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Accuracy Evaluation Between GPS Virtual Reference Station (VRS) and GPS Real Time Kinematic (RTK) Techniques

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Abstract: There are some limitations when using the GPS RTK technique such as: the limited distance for the radio transmitter; the need for base receiver; and the dependence on only one-base point to calculate the corrections. On the other hand, Virtual Reference Station VRS is an imaginary, unoccupied reference station which is only a few meters from the RTK user. The principle is to interpolate the data of several Continuous Operating Reference Station CORS in order to obtain the correction data for the rovers, which reduces the systematic influences of the RTK measurement decisively; and allowing distance between the reference station and the rover to be increased; in addition to the reliability of the system is heightened. This paper aims to comparing the X, Y and Z coordinates resulted from VRS network with the resulted coordinates for the same points resulted from RTK technique. The results supported with statistical analysis showed that horizontal positional discrepancy P_{2d} between the two techniques has a mean value of 52 mm with 18 mm standard deviation, while the spatial positional discrepancy P_{3d} has a mean value of 67 mm with standard deviation 23 mm. These findings are considered to be insignificant in the daily work of cadastral and topographic survey work, but should be taken into account for the precise surveying such as monitoring of structure deformation when combining the two techniques in the measurements.

Key words: CORS · GPS · RTK · VRS

INTRODUCTION

Typically, GPS observables are pseudoranges derived from code or phase measurements. The accuracy of code ranges is at the meter level, whereas the accuracy of the carrier phase is in the millimetre range [1]. The accuracy of the code ranges can be improved, however, by smoothing techniques. Also, the code ranges unambiguous, unlike the carrier phase. This makes code ranges immune from cycle slip. The GPS observation techniques include: Single Point Positioning SPP, Differential positioning with code measurements corrections DGPS and Relative positioning which include: static, rapid static, stop and go, kinematic and real time kinematic RTK. GPS SPP employs one GPS receiver, while DGPS and relative GPS positioning employ two or more GPS receivers, simultaneously tracking the same satellites [2]. Surveying works with GPS have conventionally been carried out in the relative and differential positioning techniques. This is mainly due to the higher positioning accuracy obtained from the relative and differential techniques, compared to that of the GPS point positioning. A major disadvantage of GPS relative and differential techniques, however, is the dependency on the measurements or corrections from the reference receiver [3]. Real Time Kinematic GPS RTK is a well-established technique to improve the positioning precision since many years. This allows even centimetre-level accurate positioning using the so-called integer ambiguity resolution technique. The basic concept is to mitigate the main error sources, ionospheric and tropospheric delay, orbit errors and satellite clock errors by receiving satellite data at a well-known location. All common errors between this reference receiver and the user receiver cancel out. Though this already works quite well for many applications, the special de-correlation (i.e. the change of the errors when moving away from the reference station) of the errors leaves large error contributions in the corrected signals [4].

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It is also well known that there are some limitations in GPS positioning survey by the traditional way of RTK, both in the cost and the precision. For example, an additional set of GPS receiver is needed for rover's own reference station, but also it is necessary to spend time and energy for setting it. Another one major drawback of the RTK is that the limit of the Radio connection which is about 5 km for most of the GPS receivers in the market now. In other words, RTK implies that the distance between the base and rover receivers should not be exceeded 5 km to maintain radio connection [5]. Moreover, the precisions of survey results are under many restrictions such as the distance of baseline between the reference station and the rover, simply the longer a baseline is, the more likely the positioning precision is to fall down due to the effects of the ionosphere and the troposphere. However, a new method of RTK-GPS positioning called Virtual Reference Station VRS, as a prevailing technique for higher precise positioning in real time with higher efficiencies and lower costs, can resolve such problems as mentioned above. The idea is to generate VRS that simulate a local reference station near by the user receiver. Thus, the errors cancel out better than by using a more distant reference station. Because of a wide network covered by multi-reference stations is used to create a VRS near rover, it is possible to do precise positioning surveying everywhere in a wide area just by receiving the correction data from a Location Business Service LBS, not need to set rover's own reference station one by one in a wide area [6].

This paper investigates the accuracy of the discrepancies in Cartesian coordinates X, Y and Z and the horizontal and spatial position P, in case of using RTK and VRS techniques up to GPS Baselines of 5-km. In this regards, GSP RTK technique will be reviewed. The concept of VRS will be presented, along with the field procedure and the accuracy of this new technique. The methodology of investigation and the description of the field test will be presented. Finally, the analysis of the obtained data supported with the statistical analysis will be shown, from which the important conclusions and recommendations will be extracted.

GPS Real Time Kinematic RTK: Real Time Kinematic RTK technique is used to determine the coordinates in real time. In this method, the base receiver remains stationary over the known point and is attached to a radio transmitter [7]. The rover receiver is normally carried out and attached to a radio receiver (Fig. 1). This method is similar to DGPS, except that in case of DGPS corrections are transmitted, however, in case of RTK the known coordinates of the base along with the receiver measurements are transmitted to the rover receiver through the communication link, using a data rate of 1Hz, which means one sample per second. The built-in software in the rover receiver combines and processes the GPS measurements collected at both the base and the rover receivers; to obtain the rover coordinates [8]. The initial ambiguity parameters are determined instantaneously using a technique called On-The-Fly OTF ambiguity resolution. Once the ambiguity parameters are fixed to integer values, the receiver will display the rover coordinates right in the field. That is, no post processing is required. The expected positioning accuracy is of the order of 10 mm + 1-2 ppm.

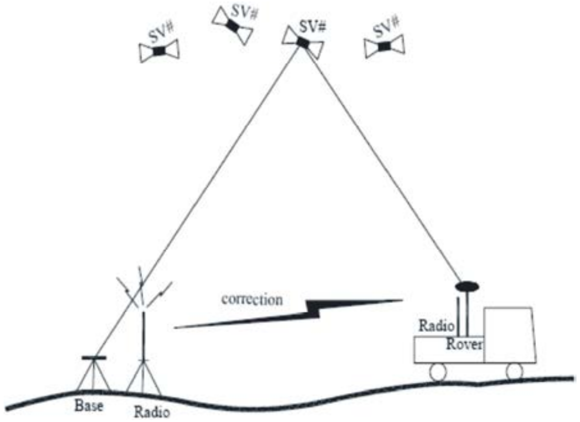

Fig. 1: GPS Real Time Kinematic RTK

GPS RTK is mainly depending on the instantaneously resolution of the ambiguity. On The Fly OTF ambiguity resolution is an advanced technique developed recently, to determine the initial ambiguity while the rover receiver in motion, which is used mainly in the GPS kinematic on the fly technique. The ambiguity resolution OTF is based mainly on the double difference code observations. Both of the satellite and receiver clock biases will be removed. In addition, the orbital errors, as well as the atmospheric biases, will be greatly reduced. However, the double difference code observations are still affected by both of the multipaths and the relatively high noise of the code observations. To overcome these two degrading factors, which may prevent the fix of the integer ambiguity OTF, a smoothing algorithm using the phase observations can be used. The main concept of this smoothing technique is based on using the phase measurements to smooth the relatively imprecise code observations. The mathematical expression of such smoothing algorithm [5]:

$$P(t_i)_{sm} = \omega . P(t_i) + (1 - \omega)(P(t_{i-1}) + \Phi(t_i) - \Phi(t_{i-1})) \quad (1)$$

which $P(t_i)_{sm}$ is the smoothed code observation at epoch t_i, $P(t_{i-1})_{sm}$ is the smoothed code observation at epoch t_i, $P(t_i)$ is the observed code at epoch t_i, $\phi(t_{i-1})$ and $\phi(t_i)$ is the phase observation in range units at epoch t_{i-1} and t_i. ω is a time dependant weight factor. For the first epoch i=1, the weight is set ω =1, thus, the full weight is given for the measured code. For consecutive epochs, the weight of the code phase is continuously reduced and thus, emphasizes the influence of the carrier phase data. Practically, a reduction of the weight by 1% is found to be superior in case of kinematic survey with epoch interval 1-second. In such a case, after 100-seconds, only the smoothed value from the previous epoch is taken into account.

The base and rover measurements are combined in the double difference mode and by the help of the smoothed codes, one can get the initial adjustment of the rover position along with real values for the unknown ambiguities with their covariance matrix. The covariance matrix can be represented geometrically to form a confidence region around the estimated real values of the ambiguities. The size of the confidence region depends on the size of the unknown ambiguities as well as the probability level. The confidence region takes the shape of an ellipse if the number of the estimated is two; an ellipsoid if the number is three; and a hyper-ellipsoid if the number is more than four, which is the case when a more than four satellites are tracked. The hyper-ellipsoid is divided into one-cycle grid lines, where each intersection is being a solution for the unknowns. Based on statistical evaluation, only one point is selected as the most likely candidate for the integer ambiguities. Once the ambiguities are correctly resolved, a final adjustment solution is performed to obtain the rover coordinates at cm accuracy level [9].

In the RTK technique, correction messages are sent to rovers in a certain format which every receiver manufacturer produces. In order to prevent the confusion that different data formats can cause, the Radio Technical Commission for Maritime Services RTCM, Special Committee 104 (RTCM SC-104) has published a standard format for broadcasting correction messages between reference and rover stations and this has been named as RTCM SC-104 [10]. Naturally, the more reference and rover stations track satellites, the faster the ambiguity resolution gets and the more the positioning accuracies increase. Augmentation of GLONASS satellite signals with GPS signals can be used in this context. Advantages of RTK GPS technique over other GPS techniques can be expressed as follows:

- It does not require post-processing of RTK data.
- Coordinates of points measured in the field can be transformed to local coordinate systems in real-time, provided that a few points (at least three) whose coordinates are also known in local coordinate system are available.
- It provides a reliable tool for positioning all points accurately. In conventional kinematic surveys in the case of (undetected) cycle slips or loss of lock in the reference station, kinematic positioning cannot be performed, whereas in RTK this is easily detected, the survey continues with a new integer ambiguity introduced and resolved in real-time.
- Known points can accurately be staked out (on a cm level) in field in real-time.

Today RTK GPS is widely being used all over the world. Due to the increased productivity in terms of time and economics RTK GPS provides, in the near future this approach can and will surely replace the DGPS method and classical terrestrial positioning techniques. On the other hand, the disadvantages of the RTK technique can be summarized as follows:

- Limited radio distance from the reference to the rover receiver, where practically this distance is about 5-km for the radio transmitter.
- The radio signal may be obstructed by some topographical constrains like mountains, or by man-made obstacles like high-voltage towers, which lead to failure in receiving the correction message in the rover receivers.
- The full dependency on the reference receiver, where any failure in this reference receiver e.g. power supply failure, will lead to a failure in the RTK technique.
- Corrections are computed based on the coordinates of the reference receiver only, which is not accurately figuring the GPS errors in the territory observations.
- The accuracy of the RTK is degraded while the distance between reference and rover receivers is increasing.

GPS Virtual Reference Station VRS: A Virtual Reference Station VRS is an imaginary, unoccupied reference station which is only a few meters from the RTK user. For this position, observation data are created from the data of surrounding Continuously Operating Reference Stations CORS as though they had been observed on that position by a GPS receiver. With classical RTK using GPS, the correction data of one reference station are transmitted to the user. However, the distance between the reference station and the mobile user rover should not exceed 5 km; otherwise the measurements will be notably affected by the systematic errors. If one wished to develop a network of GPS permanent stations over an entire country, one would need a very dense and in effect also a very expensive network [11].

Concept of VRS: The concept of VRS is to interpolate the data of several reference stations in order to obtain the correction data for the rovers, which reduces the systematic influences of the RTK measurement decisively. Not only may the allowed distance between the reference station and the rover be increased, but also the reliability of the system is heightened. Should a reference station fail temporarily for example, the correction data are computed with the surrounding reference stations. In addition, the productivity is improved by clearly shorter initialization times [6]. Several methods have been suggested for the transfer of network information to the user (Fig. 2).

Network Observations on Common Ambiguity Level: Broadcast of the observations of a master Reference Stations RS and observation differences between pairs of RS, all being on the same ambiguity level. The user performs the interpolation step on his own providing him with network corrections and valuable information on their quality. He computes VRS observations and positions in baseline mode. The necessary data formats have not been standardized yet [11].

Area Correction Parameter Model FKP: Usually the interpolation algorithms are separately applied to the dispersive (ionospheric) and to the non-dispersive (geometric, i.e. tropospheric and orbit) biases. The correction model parameters are known as Area Correction Parameters (in German Flächen Korrektur Parameter, abbreviated FKP; and broadcast the observations of a master RS and FKP. The user applies the FKP to the RS observation data set according to his position and thus obtains VRS-observations. The necessary data formats have not been standardized yet, although a manufacturer agreement exists on RTCM Type 59 proprietary message containing FKP information.

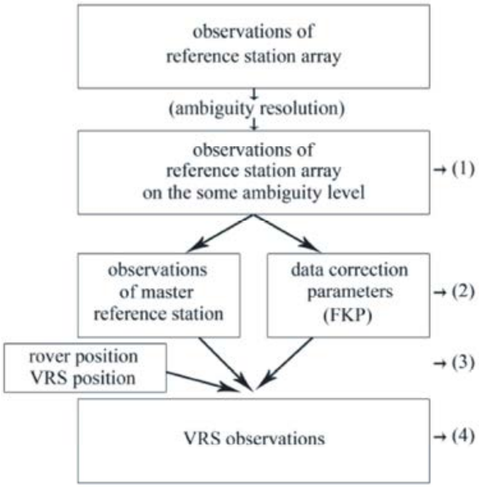

Fig. 2: Computation of VRS Observations

Gridded Corrections: Broadcast of the observations of a master RS and gridded corrections of the distance-dependent biases. The user interpolates individual corrections within the grid and applies them to the observation data set in order to obtain VRS-observations. The necessary data formats have not been standardized yet.

VRS: The user sends his approximate position to a central computing facility and by return receives VRS-observations to be used for baseline positioning. Whereas this approach uses existing data formats RTCM, RINEX, in general no information is provided on the quality of the interpolation process. A two-way communication link is required. To make the transmitted data look like it came from a different position, it has to be displaced geometrically. The pseudorange between the satellite and the virtual reference station can be approximated by:

$$\tilde{\rho}_v^s = \rho_r^s + (\tilde{R}_v^s - R_r^s) \tag{2}$$

where:

$\tilde{\rho}_v^s$: The approximate pseudorange between satellite and the virtual reference station.

ρ_r^s : The pseudorange between satellite and the original reference station.

$\tilde{R}_v^s$: The approximate geometric range between satellite and the virtual reference station.

R_r^s : The exact geometric range between satellite and the original reference station.

Applying one of the first three approaches, the user is able to extract quality information of the virtual observations either from the interpolation step or directly from the FKP. He is thus able to estimate the size of remaining observations biases which need to be considered in baseline processing. Unfortunately no standardized data formats exist for any of these methods. Existing data formats are used in the case of transferring VRS observations to the user: RTCM for real-time applications, RINEX for post-processing. VRS observations, however, do not comply with the format standards because both standards state that observation data should not be corrected for any errors. Furthermore, these standards do not provide any formats for the transmission of VRS related quality information. The baseline processing software is not even able to notice that it processes virtual reference data. As a result, the software will not come to optimal decisions in its baseline processing. Nevertheless, as long as no other standard formats exist, transferring VRS-observations will stay the preferred method of providing pre-processed network information to the user. This will hold specifically true for post-processing applications since no extensions to the RINEX-format are under discussion [6].

Field Procedure of VRS: As a quick review, a typical DGNSS setup consists of a single reference station from which the raw data (or corrections) are sent to the rover receiver (i.e., the user). The user then forms the carrier phase differences (or corrects their raw data) and performs the data processing using the differential corrections. In contrast, GNSS network architectures often make use of multiple reference stations. This approach allows a more precise modeling of distance-dependent systematic errors principally caused by ionospheric and tropospheric refractions and satellite orbit errors. More specifically, a GNSS network decreases the dependence of the error budget on the distance of nearest antenna. The general concept of network-based processing is shown in Fig. 3 [12]. The network of receivers is linked to a computation center and each station contributes its raw data to help create network-wide models of the distance-dependent errors. The computation of errors based on the full network's carrier phase measurements involves, first of all, the resolution of carrier phase ambiguities and requires knowledge of the reference station positions.

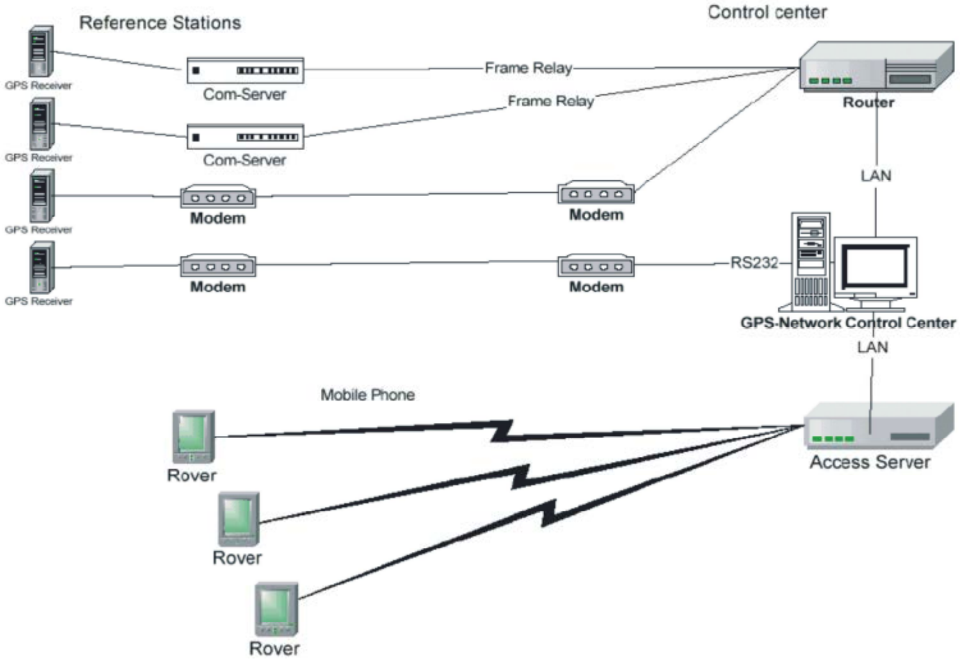

Fig. 3: Principle of VRS (after Trimble 2010)

At the same time the rover calculates its approximate position and transmits this information to the computation server, for example, via GSM or GPRS using a standard National Marine Electronics Association NMEA format. The computation center generates in real time a virtual reference station at or near the initial rover position. This is done by geometrically translating the pseudorange and carrier phase data from the closest reference station to the virtual location and then adding the interpolated errors from the network error models. This generated VRS data is then sent to the user through a wireless connection, often using Radio Technical Commission for Maritime services RTCM via Internet Protocol. Finally, just as if the VRS data had come from a physical reference station, the rover receiver uses standard single-baseline algorithms to determine the coordinates of the user's receiver, in near-real-time kinematic or post-processed modes [10].

Accuracy of VRS: The main purpose of a VRS station is to reduce the baseline distance between the rover and the reference station in order to efficiently remove spatially correlated errors especially ionosphere, troposphere and orbital error, using differential processing and to incorporate error corrections obtained from the reference stations network. The quality of VRS observations mainly depends on two aspects: firstly on the amount of station-dependent biases in the original Reference Stations observations mostly caused by carrier phase multipath; and secondly on possible ionospheric and tropospheric disturbances. Small-scale and medium-scale spatial features in ionospheric and tropospheric refraction may not be completely represented by the correction models of distance-dependent biases. Remaining errors affect ambiguity resolution and positioning accuracy of the baseline between VRS and rover station. The size of these biases and thus the quality of the VRS observations is estimated together with the correction model parameters. The VRS observation quality has to be taken into account in the baseline processing [6]. To this end, the position of the VRS plays a critical role. In particular, because the user receiver cannot, by design, distinguish a real reference station and a VRS, the distance of the VRS from the user must be commensurate with the level of errors present in the VRS data. This is what allows the receiver to use its standard data processing algorithms, which vary as a function of the baseline length (i.e., distance) to the reference station. VRS concept basically needs the resource of a physical GNSS network surrounding the measurement area of the rover, with a minimum of three reference stations to enable the modeling of errors. However, the estimation accuracy increases as more physical reference stations are added to the network, especially as the number of stations exceeds five, at which point the increased redundancy and improved network geometry provide more accurate error modeling. To conduct a survey employing a VRS network, the physical stations themselves must be installed over stable sites, preferably distributed homogeneously over the operational area. If possible, the antennas must be fixed in bedrock to ensure long term stability of the receiver's position [13]. Finally, one can say that the accuracy of the VRS is better than the accuracy of the RTK; VRS method extends the use of RTK to a whole area of a reference station network. Operational reliability and the accuracies to be achieved depend on the density and capabilities of the reference station network.

MATERIALS AND METHODS

The objective of this paper is based on comparing the X, Y and Z coordinates resulted from VRS network with the resulted coordinates for the same points resulted from RTK technique. The methodology of our investigation herein will be based on the statistical analysis of the behavior of the discrepancies in the 3-D cartesian coordinates of 10 baselines with approximate distances from 0.5 km to 5 km. The GPS field campaign was executed using Jeddah Municipality VRS network. This VRS network consists of 9 CORS stations as illustrated in Fig. 4. This GPS field campaign was conducted on 2 June 2013, in Jeddah city. 10 points were chosen in Jeddah vicinity and had been occupied by GPS dual frequency receiver of Topcon G3. These 10 points were observed two times; the first one was using Jeddah VRS network; and the second time was using RTK based on Base Control point occupied by the GPS receiver. The distances between the base and the rover receivers for the RTK for the 10-points were differing from 0.5km to 5km. The observations parameters were the same in both observations of VRS and RTK as follows: Elevation angle was 10 degree; epoch interval 1-sec; the observation time for every point was 15-seconds; and the solution type was adopted to use dual L1 and L2.

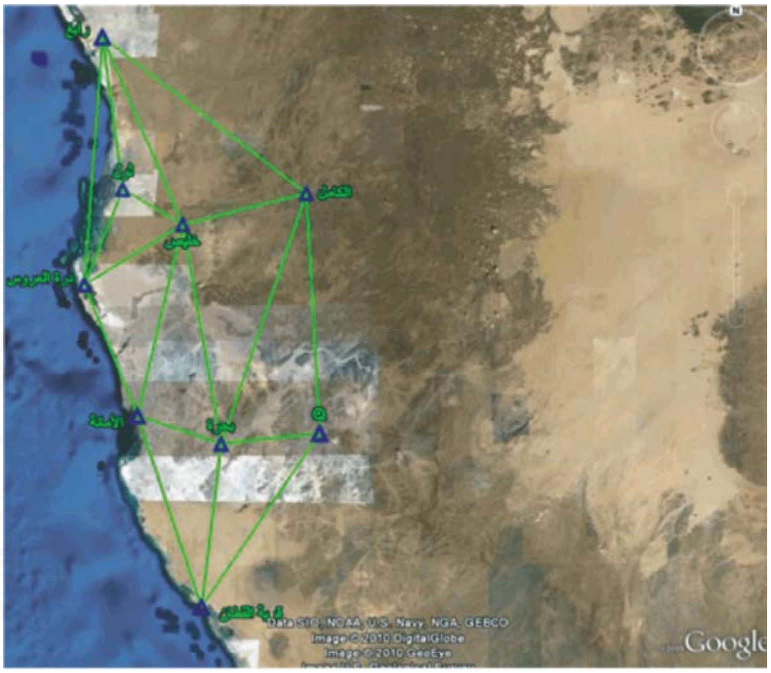

Fig. 4: Jeddah Municipality VRS Network

RESULTS AND DISCUSSION

The discrepancies in X, Y and Z coordinates between the VRS and the RTK solutions are:

$$\Delta X = X_{VRS} - X_{RTK} \quad \Delta Y = Y_{VRS} - Y_{RTK} \quad \Delta Z = Z_{VRS} - Z_{RTK} \tag{3}$$

where: X_{vRS} is the X coordinate from processing the data using VRS network. X_{RTK} is the X coordinates from processing the data using RTK technique. The same abbreviations are valid for Y and Z coordinates.

Also, the 2-d and 3-d positional discrepancy and the standard deviation can be calculated from [14]:

$$\Delta P_{2d} = \sqrt{(\Delta X)^2 + (\Delta Y)^2} \quad \Delta P_{3d} = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2} \tag{4}$$

$$\sigma^2_{\Delta p_{2d}} = \sigma^2_{\Delta X} + \sigma^2_{\Delta Y} \quad \sigma^2_{\Delta p_{3d}} = \sigma^2_{\Delta X} + \sigma^2_{\Delta Y} + \sigma^2_{\Delta Z} \tag{5}$$

The discrepancies in X, Y, Z and the 2-d and 3-d position P, as well as the approximate RTK baselines lengths of the 10 points are shown in Table 1.

Fig. 5 shows the X, Y and Z coordinate discrepancies for the 10 points derived from VRS and RTK techniques. In addition, Fig. 6 shows the 2-d and 3-d positional discrepancies P for the same points.

The previous figures are supported by descriptive statistics to measure the quality of the obtained results. Table 2 shows these descriptive statistics. For instance, the X-coordinate discrepancies have mean value 7mm and SD 40mm for single determination. The Y-coordinate discrepancies have mean value of 7mm and 41mm SD for single determination. The Z-coordinate discrepancies have 14mm mean value and 44mm SD for single determination. Finally, the 2-d and 3-d positional discrepancies have 52mm, 67mm mean value respectively; with SD 57mm and 72 mm, respectively.

In order to check any systematic errors in the sample of the resulted discrepancies, the sample mean should be examined according to its deviation from the mean of the population which equal to zero [14]. This random sample is assumed to be normally distributed and had taken from a population of unknown true mean ì equal to zero. The following confidence interval can be used:

$$\mu - T.\ Sd_{mean} < m < \mu + T.\ SD_{mean} \tag{6}$$

where μ is the population mean (μ =0), SD_{mean} is the sample standard deviation, T is the Student t-distribution corresponding to sample size n and degree of freedom df equal to n-1, assuming that the confidence level is 95%. Table 3 summarizes the results of this test which indicates that no significant systematic errors are existing in the discrepancies between the VRS and RTK solutions.

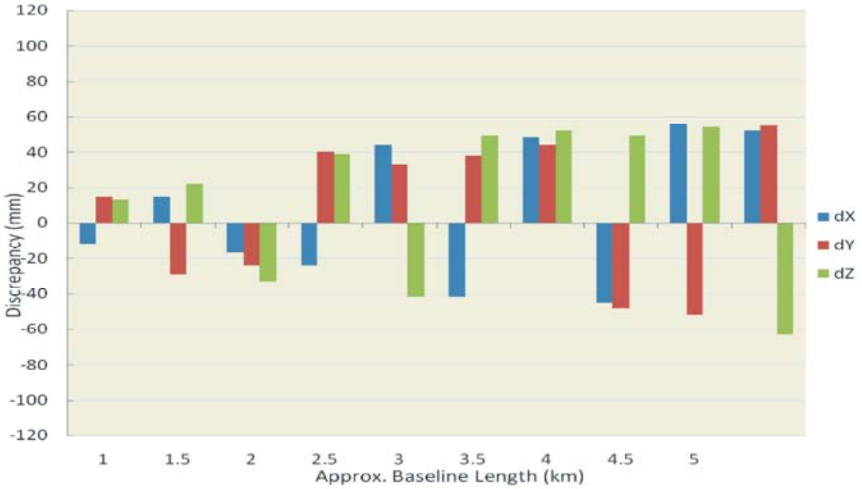

Fig. 5: Variation of the X, Y and Z coordinate discrepancies

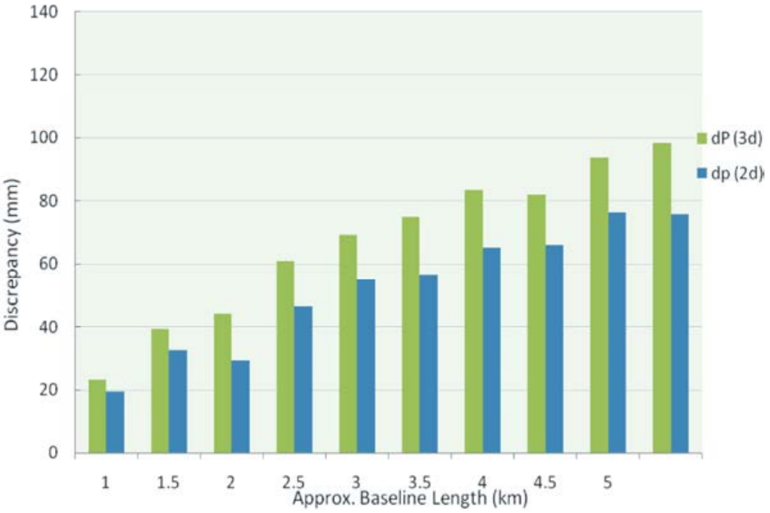

Fig. 6: Variation of the Positional discrepancies

Table 1: The discrepancies in X, Y, Z and position P

Pt	Approx. RTK baseline (km)	ΔX (mm)	ΔY (mm)	ΔZ (mm)	ΔP_{2d} (mm)	ΔP_{3d} (mm)
1	0.5	-12	15	13	19	23
2	1.0	15	-29	22	33	39
3	1.5	-17	-24	-33	29	44
4	2.0	-24	40	39	47	61
5	2.5	44	33	-42	55	69
6	3.0	-42	38	49	57	75
7	3.5	48	44	52	65	83
8	4.0	-45	-48	49	66	82
9	4.5	56	-52	54	76	94
10	5.0	52	55	-63	76	98

Table 2: Descriptive statistics of the discrepancies (mm)

Disc.	Max.	Min.	Range	Mean	$S.D_{single}$	$S.D_{mean}$
ΔX	56	-45	101	7	40	13
ΔY	55	-52	107	7	41	13
ΔZ	54	-63	117	14	44	14
ΔP_{2d}	76	19	57	52	57	18
ΔP_{3d}	98	23	75	67	72	23

Table 3: Testing of deviation of the sample mean

Disc.	df	T	SD_{mean}	Lower limit	Mean	Upper limit	Test result
ΔX	9	2.26	13 mm	-29.4 mm	7 mm	29.4 mm	pass
ΔY			13 mm	-29.4 mm	7 mm	29.4 mm	pass
ΔZ			14 mm	-31.6 mm	14 mm	31.6 mm	pass

CONCLUSIONS

The present study focuses on making comparative study for the resulted Cartesian coordinates X, Y and Z between the VRS and RTK techniques. In this regard, a field test was executed to observe 10 points using the VRS network of Jeddah city municipality and by observing the same points using the RTK technique. The baselines for the RTK campaign for the 10 points were ranged from 0.5 to 5 km. The results supported with statistical analysis showed that the difference between VRS and RTK techniques gives discrepancies of mean values 7 mm, 7 mm and 14 mm in X, Y and Z coordinates, respectively; with standard deviation of 13 mm, 13 mm and 14 mm for the 3-caretesian components respectively. The horizontal positional discrepancy P_{2d} between the two techniques has a mean value of 52 mm with 18 mm standard deviation, while the spatial positional discrepancy P_{3d} has a mean value of 67 mm with standard deviation 23 mm. The above findings are considered to be insignificant in the daily work of cadastral and topographic survey work, but should be taken into account for the precise surveying such as monitoring of structure deformation when using the two techniques together. In addition, using the VRS technique instead of RTK has many advantages such as independency of base receiver, consistent error modeling, more productivity and no limitation for radio communication.

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