = 10$$

Consequently, the probability of the company not going bankrupt approximately equals

$$P\{\xi \le u\}$$

where ξ is the Poisson chance quantity with the parameter $\lambda = 10$;

u is the company's capital.

The problem says it has to be

$$P\{\xi \le u\} \ge 0,95$$

From this condition we have

$$u = x_{0,95}$$

where $x_{0,95}$ is the quintile of the level 0,95 of the Poisson distribution with the parameter 10.

Using the quintile table, we find that

$$u = x_{0,95} = 15$$

The net premium for the first policyholder group equals

$$b_1 + q_{38} = 0{,}003$$

and for the second group

$$b_1 + q_{18} = 0{,}001$$

Thus, through net premiums the company will get the amount

$$3000 \cdot 0{,}003 + 1000 \cdot 0{,}001 = 10$$

The remaining amount needed

15 - 10 = 5

will be constituted by the insurance mark-up.

Since the insurance mark-up constitutes 50% of the size of net premiums, the company has to increase net premiums by 50%.

Consequently, for the first policyholder group the size of a premium will be
0,003 • 1,5 = 0,0045 Aand for the second group 0,001 • 1,5 = 0,0015

The long-term insurance model is characterized by that in this case we count in changes in the value of money over time.

With full life insurance, there is no uncertainty in the fact of presenting a claim by the policyholder: the claim will be filed for sure the moment the policyholder passes away.

There is no fortuity to the size of the claim either: the company has to pay the policyholder's heirs the amount b, as is required by the contract.

This is what sets full insurance apart from short-term types of insurance.

It's clear that the size of the insurance premium p has to be much higher than the size of the insurance disbursement b.

In this regard, the question then arises: how does the company manage to come up with the money for the disbursement of claims?

Indeed, with full life insurance, all policyholders' heirs will file claims with the company for sure.

The thing is the company receives the insurance premium p at the moment the contract is being entered into and it pays the disbursement b a lot later.

Within this period of time (which equals the remaining term of life $T(x)$ of the policyholder, where x is his/her age) the insurance premium makes the company profit and grows via the following by the amount

$$pe^{\delta T(x)}$$

where δ is the intensity, %.

Thus, the company's profit from entering one full insurance contract is

$$\varepsilon = pe^{\delta T(x)} - b$$

In order to get the amount b at the moment of the policyholder's death (i.e. in $T(x)$ years after the conclusion of the contract), the insurance company, according to the formula for changes in the value of money over time, has to get from him/her the amount $z := be^{-\delta T(x)}$ at the moment the contract is being entered into.

The amount z is a chance quantity that expresses the up-to-date (i.e. as of the moment the contract is being entered into) size of the future insurance disbursement.

The average size p_0 of this quantity is designated by the insurance company as the net premium.

Numeric calculations are done using the Erlang model.

Example: Let's assume that life expectancy is described using the Erlang model and the average life expectancy is 80 years.

We'll calculate the net premium for policyholders aged 20 and 30, the intensity equaling 10%.

Solution: The average life expectancy in the Erlang model equals 2*A*.

Then

$$\overline{A_x} = \frac{1}{(x+a)\cdot(a\cdot\delta+1)}\cdot\left(x+\frac{a}{a\cdot\delta+1}\right) = \frac{x\cdot a\cdot\delta+x+a}{(x+a)\cdot(a\cdot\delta+1)^2}$$

the value

$A = 40$ $\delta = 0{,}1$

we get

$$\overline{A_{20}} \approx 0{,}0933$$
$$\overline{A_{30}} \approx 0{,}1086$$

We can demonstrate that the function of the distribution $F_z(t)$ of the up-to-date size of the future insurance disbursement z, under full life insurance, is expressed via the following formula (using the Erlang model):

$$F_z(t) = \begin{cases} t^{\frac{1}{a\delta}}\cdot\left(1-\frac{\ln t}{\delta\cdot(x+a)}\right), t\in(0,1) \\ 1, t\geq 1 \end{cases}$$

The exponential smoothing method is used for short-term and long-term forecasts and is predicated on the mean-weighted value of sales over a certain number of periods passed.

The forecast value is calculated via the formula

$$Q = \alpha\cdot Q_t + (1-\alpha)\cdot Q_{t-1}$$

где Q is the forecast policy sales volume within the current period;

α is the smoothing constant;
Q_t is the sales volume over the period t;
Q_{t-1} is the smoothed sales volume for the period t - 1.

The smoothing constant is taken by an analyst interactively within the interval from 0 to 1. Its value is low when there is little change in sales and tends to 1 when there are strong fluctuations on the insurance market.

Example: We need to calculate the forecast value of possible short-term policy sales in May 2012, if in April 2012 the company entered into 121 insurance contracts and in March forecast sales constituted 119 contracts.

Solution: The forecast value is calculated via the formula

$$Q = \alpha\cdot Q_t + (1-\alpha)\cdot Q_{t-1}$$

where Q is the forecast volume of policy sales for May 2012;

α is the smoothing constant;
Q_t is the sales volume for April 2012;
Q_{t-1} is the forecast sales volume for March 2012.

As we can see, the value of the smoothing constant is low because there is little change in sales over the given period. Then

$$Q = 0{,}1\cdot 121 + (1-0{,}1)\cdot 119 = 119{,}2 \approx 119$$

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